

Relations between skylight scattering angle and degree of polarization under different sky conditions

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Received November 11, 2009

The relations between scattering angle (SA) and the degree of polarization (DOP) of skylights are studied. Measurements under different sky conditions demonstrate that all relation curves between SA and DOP can be described as parabolas. DOP reaches its peak when SA is 90° and the sizes of scattering particles are much smaller than the wavelengths of skylight. The peak value of DOP moves by a small drift when the size of the particle increases. We propose and analyze a polarization dependence model for SA and DOP. Results from simulation are in good agreement with experimental results.

OCIS codes: 010.1310, 290.5855, 290.1310, 290.4020.

doi: 10.3788/COL20100806.0546.

Light emitted by the sun is not polarized. When it enters the atmosphere, light is scattered by atmospheric constituents (gases like N₂ and O₂, etc.) and solid and liquid particles (aerosols, water, and ice crystals)^[1,2]. Scattered light is polarized and a polarization pattern of the skylight is thus formed. Observations and measurements of polarization patterns are important in the research on polarization navigation^[3,4], atmospheric scattering radiation^[5,6], distributions and kinds of atmospheric particle, etc.^[6-8]

Polarization patterns mainly depend on the position and distribution of various components in the atmosphere^[9]. Polarization patterns of the clear sky have been studied thoroughly, but research on overcast skies remains inadequate. Brines *et al.* showed that polarization patterns were stable and predictable for clear skies but very hazy under overcast conditions^[10]. Hegedüs *et al.* found that celestial distributions of polarization had very robust patterns and always obtained the same values under all possible sky conditions^[11]. Liu *et al.* presented patterns of polarization that were dependent on wavelength, surface albedo, etc., which were in agreement with the plane-parallel Rayleigh atmospheric model^[12]. Generally, polarization of skylight is generated by scattering, and thus, polarization patterns are dependent on scattering angles (SAs). In this letter, we study the relations between SA and polarization patterns theoretically and experimentally. A new polarization dependence model for SA and degree of polarization (DOP) is then proposed and analyzed.

We used a four-channel, high-precision polarizing radiometer to measure relative intensity of polarization, DOP, and orientation of polarization plane of the sky. Spectral response range was from 400 to 540 nm. The measured point of the sky was 30° to the zenith. Real-time solar elevation was obtained from astronomical data. Real-time SA was obtained from the latitude and longitude of the measurement system and the solar elevation^[13]. DOP and SA values under all sky conditions were measured from 10:00 AM to 1:00 PM for over

four months.

Figure 1 shows a typical curve between DOP and SA under clear sky conditions. When SA is 90°, DOP reaches its peak value at 0.491. The relation curve between DOP and SA is fitted to a clear parabola.

A typical curve in the sky covered by some thin clouds is shown in Fig. 2. The curve is also fitted to a clear parabola, although DOP values fluctuate slightly. The parabola is different compared with that of the clear sky. The peak value of DOP is 0.357 and it is smaller than the

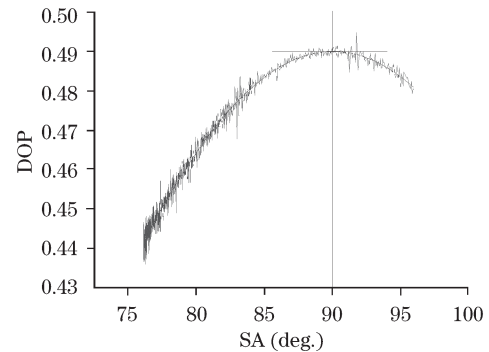


Fig. 1. Relation curve between DOP and SA under clear sky conditions.

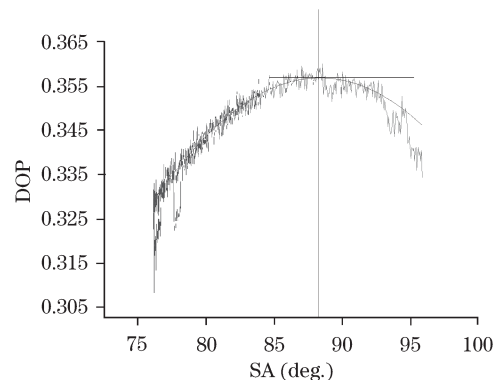


Fig. 2. Relation curve between DOP and SA in sky covered by some thin clouds.

