Preparation of entangled states of atomic qubits via atom-cavity-laser interaction

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We propose a scheme to prepare the Bell states for atomic qubits trapped in separate optical cavities via atom-cavity-laser interaction. The quantum information of each qubit is encoded on the degenerate ground states of the atom, so the entanglement between them is relatively stable against spontaneous emission. The proposed scheme consists of a Mach-Zehnder interferometer (MZI) with two arms, and each arm contains a cavity with an N-type atom in it. It requires two classical fields and a single-photon source. By controlling the sequence and time of atom-cavity-laser interaction, the deterministic production of the atomic Bell states is shown. We also introduce the generalization of the present scheme to generate the $2N$-atom Greenberger-Horne-Zeilinger state.

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Quantum entanglement is a characteristic feature of quantum states. It has important applications in quantum science and technology, such as quantum communication and quantum information processing[1], quantum teleportation[2], quantum dense coding[3], and quantum cryptography[4,5]. Consequently, quantum entanglement is considered as an absolutely necessary resource for quantum information processing and a common platform of researchers in the fields of quantum optics, quantum information, and condensed matter physics. There are numerous schemes for generating various kinds of entanglement for different particles, such as the entanglement between photons[6−9], atoms[10−14], trapped ions[15], atoms and photons[16,17], respectively.

In addition to the entanglement between two particles, there is also entanglement of multiparticles. Dür *et al.* have shown that there are two inequivalent classes of tripartite entangled states under stochastic local operation and classical communication (LOCC)[18]. One is the Greenberger-Horne-Zeilinger (GHZ) type entanglement, and the other is the W-type entanglement. These two types of states cannot be converted to each other by LOCC with nonzero success probability. As is well known, the multiparticle entangled state is an essential component in quantum information processing in quantum networks, such as in quantum communication between the communication center and the multiuser. Therefore, the development of an experimentally feasible method to generate different types of multipartite entanglement is necessary. Over the past decades, many significant advancements[19,20] have shown that the generation of entanglement of two particles is no longer a major technological challenge. However, the creation of entanglement of multiparticles, such as the GHZ and W states, remains to be a great challenge for both theoretical and experimental physicists despite some notable recent achievements[21,22] along this direction.

On the other hand, cavity quantum electrodynamics (QED) has become an important platform to demonstrate the quantum characteristics of atoms and photons in recent years and provides an almost ideal system for the generation of entanglement between atoms or ions. In this letter, we propose a method to generate the Bell states and multipartite entangled states of atoms trapped in cavities via the cavity-QED-based approach. The scheme uses linear optical devices and has an ideal success probability of 100%. In our scheme, we utilize the N–type atomic off-resonant interaction with two classical fields and a single-photon pulse to realize the Hadamard operation and control Z operation[23]. By controlling the interaction sequence and properly preparing the initial state of the atoms, we can obtain the atomic entangled states.

We show the basic building block of our scheme, i.e., the model of atom-cavity-laser interaction with the photon pulse[23]. We consider an N-type four-level atom[24], as shown in Fig. 1, which is trapped in a single-mode cavity. The atom has two degenerate lower states $|0\rangle$ and $|1\rangle$, which act as the basic qubit states and are coupled to the upper state $|2\rangle$ via two classical fields Ω_1 and Ω_2, respectively, with the same detuning Δ_1. The cavity field couples the transition $|1\rangle \leftrightarrow |3\rangle$ with detuning Δ_2 and coupling constant g. Under the dipole and rotating wave approximation, the total Hamiltonian of this atom-cavity-laser system can be written as $\hat{H} = \hat{H}_0 + \hat{H}_{\rm int}$ with

$$\hat{H}_0 = \sum_{i=0}^{3} \omega_i |i\rangle\langle i| + \omega_{\rm c} \hat{a}^\dagger \hat{a}, \tag{1}$$

and

$$\begin{aligned}\hat{H}_{\rm int} = \ & \Omega_1 {\rm e}^{{\rm i}\nu_1 t}|0\rangle\langle 2| + \Omega_2 {\rm e}^{{\rm i}\nu_2 t}|1\rangle\langle 2| \\ & + g\hat{a}^\dagger |1\rangle\langle 3| + H_{\rm c},\end{aligned} \tag{2}$$

where $\hat{H}_0$ gives the free evolution of the cavity field and four-level atom, and $\hat{H}_{\rm int}$ includes the interaction Hamiltonian between the cavity field and the atom, and the interaction Hamiltonian between two classical fields and the atom. Here we let $\hbar = 1$, and the symbol $H._{\rm c}$. means Hermitian conjugate; $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators, respectively, associated with the cavity mode with frequency $\omega_{\rm c}$; ω_i $(i = 0,1,2,3)$ are the Bohr frequencies of atomic states $|i\rangle$; and ν_1 and ν_2 are the frequencies of the two classical fields Ω_1 and Ω_2, respectively.

