Characterization of tightly focused partially coherent radially polarized vortex beams

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Tight focusing properties of partially coherent radially polarized vortex beams are studied based on vectorial Debye theory. We focus on the focal properties including the intensity and the partially coherent and polarized properties of such partially coherent vortex beams through a high numerical aperture objective. It is found that the source coherence length and the maximal numerical aperture angle have direct influence on the focal intensity, as well as coherence and polarization properties. This research is important in optical micromanipulation and beam shaping.

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Radially polarized beams are well known to create a strong longitudinally polarized field component near the focal region by a high numerical aperture (NA) objective, hence exhibiting special properties[1−4]. This focused light has a tighter spot and can be applied in many fields such as high resolution microscopy, lithography, optical data storage, material processing, and optical trapping and acceleration[5]. In 2003, direct detection and characterization of this sharp longitudinal field by experimental demonstration was reported[5]. Because of the unique characteristics of vortex beams[6,7], the tight focusing of optical vortex beams, such as linearly, circularly, and elliptically polarized vortex beams, as well as cylindrically polarized vortex beams, have been extensively studied[8−11].

However, because of the universality and characteristics of partially coherent light[12,13], the partially coherent radially polarized beam seems more important and promising. Seshadri introduced the cross-spectral density of partially coherent azimuthally or radially polarized symmetric beams and investigated the effect of the coherence length on its average Poynting vectors[13]. Recently, the propagation properties of partially coherent radially polarized beams in free space and a turbulent atmosphere have been studied[14,15]. However, to the best of our knowledge, there is no paper dealing with the tight focusing of partially coherent radially polarized vortex beams. Moreover, it is of great importance to explore the coherence and polarization properties of the tightly focused vortex wave fields due to their increasing applications. In this letter, the focal characteristics including intensity, and coherence and polarization properties of partially coherent radically polarized vortex beams are studied based on vectorial Debye theory. The influence of the coherence length and the maximal NA angle on these focusing properties are analyzed in detail.

The electric field of a completely coherent radially polarized vortex beam focused by a high NA objective can be expressed as[3−11]

$$E(r,\psi,z)=\begin{bmatrix}E_x(r,\psi,z)\\E_y(r,\psi,z)\\E_z(r,\psi,z)\end{bmatrix}=-\mathrm{i}^{n+1}E_0\begin{bmatrix}\mathrm{i}\left(I_{n+1}\mathrm{e}^{\mathrm{i}\psi}-I_{n-1}\mathrm{e}^{-\mathrm{i}\psi}\right)\\I_{n+1}\mathrm{e}^{\mathrm{i}\psi}+I_{n-1}\mathrm{e}^{-\mathrm{i}\psi}\\2I_n\end{bmatrix}\mathrm{e}^{\mathrm{i}n\psi},\quad(1)$$

where n is the topological charge, E_0 is a constant related to the intensity of the beam, r, ψ, and z are the cylindrical coordinates of an observation point in the focal region. The definition of the variables I_n and $I_{n\pm1}$ is given by

$$I_n(r,z)=\int_0^\alpha P(\theta)\sqrt{\cos\theta}\sin^2\theta\, J_n(kr\sin\theta)\exp(\mathrm{i}kz\cos\theta)\,\mathrm{d}\theta,\quad(2)$$

$$I_{n\pm1}(r,z)=\int_0^\alpha P(\theta)\sqrt{\cos\theta}\sin\theta\cos\theta\, J_{n\pm1}(kr\sin\theta)\exp(\mathrm{i}kz\cos\theta)\,\mathrm{d}\theta,\quad(3)$$

where $P(\theta)$ is the pupil apodization function and J_n is the nth Bessel function of the first kind.

Assuming that the field wave is quasi-monochromatic, the second-order correlation properties of the beam can be characterized by the 3×3 electric cross-spectral density matrix $W(\boldsymbol{r}_1,\boldsymbol{r}_2)$. The elements of the 3×3 matrix are given by[16]

$$W_{ij}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\left\langle E_i^*(\gamma_1,\psi_1,z_1)E_j(\gamma_2,\psi_2,z_2)\right\rangle\quad(i,j=x,y,z),\quad(4)$$

where $E_i(\gamma,\psi,z)$ and $E_j(\gamma,\psi,z)$ denote Cartesian components of the electric field, the asterisk stands for the complex conjugate, and the angle brackets represent an ensemble average.

