Analysis of average capacity for free-space optical links with pointing errors over gamma-gamma turbulence channels

Chao Liu (刘 超), Yong Yao (姚 勇)*, Yunxu Sun (孙云旭), and Xinhui Zhao (赵新辉)

Department of Electronic and Information Engineering, Shenzhen Graduate School,
Harbin Institute of Technology, Shenzhen 518055, China
**E-mail: yaoyong@hit.edu.cn*

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A novel closed-form expression for average capacity is derived for free-space optical (FSO) links over Gamma-Gamma turbulence channels by considering the effect of misalignment (pointing errors). The simulation results show that the average capacity of the FSO links can be analyzed with the effects of atmospheric turbulence condition, beam width, detector size, jitter variable, and transmitted optical power. Meanwhile, the results are further provided to verify the accuracy of our mathematical analysis. This work is useful for the FSO designer.

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Free-space optical (FSO) communication systems have several advantages over conventional radio frequency (RF) communication systems, such as high bandwidth, secure transmission, smaller size, and more freedom from interferences[1−3]. However, long distance outdoor FSO links are always affected by the atmospheric turbulence and pointing errors. Atmospheric turbulence can cause severe degradation in the received signals, known as fading or scintillation. Pointing errors due to building sway is another concern in outdoor FSO links. Thermal expansion, dynamic wind loads, and weak earthquakes result in the sway of high-rise buildings, which causes vibrations of the transmitted beam, so the effect of misalignment (pointing errors) occurs between the transmitter and receiver[4].

Various statistical models have been proposed to describe atmospheric turbulence channels with respect to the strength of turbulence[1]. It was found that for moderate-to-strong turbulence conditions the Gamma-Gamma (GG) distribution almost accurately predicts the probability of fade, the expected number of fades per second, and the mean fading time for the irradiance fluctuations collected by finite-sized apertures that are significantly smaller than the coherence radius[5]. Nistazakis *et al*. used the GG turbulence model to evaluate the average capacity of FSO links by considering the different link length, operation wavelength, and strength of turbulence[6]. The average capacity over generalized-K fading channels has been analyzed and evaluated[7]. In Ref. [8], the ergodic capacity of a FSO communication system under strong turbulence regime that follows the K distribution was evaluated. However, the combined effects of atmospheric turbulence and pointing errors on average capacity were not considered. In Ref. [9], the probability density function (PDF) in no closed-form solution for the combined effects has been investigated based on GG distribution and the outage capacity has been studied considering beam width, pointing error variance, and detector aperture size. The bit-error rate (BER) performance of FSO links has been studied in log-normal (LN) distribution and K distribution in turbulence channels with pointing errors[6,10]. The analyses of ergodic capacities were presented in LN distribution and GG distribution without considering the effects of beam width and detector aperture size[11]. Unlike these earlier works, our theoretical model considers the average capacity in closed-form solution for FSO links over GG turbulence channels with the combined effects of atmospheric turbulence and pointing errors. The closed-form expression of average capacity is presented by considering the effects of atmospheric turbulence condition, beam width, detector size, jitter variable, and transmitted optical power.

In this letter, we consider a FSO communication system using intensity modulation/direct detection (IM/DD) with on-off keying (OOK). The laser beams propagate along a horizontal path through GG turbulence channel with additive white Gaussian noise (AWGN) in the presence of pointing errors. The channel is assumed to be memoryless, stationary, and ergodic, with independent and identically distributed intensity fast fading statistics. We also consider that the channel state information is available at both transmitter and receiver. The statistical channel model is given by

$$y = hx + n, \tag{1}$$

where y is the electrical signal at the receiver, h is normalized channel fading coefficient considered to be constant over a large number of transmitted bits, and n is AWGN with variance σ_{n}^2. Since the atmospheric turbulence and pointing errors are random factors which cause the channel fading, h can be expressed as

$$h = h_{\mathrm{a}} h_{\mathrm{p}}, \tag{2}$$

where h_{a} is the attenuation due to atmospheric turbulence and h_{p} is the attenuation due to pointing errors.