In the interaction picture with respect to $\hat{H}_0$, the associated Hamiltonian reads

$$\begin{aligned}\hat{H}_I &= \Omega_1 {\rm e}^{-{\rm i}\Delta_1 t}|0\rangle\langle 2| + \Omega_2 {\rm e}^{-{\rm i}\Delta_1 t}|1\rangle\langle 2| \\ &\quad + g\hat{a}^\dagger {\rm e}^{-{\rm i}\Delta_2 t}|1\rangle\langle 3| + H._{\rm c},\end{aligned} \tag{3}$$

where $\Delta_1 = \omega_0 - \omega_2 + \nu_1 = \omega_1 - \omega_2 + \nu_2$ and $\Delta_2 = \omega_1 - \omega_3 + \omega_{\rm c}$. We consider the detunings $\{|\Delta_1 - \Delta_2|, |\Delta_1|, |\Delta_2|\} \gg g, |\Omega_1|, |\Omega_2|$, then we use adiabatic elimination and obtain the effective Hamiltonian

$$\begin{aligned}\hat{H}_{\rm eff} &= \frac{g^2\hat{a}\hat{a}^\dagger}{\Delta_2}(|3\rangle\langle 3| - |1\rangle\langle 1|) \\ &\quad - \frac{\Omega_1\Omega_2}{\Delta_1}(|0\rangle\langle 1|{\rm e}^{{\rm i}\varphi} + |1\rangle\langle 0|{\rm e}^{-{\rm i}\varphi}),\end{aligned} \tag{4}$$

where φ is the phase factor[25].

We assume that the cavity field is prepared in vacuum state $|0\rangle_{\rm c}$, the atom is in the ground state $|0\rangle$ or $|1\rangle$, and the two classical fields Ω_1 and Ω_2 are turned on at $t = 0$. Then

$$\begin{aligned}&\hat{M}|0\rangle|0\rangle_{\rm c} \rightarrow |\phi\rangle|0\rangle_{\rm c}, \\ &\hat{M}|1\rangle|0\rangle_{\rm c} \rightarrow |\phi'\rangle|0\rangle_{\rm c},\end{aligned} \tag{5}$$

where

$$\hat{M} = \exp(-{\rm i}\hat{H}_{\rm eff}t), \tag{6}$$

$$|\phi\rangle = (|0\rangle - |1\rangle)/\sqrt{2}, \tag{7}$$

$$|\phi'\rangle = (|0\rangle + |1\rangle)/\sqrt{2}, \tag{8}$$

and we choose the interaction time $\Omega_1\Omega_2 t/\Delta_1 = \pi/4$ and phase factor $\varphi = -\pi/2$, which indicates that the two classical fields can be used to perform the Hadamard operation for atomic qubit.

After turning off the classical fields, a single photon is adiabatically put into the cavity, which induces a light shift on the state $|1\rangle$. The magnitude of the shift is well known and is given by $\gamma = -g^2/\Delta_2$ for $\Delta_2 \gg g$[23,26]. Therefore, in order to obtain a phase shift of θ, the photon must be present in the cavity for a time given by $t = \theta\Delta_2/g^2$. We take $\theta = \pi$ to ensure that the state $|\phi\rangle \leftrightarrow |\phi'\rangle$ and to realize the control Z operation on the atomic qubits. Therefore for a single photon passing through the optical cavity, we use the operator $\hat{P}$ to describe the process:

$$\begin{aligned}&\hat{P}|\phi\rangle|0\rangle_{\rm i} \rightarrow |\phi\rangle|0\rangle_{\rm o}, \\ &\hat{P}|\phi\rangle|1\rangle_{\rm i} \rightarrow |\phi'\rangle|1\rangle_{\rm o}, \\ &\hat{P}|\phi'\rangle|0\rangle_{\rm i} \rightarrow |\phi'\rangle|0\rangle_{\rm o}, \\ &\hat{P}|\phi'\rangle|1\rangle_{\rm i} \rightarrow |\phi\rangle|1\rangle_{\rm o},\end{aligned} \tag{9}$$

where the indices $\{{\rm i}, {\rm o}\}$ represent the input and output optical modes.