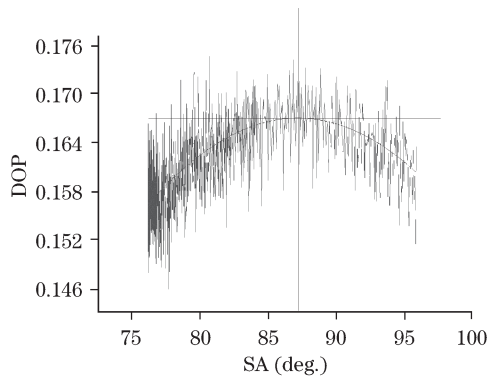


Fig. 3. Relation curve between DOP and SA in sky covered by heavy clouds.

value under clear sky conditions. The peak has drifted to the left; it reaches SA at about 88°.

Figure 3 shows a typical curve between DOP and SA in sky covered by heavy clouds. Although the fluctuation of DOP values is more severe, the curve is also fitted to a parabola. DOP values are much smaller than the values under clear sky conditions. Its peak value is 0.167, which is only one-third of the value of the clear sky. The peak of DOP drifts more to the left relative to the position when SA is 87°.

Polarization patterns are not stable and the values of patterns change under different sky conditions (Figs. 1–3). Results are not in agreement with previous reports^[10–12]. A polarization dependence model between SA and DOP is then proposed to analyze the observations.

Generally speaking, DOP is defined as

$$P = \frac{|s_1(\theta)|^2 - |s_2(\theta)|^2}{|s_1(\theta)|^2 + |s_2(\theta)|^2}, \quad (1)$$

where θ is SA, $s_1(\theta)$ and $s_2(\theta)$ are complex amplitude functions. According to the Mie theory, $s_1(\theta)$ and $s_2(\theta)$ can be defined as

$$s_1(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (\pi_n a_n + \tau_n b_n), \quad (2)$$

$$s_2(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (\tau_n a_n + \pi_n b_n), \quad (3)$$

where a_n and b_n are Mie scattering functions described as

$$a_n = \frac{\psi_n(x)\psi'_n(mx) - m\psi'_n(x)\psi_n(mx)}{\zeta(x)\psi'_n(mx) - m\zeta'_n(x)\zeta(mx)}, \quad (4)$$

$$b_n = \frac{m\psi_n(x)\psi'_n(mx) - \psi'_n(x)\psi_n(mx)}{m\zeta(x)\zeta'_n(mx) - \zeta'_n(x)\psi_n(mx)}, \quad (5)$$

where $\psi_n(x)$ and $\zeta_n(x)$ are Riccati-Bessel functions. These are defined as

$$\psi_n(x) = \sqrt{\pi x/2} \cdot J_{n+\frac{1}{2}}(x), \quad (6)$$

$$\zeta_n(x) = \sqrt{\pi x/2} \cdot H_{n+\frac{1}{2}}(x), \quad (7)$$

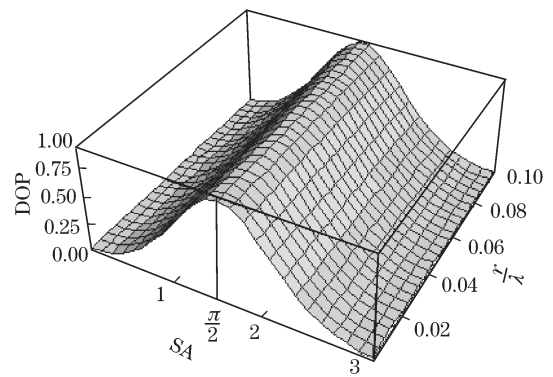


Fig. 4. Simulation results based on the polarization dependence model between SA and DOP when $0 < \frac{r}{\lambda} < 0.1$.

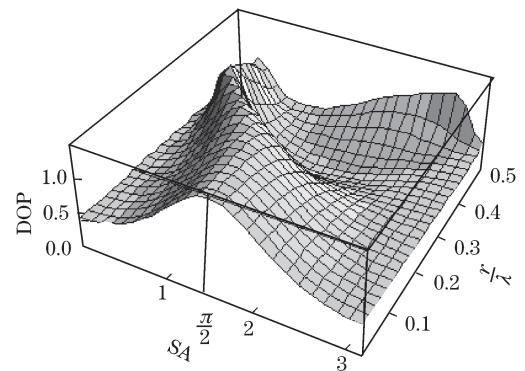


Fig. 5. Simulation results based on the polarization dependence model between SA and DOP when $0 < \frac{r}{\lambda} < 0.5$.

where $J_{n+\frac{1}{2}}(x)$ is the half-order Bessel function of the first kind and $H_{n+\frac{1}{2}}(x)$ is the half-order Hankel function of the second one. Additionally, π_n and τ_n are the derivatives of associated Legendre function, defined as

$$\pi_n = \frac{dP_n(\cos \theta)}{d(\cos \theta)}, \quad (8)$$

$$\tau_n = \frac{dP_n^{(1)}(\cos \theta)}{d\theta}, \quad (9)$$

$$x = \frac{2\pi r}{\lambda}, \quad (10)$$

where r is the radius of the scattering particle and λ is the wavelength of the incident light^[2,14,15].