In the GG turbulence channel, the PDF of h_{a} is given by[1]

$$f_{h_{\mathrm{a}}}(h_{\mathrm{a}}) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} (h_{\mathrm{a}})^{\frac{(\alpha+\beta)}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_{\mathrm{a}}}\right), \tag{3}$$

where $K_n[\cdot]$ is the modified Bessel function of the second kind of order n. Positive parameters α and β are defined for the case of a plane wave by[1]

$$\alpha = \left[\exp\left(\frac{0.49\sigma_{\mathrm{R}}^2}{\left(1+0.18d^2+0.56\sigma_{\mathrm{R}}^{12/5}\right)^{7/6}}\right)-1\right]^{-1}, \quad (4)$$

$$\beta = \left[\exp\left(\frac{0.51\sigma_{\mathrm{R}}^2}{\left(1+0.9d^2+0.62d^2\sigma_{\mathrm{R}}^{12/5}\right)^{5/6}}\right)-1\right]^{-1}, \quad (5)$$

where $\sigma_{\mathrm{R}}^2 = 1.23C_n^2 k^{7/6} L^{11/6}$ is the Rytov variance, $d=\sqrt{kD^2/4L}$ is the receiver aperture, $k=2\pi/\lambda$ is the optical wave number, D is the receiver aperture diameter, λ is the communication wavelength, and L is the link length. Here, C_n^2 is the refractive-index structure parameter. In general, C_n^2 varies from 10^{-17} $\mathrm{m}^{-2/3}$ for weak turbulence to 10^{-13} $\mathrm{m}^{-2/3}$ for strong turbulence with a defined typical average value of 10^{-15} $\mathrm{m}^{-2/3}$[6].

Independent identical Gaussian distributions for elevation and horizontal displacement are considered. The channel fading will vary by the combined effects of atmospheric turbulence and pointing errors. Assuming a circular detection aperture of radius r and a Gaussian beam, the combined PDF of h is given as[4,9]

$$f_h(h) = \int \frac{\gamma^2}{A_0^{\gamma^2} h_{\mathrm{a}}}\left(\frac{h}{h_{\mathrm{a}}}\right)^{\gamma^2-1} f_{h_{\mathrm{a}}}(h_{\mathrm{a}})\,\mathrm{d}h_{\mathrm{a}}, \quad 0 \le h \le A_0 h_{\mathrm{a}}, \quad (6)$$

where $\gamma = W_{z\mathrm{eq}}/2\sigma_{\mathrm{s}}$ is the ratio of the equivalent beam radius at the receiver to the pointing error displacement standard derivation (jitter) at the receiver, $A_0 = [\mathrm{erf}(\upsilon)]^2$ is the fraction of the collected power at $r=0$, and $W_{z\mathrm{eq}}$ is the equivalent beam width with $W_{z\mathrm{eq}}^2 = W_z^2\sqrt{\pi}\mathrm{erf}(\upsilon)/[2\upsilon\exp(-\upsilon^2)]$, where $\mathrm{erf}(\cdot)$ is the error function, W_z is the beam waist at distance z, and $\upsilon=\sqrt{\pi}r/\sqrt{2}W_z$.

A closed-form expression for the combined effects of atmospheric turbulence and pointing errors is presented as[12]

$$f_h(h) = \frac{(\alpha\beta)^{(\alpha+\beta)/2}\gamma^2 h^{(\alpha+\beta)/2-1}}{\Gamma(\alpha)\Gamma(\beta)A_0^{(\alpha+\beta)/2}} \times G_{1,3}^{3,0}\left[\frac{\alpha\beta h}{A_0}\middle| \begin{matrix} \gamma^2 \\ \gamma^2-1, \alpha-1, \beta-1 \end{matrix}\right]. \quad (7)$$

The channel state is randomly time-variant. The received instantaneous electrical signal-to-noise ratio (SNR) is a random variable and can be defined as[9]

$$\mathrm{SNR}(h) = \frac{2P_{\mathrm{t}}^2 h^2}{\sigma_{\mathrm{n}}^2}, \quad (8)$$

where P_{t} is the average transmitted optical power. Thus, the average capacity can be given by

$$\langle C\rangle = \int_0^\infty B\log_2[1+\mathrm{SNR}(h)]f_h(h)\,\mathrm{d}h, \quad (9)$$

where B is the bandwidth. By substituting Eqs. (7) and (8) into Eq. (9), using the Meijer G function of $\ln(1+x)$[13], and utilizing Eq. 07.34.21.0013.01[14], a closed-form expression for the average capacity of the GG distribution channel will be derived as

$$\langle C\rangle = \frac{2^{\alpha+\beta-2}B\gamma^2}{\Gamma(\alpha)\Gamma(\beta)\pi\ln 2} \times G_{8,4}^{1,8}\left[\frac{32P_{\mathrm{t}}^2A_0^2}{(\alpha\beta)^2\sigma_{\mathrm{n}}^2}\middle| \begin{matrix} 1, 1, \frac{1-\gamma^2}{2}, \frac{2-\gamma^2}{2}, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-\beta}{2}, \frac{2-\beta}{2} \\ 1, 0, -\frac{\gamma^2}{2}, \frac{1-\gamma^2}{2} \end{matrix}\right]. \quad (10)$$