The explicit expressions of the diagonal elements of W_{ij} can be derived from Eq. (1):

$$W_{xx}(r_1,r_2,z) = E_0^2\left[I_{n+1}^*(r_1,z)\,\mathrm{e}^{-\mathrm{i}\psi_1} - I_{n-1}^*(r_1,z)\,\mathrm{e}^{i\psi_1}\right] \times \left[I_{n+1}(r_2,z)\,\mathrm{e}^{\mathrm{i}\psi_2} - I_{n-1}(r_2,z)\,\mathrm{e}^{-\mathrm{i}\psi_2}\right]\exp\left[\mathrm{i}n\left(\psi_2-\psi_1\right)\right], \tag{5}$$

$$W_{yy}(r_1,r_2,z) = E_0^2\left[I_{n+1}^*(r_1,z)\,\mathrm{e}^{-\mathrm{i}\psi_1} + I_{n-1}^*(r_1,z)\,\mathrm{e}^{\mathrm{i}\psi_1}\right] \times \left[I_{n+1}(r_2,z)\,\mathrm{e}^{\mathrm{i}\psi_2} + I_{n-1}(r_2,z)\,\mathrm{e}^{-\mathrm{i}\psi_2}\right]\exp\left[\mathrm{i}n\left(\psi_2-\psi_1\right)\right], \tag{6}$$

$$W_{zz}(r_1,r_2,z) = 4E_0^2 I_n^*(r_1,z)\,I_n(r_2,z)\exp\left[\mathrm{i}n\left(\psi_2-\psi_1\right)\right], \tag{7}$$

where

$$I_p^*(r_1,z)\,I_q(r_2,z) = \int_0^\alpha\int_0^\alpha A(\theta_1,\theta_2)\sqrt{\cos\theta_1\cos\theta_2}\sin\theta_1\sin\theta_2 g(\theta_1)\,g(\theta_2) \times J_p(kr_1\sin\theta)\,J_q(kr_2\sin\theta)\exp\left[\mathrm{i}kz\left(\cos\theta_2-\cos\theta_1\right)\right]\mathrm{d}\theta_1\mathrm{d}\theta_2, \tag{8}$$

where

$$g(\theta_i) = \begin{cases}\sin\theta & p,q=n\\ \cos\theta & p,q=n\pm1.\end{cases} \tag{9}$$

It is assumed that the field in the high NA objective is a Gaussian model with an optical vortex:

$$E(r) = \exp\left(-r^2/\omega_0^2\right)\exp\left(\mathrm{i}n\varphi\right)\exp(\mathrm{i}\beta), \tag{10}$$

where ω_0 is the beam size and β is an arbitrary phase. The cross-spectral density of such a partially coherent vortex beam can be expressed as[8]

$$W(\boldsymbol{r}_1,\boldsymbol{r}_2,0) = A(r_1,r_2)\exp\left[\mathrm{i}n\left(\varphi_2-\varphi_1\right)\right], \tag{11}$$

where

$$A(r_1,r_2) = \exp\left[-\left(\boldsymbol{r}_1^2+\boldsymbol{r}_2^2\right)/\omega_0^2\right] \times\exp\left[-\left(\boldsymbol{r}_1-\boldsymbol{r}_2\right)^2\Big/L_c^2\right], \tag{12}$$

where L_c is the source coherence length.

Under the sine condition, $r = f\sin\theta$, where f is the focal length of the objective. The cross-spectral density of such a partially coherent vortex beam of the pupil can be expressed as

$$A(\theta_1,\theta_2) = \exp[-f^2(\sin^2\theta_1+\sin^2\theta_2)/w_0^2) \times\exp(-f^2(\sin\theta_1-\sin\theta_2)^2/L_\mathrm{c}^2]. \tag{13}$$

The intensity distribution $I(r,\psi,z)$, the degree of coherence $\mu_{ij}(r,\psi,z)$, and the degree of polarization $P(r,\psi,z)$ of the field in the focal region are given by[16,17]

$$\begin{aligned}I(r,\psi,z) &= \mathrm{Tr}W(r,\psi,z)\\ &= W_{xx}(r,\psi,z)+W_{yy}(r,\psi,z)\\ &\quad + W_{zz}(r,\psi,z),\end{aligned} \tag{14}$$

$$\mu_{ij}(r,\psi,z) = \frac{W_{ij}(r,\psi,z)}{\sqrt{W_{ii}(r,\psi,z)W_{jj}(r,\psi,z)}}(i,j=x,y,z), \tag{15}$$

$$P_{3\mathrm{D}}(r,\psi,z) = \sqrt{\frac{3}{2}\left\{\frac{I_x(r,\psi,z)^2+I_y(r,\psi,z)^2+I_z(r,\psi,z)^2}{\left[I_x(r,\psi,z)+I_y(r,\psi,z)+I_z(r,\psi,z)\right]^2}-\frac{1}{3}\right\}}, \tag{16}$$

where Tr denotes the trace of the electric cross-spectral density matrix $W(\boldsymbol{r}_1,\boldsymbol{r}_2)$.