We present the actual arrangement of our scheme as shown in Fig. 2 to generate two atomic qubit Bell states. Two beam splitters (BS1, BS2), some mirrors, and fibers form a Mach-Zehnder interferometer (MZI); two optical cavities are placed into the arms of the interferometers. Each cavity contains an N-type atom. Two photon detectors D1 and D2 are used to detect the output photon state. The two 50:50 linear beam splitters are the main components of the MZI. The input optical field is bifurcated at the first beam splitter, steered to interact with the atomic qubits, and then recombined at the second beam splitter. We employ the idea of cavity Q switching in order to control the input photon into the cavity. After interaction, the photon is then Q-switched out of the cavity back into the optical mode using the appropriate shaping techniques[27,28]. For convenience of description, we divide our scheme into the following three steps.

Step 1: The atomic qubits are initialized in the state $|00\rangle_{1,2}$, and then transformed to $|\phi\rangle_1|\phi\rangle_2$ via the two classical fields Ω_1 and Ω_2, subsequently interacting with the two atoms.

Step 2: For the single photon input, the upper channel is described by $|1\rangle_i$, while the lower channel is in the vacuum state. The initial state of the optical field is $|10\rangle_i$, then the initial state of the whole system is

$$|\Phi_{\rm i}\rangle = |\phi\rangle_1|\phi\rangle_2 \otimes |10\rangle_i. \tag{10}$$

The propagator of the whole system can be described by $\hat{U} = \hat{U}_{\rm BS}\hat{P}_1\hat{P}_2\hat{U}_{\rm BS}$, where $\hat{U}_{\rm BS} = \exp[-{\rm i}\pi(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1)/4]$ is the beam splitter propagator, $\hat{a}_{1,2}^\dagger$ and $\hat{a}_{1,2}$ are the creation and annihilation operators of the optical field corresponding to the upper arm and the lower one, and $\hat{P}_{1,2}$ refers to two control Z operations mentioned

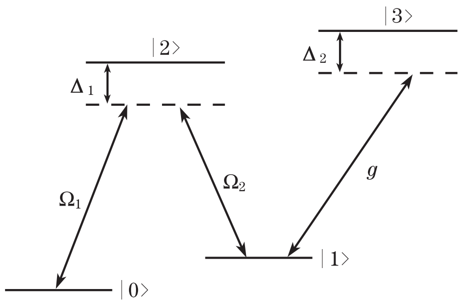

Fig. 1. N-type four-level atom with two degenerate lower states $|0\rangle$ and $|1\rangle$ as the basic qubit states required for the scheme.

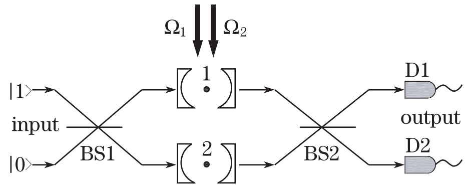

Fig. 2. Schematic diagram to generate two atomic qubit Bell states.

above. The state of the system at the interferometer output is given by

$$|\Phi_f\rangle = \hat{U}|\Phi_i\rangle = \frac{1}{\sqrt{2}}|10\rangle_o \otimes \frac{1}{\sqrt{2}}(|\phi'\rangle_1|\phi\rangle_2 + |\phi\rangle_1|\phi'\rangle_2) - \mathrm{i}\frac{1}{\sqrt{2}}|01\rangle_o \otimes \frac{1}{\sqrt{2}}(|\phi'\rangle_1|\phi\rangle_2 - |\phi\rangle_1|\phi'\rangle_2). \quad (11)$$

From the state given by Eq. (11), the two Bell states can be obtained through performing the detection of the photon at detectors D1 and D2. In fact, if detector D1 clicks but not D2, the atomic state will collapse into $(|\phi'\rangle_1|\phi\rangle_2 + |\phi\rangle_1|\phi'\rangle_2)/2$. However if the detector D2 clicks but not D1, the resulting state is reduced to $(|\phi'\rangle_1|\phi\rangle_2 - |\phi\rangle_1|\phi'\rangle_2)/2$.

Step 3: In order to obtain the Bell states, we perform the second transform on the atomic state by using the two classic fields Ω_1 and Ω_2, in the same way as step 1. This takes the state $|\phi\rangle \to |0\rangle$ and $|\phi'\rangle \to |1\rangle$. Therefore, we obtain the perfect atomic qubit Bell states

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|10\rangle \pm |01\rangle). \quad (12)$$