Equation (10) can be used to evaluate the average capacity of a FSO link for the cases of GG distribution channels. Moreover, the closed-form expression takes into account the atmospheric turbulence conditions, beam width, detector size, jitter variable, and transmitted optical power. Therefore, it can be used to simulate a specific FSO system.

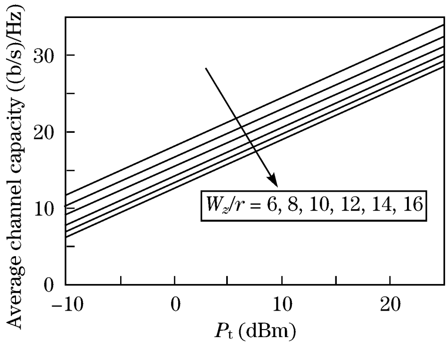

Fig. 1. Average capacity versus transmitted power for several values of the normalized beam width (W_z/r = 6, 8, 10, 12, 14, 16), assuming $L=1000$ m, $\lambda=850$ nm, $D=0.01$ m, $\sigma_{\mathrm{R}}^2=0.26$, and $\sigma_{\mathrm{s}}/r=0.1$.

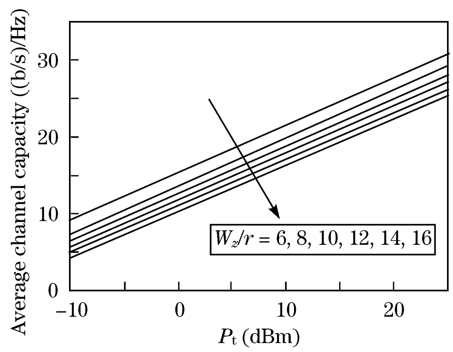

Fig. 2. Average capacity versus transmitted power for several values of the normalized beam width (W_z/r = 6, 8, 10, 12, 14, 16), assuming $L=1000$ m, $\lambda=850$ nm, $D=0.01$ m, $\sigma_{\mathrm{R}}^2=2.6$, and $\sigma_{\mathrm{s}}/r=0.1$.

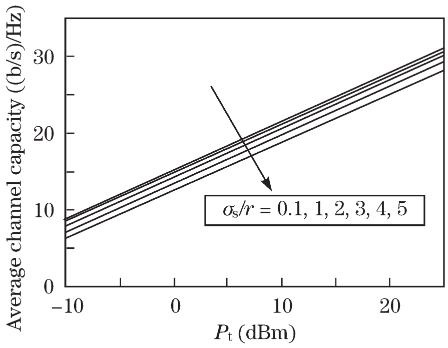

Fig. 3. Average capacity versus transmitted power for several values of the normalized jitter ($\sigma_s/r = 0.1,\ 1,\ 2,\ 3,\ 4,\ 5$), assuming $L = 1000$ m, $\lambda = 850$ nm, $D = 0.01$ m, $\sigma_R^2 = 0.26$, and $W_z/r = 10$.

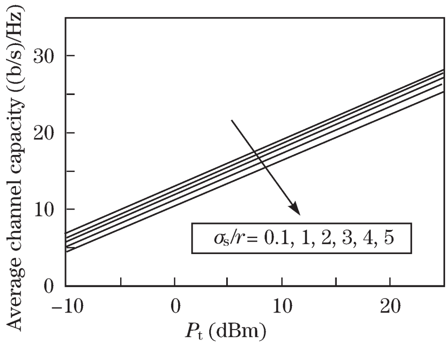

Fig. 4. Average capacity versus transmitted power for several values of the normalized jitter ($\sigma_s/r = 0.1,\ 1,\ 2,\ 3,\ 4,\ 5$), assuming $L = 1000$ m, $\lambda = 850$ nm, $D = 0.01$ m, $\sigma_R^2 = 2.6$, and $W_z/r = 10$.