Now we present some numerical results to show the focusing properties of partially coherent radially polarized vortex beams through a high NA objective and the influences of source coherent length L_c and maximal NA angle α on the focal properties including intensity, and coherence and polarization properties.

Figures 1(a) and (b) show the total intensity distribution of the partially coherent radially polarized vortex beam near the focal region. It is found that the core intensity is nonzero and the intensity distributions are symmetric to both the r axis and the z axis. Figures 2(a) and (b) show the influences of source coherent length L_c and maximal NA angle α on the intensity distribution of the partially coherent radially polarized vortex beam. It is also found that the increments of L_c and α will increase the intensity in the focal plane. From Figs. 2(a) and (b), it is clear that with a large L_c or an appropriate α, the intensity distribution in the focal plane is flatter.

Then we focus our attention on studying the spectral degree of coherence of tightly focused partially coherent radially polarized beams, as illustrated in Fig. 3(a). It shows that the curve of $|\mu_{xz}|$ is smoother than those of $|\mu_{xy}|$ and $|\mu_{yz}|$, which indicates that the coherence between x and y directions is less affected by the increment of x. We can also find in Fig. 3(a) that in the region close to the focus, the coherence remains nearly unchanged while being focused. However, for the region far away from the propagation axis, tight focusing of a vortex beam through a high NA will decrease the degree of coherence. Figures 3(b) and (c) show the influence of L_c and α on the polarization distribution of the partially coherent radially polarized vortex beam in the focal

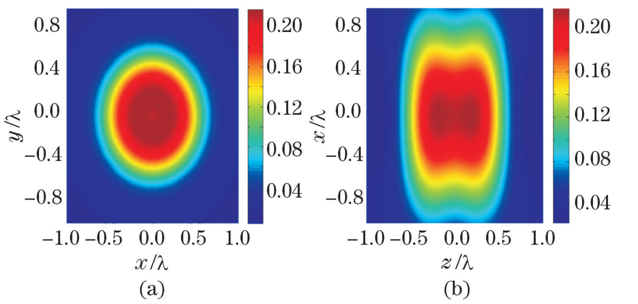

Fig. 1. Contour maps of the intensity distribution of the partially coherent radially polarized vortex beam (a) at focus and (b) through focus. The parameters are chosen as $\lambda = 633$ nm, $\omega_0 = 1$ cm, $f = 1$ cm, $L_c = 1$ cm, $\alpha = 70°$, and $n = 1$.

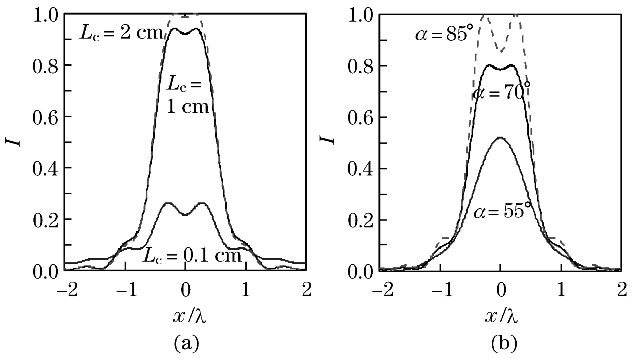

Fig. 2. Intensity (I) distribution of a partially coherent radially polarized vortex beam with (a) different source coherent lengths L_c and (b) different maximal angles α. The other parameters are the same as those in Fig. 1.