The probability of success in obtaining the above Bell states is next examined. From Eq. (11), it is straightforward to see that when the first photodetector D1 clicks, the photodetector D2 detects the null result at the same time, and the atomic-qubit state will collapse onto the Bell state $|\Psi^+\rangle$ with 1/2 probability of success. When the second photodetector D2 clicks, the photodetector D1 detects the null result at the same time, and the atomic-qubit state will collapse onto the Bell state $|\Psi^-\rangle$ with 1/2 probability of success as well. The Bell state $|\Psi^+\rangle$ (or $|\Psi^-\rangle$) can be obtained by applying the Pauli operator $\hat{\sigma}^z$ to the first atomic qubit of the Bell state $|\Psi^-\rangle$ (or $|\Psi^+\rangle$), i.e.,

$$\hat{\sigma}_1^z|\Psi^+\rangle = |\Psi^-\rangle, \quad \hat{\sigma}_1^z|\Psi^-\rangle = |\Psi^+\rangle, \quad (13)$$

which implies that the Bell states $|\Psi^+\rangle$ and $|\Psi^-\rangle$ can be produced with the probability of success being unity through using the $\hat{\sigma}^z$ operation on the first atomic qubit. Hence, in our scheme, the Bell states can be produced deterministically through making single photon detections of output fields and unitary transformation ($\hat{\sigma}^z$) upon the first atomic qubit.

We consider the influence of the imperfection of photon detections in the present scheme. An ideal photon detection with quantum efficiency $\eta = 1$ can be described by the positive operator-valued measure (POVM) of each detector $\{\Pi_0 = |0\rangle\langle 0|, \Pi_1 = I - \Pi_0\}$. In the realistic case, an incoming photon cannot be detected with the success probability of 1. If the quantum efficiency of the photodetector is η, the POVM is given by[29,30]

$$\Pi_0(\eta) = \sum_{i=0}(1-\eta)^i|i\rangle\langle i|, \quad \Pi_1(\eta) = I - \Pi_0(\eta), \quad (14)$$

from which it is straightforward to find that the inefficiency of photodetectors does not affect the quality of the generated entangled states, but rather it decreases the success probability. In our scheme, the success probability of obtaining the Bell state is η^2 when the quantum efficiency of each photodetector is η.

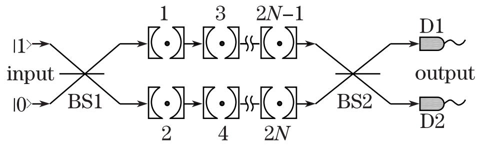

Fig. 3. Schematic diagram to generate the $2N$-atom GHZ state.

The scheme to generate multiparticle entangled states can easily be constructed with some rearrangements of the system configuration. We now add some cavities in our scheme as shown in Fig. 3 to generate the GHZ state among $2N$ atoms each trapped in a cavity. The $2N$ atoms are prepared initially in state $|0000\cdots\rangle$. As before, we use two classic fields to transform each qubit $|0\rangle \to |\phi\rangle$, and then a single photon is directed to beam splitter BS1. Thus, if the photon is transmitted (reflected), all even (odd) numbered atoms in the lower (upper) row will experience a state transform $|\phi\rangle \to |\phi'\rangle$. Finally, we perform a second transform on the atomic state by using the two classic fields again to take the state $|\phi\rangle \to |0\rangle$ and $|\phi'\rangle \to |1\rangle$, and then we can easily obtain the detection of a photon at detector D1 or D2, with which signals of a $2N$-atom entangled state $(|101010\cdots\rangle + |010101\cdots\rangle)/\sqrt{2}$ or $(|101010\cdots\rangle - |010101\cdots\rangle)/\sqrt{2}$ are produced. To derive the standard $2N$-atom GHZ states $(|111111\cdots\rangle \pm |000000\cdots\rangle)/\sqrt{2}$, NOT operations on all odd- or even-numbered atoms are needed.

We give a brief discussion of the experimental feasibility of our scheme. For the atomic-level structure, we can choose the alkali atoms, such as ^{87}Rb atoms. For the single photon source, many single-photon sources have been developed in the last decade, such as a single atom or ion in a trap, a cold atomic ensemble, a single quantum dot, and a single color center in diamond. The imperfection of the single-photon source and the photon detectors will reduce the success probability but will not affect the quality of the states to be generated. The realization of the $\hat{\sigma}^z$ operation and NOT operation is also shown. The $\hat{\sigma}^z$ operation in our scheme is a single atomic qubit rotational operation that can be implemented by the far-off-resonant interaction between the atom with levels $|0\rangle$ and $|2\rangle$ and a classical field. Meanwhile, the NOT operation can be implemented by two large detuning classical fields interacting with the atom with levels $|0\rangle$, $|1\rangle$, and $|2\rangle$.

In conclusion, we propose an optical scheme to generate the Bell and GHZ states of atoms qubits in separated optical cavities by using atom-cavity-laser interaction under the condition of large detuning and photon detections. The scheme has several key features. It requires only linear optical devices in addition to atom-cavity-laser interaction and photon detections. The scheme can be constructed to generate entanglement between an arbitrary number of atoms. The success probability with the scheme can ideally reach unity.

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