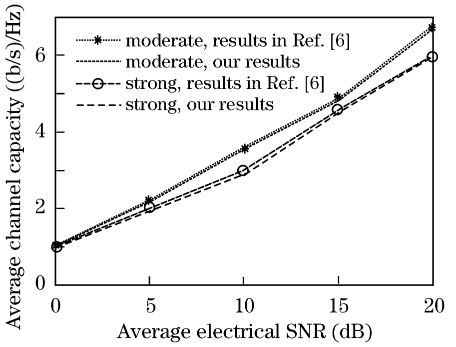

Fig. 5. Average capacity versus average electrical SNR in our analysis and Ref. [6], for moderate and strong turbulence strength ($C_n^2 = 9 \times 10^{-15}$ m$^{-2/3}$ and $C_n^2 = 3 \times 10^{-14}$ m$^{-2/3}$), assuming $L = 1000$ m, $\lambda = 850$ nm, $D = 0.01$ m, and $\sigma = 10^{-7}$ A/Hz.

Simulation results are based on parameters of $L = 1000$ m, $\lambda = 850$ nm, $D = 0.01$ m, and noise standard deviation $\sigma_n = 10^{-7}$A/Hz. These parameters are acceptable values in Refs. [4,6]. We consider the different atmosphere turbulence conditions, (i.e., $C_n^2 = 6.5 \times 10^{-15}$ m$^{-2/3}$ and $C_n^2 = 6.5 \times 10^{-14}$ m$^{-2/3}$, for moderate and strong turbulence conditions, respectively), then the Rytov variances can be calculated as $\sigma_R^2 = 0.26$ and $\sigma_R^2 = 2.6$. The normalized beam width and normalized jitter can be expressed as W_z/r and σ_s/r, respectively[4]. Figure 1 shows the average capacity in terms of the transmitted optical power for $\sigma_R^2 = 0.26$ with various values of the normalized beam width in six steps ($W_z/r = 6,\ 8,\ 10,\ 12,\ 14,\ 16$) and normalized jitter $\sigma_s/r = 0.1$. It is observed that using a narrow beam width, a higher average capacity is achieved with the increase of transmitted optical power. In Fig. 2, just changing the Rytov variance to $\sigma_R^2 = 2.6$ and keeping the other parameters the same as Fig. 1, it is found that the change in Rytov variance induces a decrement in average capacity about 2.98 (b/s)/Hz at $P_t = 25$ dBm and $W_z/r = 6$. For the case of $P_t = -10$ dBm and $W_z/r = 6$ in Fig. 2, the decrement is 2.63 (b/s)/Hz. It is obvious that a larger transmitted optical power corresponds to larger increment in average capacity when the strength of turbulence becomes weaker. For different values of W_z/r, the same results can be gotten. These results can also be investigated by comparing the differences in line slope between Figs. 1 and 2.

In Fig. 3, the average capacity versus transmitted optical power is presented for various values of the normalized jitter (i.e., $\sigma_s/r = 0.1,\ 1,\ 2,\ 3,\ 4,\ 5$) under the condition of $\sigma_R^2 = 0.26$ and the normalized beam width $W_z/r = 10$. In Fig. 4, Rytov variance is changed to $\sigma_R^2 = 2.6$ and the other parameters are the same as Fig. 3. From these figures, it is shown that both turbulence and pointing errors degrade the average capacity of FSO links.

In the absence of pointing errors, we compare our simulation results of the limited case with the results in Ref. [6] where the pointing errors are not taken into account. Simulation results are based on the parameters in Ref. [6] (i.e., $C_n^2 = 9 \times 10^{-15}$ m$^{-2/3}$ and $C_n^2 = 3 \times 10^{-14}m^{-2/3}$ for moderate and strong turbulence, $L = 1000$ m, $\lambda = 850$ nm, and $D = 0.01$ m) and $\sigma_n = 10^{-7}$ A/Hz. Figure 5 shows the average capacity versus average electrical SNR in the absence of pointing errors for our results and the results in Ref. [6]. It is found that our results almost match the results in Ref. [6]. Thus, the accuracy of our mathematical analysis is further proved.

In conclusion, the average capacity of a FSO link with pointing errors over GG turbulence channel is studied. A closed-form average capacity expression is derived for this fading channel. It can be used to analyze the average capacity with the effects of turbulence strength, beam width, detector size, jitter variable, and transmitted optical power. It is useful for the designer of FSO communication systems to consider these factors affecting the channel capacity.

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