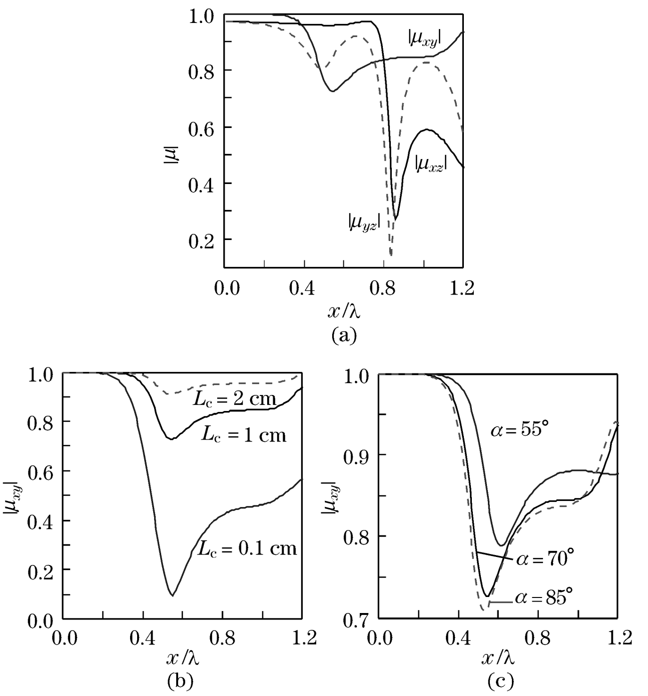

Fig. 3. Modulus of complex coherent coefficient $|\mu|$ distribution in the focal plane. (a) Distributions of $|\mu_{xy}|$, $|\mu_{xz}|$, and $|\mu_{yz}|$; (b) $|\mu_{xy}|$ as a function of source coherent length L_c; (c) $|\mu_{xy}|$ as a function of the maximal NA angle α. The other parameters are the same as those in Fig. 1.

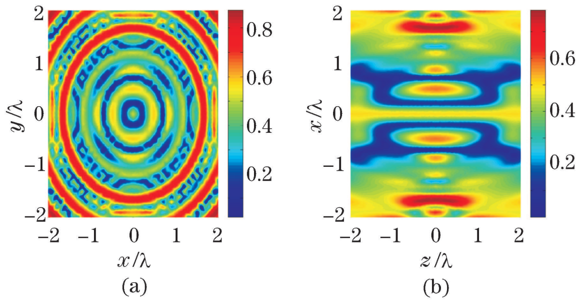

Fig. 4. Contour maps of degree of polarization distribution (a) at focus and (b) through focus. The parameters are the same as those in Fig. 1.

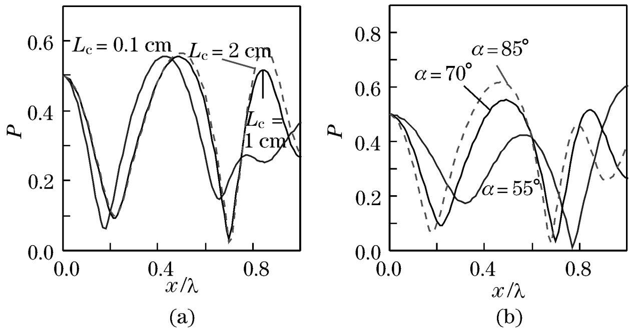

Fig. 5. Distributions of degree of polarization in P the focal plane. (a) P as a function of source coherent length L_c; (b) P as a function of the maximal angle α. The other parameters are the same as those in Fig. 1.

plane. However, when x is large enough, the influence of L_c and α becomes obvious and a larger L_c or α will cause a larger oscillation.

Now we focus on the polarization properties of the partially coherent radially polarized vortex beam when they are focused by a high NA objective. Figures 4(a) and (b) show that the polarization distributions are symmetric to the focal plane and the z axis. The increase of α will enhance the focused intensity in the axial directions, which would make the field more depolarized. The polarization patterns in Fig. 4 reveal that the degrees of polarization are all smaller than unity and oscillate sharply, which means that the beam is depolarized in the focused field. However, for the region far away from the propagation axis, the degree of polarization will increase, which indicates that the polarization of the beam increases when the beam is tightly focused, as shown in Figs. 4(a) and (b).

Next, the influence of α and L_c on the polarization distribution of the partially coherent radially polarized vortex beam in the focal plane is studied. In Fig. 5, in the focal region, we can find that the influence of L_c and α is obvious and a larger L_c or α will cause a larger polarization oscillation. However, the increase of L_c causes little change to the polarization distribution when L_c reaches 1 cm.

In conclusion, the focusing properties of radially polarized partially coherent vortex beams through a high NA objective are studied. The expressions for intensity distribution, spectral degree of coherence, and degree of polarization are derived based on the vectorial Debye theory. Numerical calculations are performed and the results are discussed in detail. It is shown that the coherence length and the maximal NA angle have direct influence on the intensity and the partially coherent and polarized properties. So it is possible to tune the focal characteristics of the beams by changing the coherence length and the maximal NA angle. These results are very important for the applications of vortex beams in many fields.

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