Figure 4 shows the relations among DOP, SA, and $\frac{r}{\lambda}$ when the size of a scattering particle is much smaller than the wavelength of the skylight ($0 < \frac{r}{\lambda} < 0.1$). All curves are fitted to the parabolas. The values of DOP reach their peaks when the scattering angle is 90°. When the range of $\frac{r}{\lambda}$ increases from 0 to 0.5, the curves change, as shown in Fig. 5. When the size of the particles increases, the peak values of DOP decrease and the position of the peaks drifts to the left. SA with smaller values could make the DOP values reach their maxima.

A polarization dependence model between SA and DOP was then used to explain the experiments. Under

clear sky conditions, sunlight is mainly scattered by atmospheric molecules and particles. Their sizes are much smaller than the wavelengths of skylight. DOP values reached their maxima when SA was 90° . The curves were described as parabolas (Figs. 1 and 4). When the sky was covered by thin clouds, the number of particles (e.g., aerosols and cloud droplets) increased, and their sizes were similar to the wavelength of the skylight. The maximum values of DOP decreased and drifted to the position of the smaller SA (Figs. 2 and 5). When the sky was covered with heavy clouds, the number of particles was larger and their sizes increased. Additionally, the maximum values of DOP decreased and drifted to the left (Figs. 3 and 5).

In conclusion, We have studied and measured experimentally DOP and SA relations under different sky conditions. Regardless of the conditions, the curves have always been parabolic. When the sky is clear, sizes of scattered particles are much smaller than the wavelengths of the skylight. DOP has reached its maximum when SA is 90° . In cloudy days when the scattering particles are large, the maximum DOP decreases and the SA value (i.e., where DOP reaches its peak) shifts to the left and to less than 90° . With heavier clouds, the maximum DOP decreases and the drift of SA value increases. We have proposed and analyzed the theoretical polarization dependence model for SA and DOP. Simulation results based on the model match experimental results.

This work was supported by the Key Basic Research Program (No. 07JC14056) and the International Cooperation Program Project (No. 08520704200) of the

Shanghai Municipal Science and Technology Commission, and the National Natural Science Foundation of China (Nos. 60878029 and 60938003).

References

1. K. N. Liou, *An Introduction to Atmospheric Radiation* (Academic Press, New York, 1980).
2. H. C. van de Hulst, *Light Scattering by Small Particles* (Dover Publications, New York, 1981).
3. G. P. Können, *Polarized Light in Nature* (Cambridge University Press, Cambridge, 1985).
4. G. Horváth and D. Varjú, *Polarized Light in Animal Vision: Polarization Patterns in Nature* (Springer, Berlin, 2003).
5. D. T. Phillips and P. J. Wyatt, *Appl. Opt.* **11**, 2082 (1972).
6. L. Ma, Y. Gu, P. Shi, and X. Li, *Acta Opt. Sin.* (in Chinese) **28**, (S2) 68 (2008).
7. Z. Sekera, *J. Opt. Soc. Am. A* **47**, 484 (1957).
8. D. L. Hofeldt, *Appl. Opt.* **32**, 7551 (1993).
9. K. L. Coulson, *Polarization and Intensity of Light in the Atmosphere* (Deepak, Hampton, 1988).
10. M. L. Brines and J. L. Gould, *J. Exp. Biol.* **96**, 69 (1982).
11. R. Hegedüs, S. Åkesson, and G. Horváth, *J. Opt. Soc. Am. A* **24**, 2347 (2007).
12. Y. Liu and K. Voss, *Appl. Opt.* **36**, 8753 (1997).
13. J. A. van Allen, *Am. J. Phys.* **72**, 1418 (2004).
14. J. R. Probert-Jones, *J. Opt. Soc. Am. A* **1**, 822 (1984).
15. F. Zheng, D. Hua, and A. Zhou, *Acta Opt. Sin.* (in Chinese) **36**, 5 (2009).