Photonic band gap from periodic structures containing anisotropic nonmagnetic left-handed metamaterial

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We demonstrate a photonic band gap (PBG) from one-dimensional (1D) periodic structures created by a double-layer unit cell with an air layer and an anisotropic nonmagnetic left-handed metamaterial (LHM) layer whose permittivity elements are partially negative. The requirements imposed on the materials and structures to realize a PBG are derived when the frequency is above or below the cutoff frequency, and the transmission properties of the PBG are discussed by utilizing 4×4 transfer-matrix method with dispersive semiconductor metamaterial.

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Photonic band gap (PBG) material[1,2] is a type of artificial material with periodically dielectric modulated function, and it has received considerable attention in recent years because of its property for stopping photons with forbidden frequencies from propagating in the structure. Left-handed metamaterial (LHM) is another type of artificial composite which has a negative refractive index[3−5], and it can be achieved by simultaneously or partially negative permittivity and permeability. It has a large number of potential applications in optics, material science, biology, and biophysics because of its negative phase velocity for light propagating in it. In particular, PBG structures containing LHM have several extraordinary properties. For example, a new type of PBG corresponding to a vanishing effective phase, denoted by zero-Φ_{eff} gap, can be realized in a layered system combing both positive- and negative-index materials; it is robust against weak disorder and lattice scaling and possesses omnidirectional gaps[6,7]. Shadrivov *et al*. proposed a complete three-dimensional (3D) PBG structure with isotropic LHM and common material[8]. Because most LHMs are dispersive and anisotropic, the one-dimensional (1D) PBG with dispersive LHM[9,10] and two-dimensional (2D) PBG with anisotropic LHM[11] are quickly presented. All elements of permittivity and permeability tensors of the anisotropic LHM discussed are assumed to be negative in Ref. [11]. However, this is not generally practical. A novel nonmagnetic LHM is proposed in Ref. [12] which is based on the strong anisotropy of the dielectric response, and it does not have any magnetic response ($\mu = \mu_0$). It has an anisotropic uniaxial dielectric constant $\boldsymbol{\varepsilon}$, with $\varepsilon_x = \varepsilon_y = \varepsilon_0\varepsilon_\parallel > 0$ and $\varepsilon_z = \varepsilon_0\varepsilon_\perp < 0$.

In this letter, we propose and demonstrate a PBG in a layered system containing air and the above nonmagnetic anisotropic LHM with partially negative elements of $\boldsymbol{\varepsilon}$ tensor. Here we call it as an anisotropic nonmagnetic (ANM) gap which differs from traditional Bragg gap and zero-Φ_{eff} gap. Our structure is different from the structure in Ref. [11] because it comprises LHM just with partially negative elements of $\boldsymbol{\varepsilon}$, while the latter just considers the situation that all elements of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ tensors are negative. Moreover, the ANM gap never needs any magnetic resonance which is necessary in all the PBG structures mentioned above. By analyzing the trace of the transfer matrix, the requirements imposed on the materials and structures to realize an ANM gap are derived. By utilizing the 4×4 transfer-matrix method[13], the transmission properties of the PBG are also discussed with an example of the dispersive semiconductor metamaterial[14].

We consider a 1D periodic layered stack created by a double-layer unit cell with an air layer (transparent area) of a thickness d_1 and an anisotropic LHM layer (gray area) of a thickness d_2, as shown in Fig. 1. The latter is characterized by a permeability μ_0 and a permittivity tensor

$$\boldsymbol{\varepsilon} = \varepsilon_0 \begin{pmatrix} \varepsilon_\parallel & 0 & 0 \\ 0 & \varepsilon_\parallel & 0 \\ 0 & 0 & \varepsilon_\perp \end{pmatrix},$$

where $\varepsilon_\perp < 0$ and $\varepsilon_\parallel > 0$. In our case, we focus on the propagation of extraordinary TM-polarized waves which are affected by both $\varepsilon_\parallel$ and $\varepsilon_\perp$[12], while the propagation of ordinary TE wave depends only on $\varepsilon_\parallel$.

When an eigen TM electromagnetic wave propagates in a PBG, it possesses a Bloch wave vector $\mathbf{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$ according to the Bloch theory, and k_z can be found explicitly for two-layered periodic structures by the trace of a transfer-matrix[11]:

$$\mathrm{Tr}(T) = 2\cos(k_z a) = \frac{1}{4}\left\{ \begin{array}{l} \left(2+\Omega+\Omega^{-1}\right)\left[\mathrm{e}^{\mathrm{i}(\beta d_2+\alpha d_1)} + \mathrm{e}^{-\mathrm{i}(\beta d_2+\alpha d_1)}\right] \\ +\left(2-\Omega-\Omega^{-1}\right)\left[\mathrm{e}^{\mathrm{i}(\beta d_2-\alpha d_1)} + \mathrm{e}^{-\mathrm{i}(\beta d_2-\alpha d_1)}\right] \end{array} \right\}, \quad (1)$$

where $\alpha = \sqrt{k_0^2 - k_x^2 - k_y^2}$, $\beta = \sqrt{\varepsilon_\parallel k_0^2 - \frac{\varepsilon_\parallel}{\varepsilon_\perp}\left(k_x^2 + k_y^2\right)}$, $\Omega = \frac{1}{\varepsilon_\parallel} \cdot \frac{\alpha}{\beta}$, and $a = d_1 + d_2$. It is easy to find that when $|\mathrm{Tr}(T)| > 2$, no propagating wave is allowed inside the system and then a PBG is generated. Because of the permittivity distribution of the LHM $\left(\varepsilon_\parallel > 0, \varepsilon_\perp < 0\right)$, β is always real but α can be real or imaginary. We define the cutoff frequency as $\omega_\mathrm{c} = c\sqrt{k_x^2 + k_y^2}$ (c is the light velocity in vacuum), and the whole frequency domain can be divided into two ranges: $\omega > \omega_\mathrm{c}$ and $\omega < \omega_\mathrm{c}$. In what follows, we will study the condition $|\mathrm{Tr}(T)| > 2$ in two separate cases to search for the requirements imposed on material and structure parameters.

The first case is $\omega > \omega_\mathrm{c}$. In this case, both α and β are real, and Eq. (1) becomes

$$\mathrm{Tr}(T) = 2\cos(\alpha d_1 + \beta d_2) - \left(\Omega + \Omega^{-1} - 2\right)\sin(\alpha d_1)\sin(\beta d_2). \quad (2)$$

We take into account the condition

$$\alpha d_1 + \beta d_2 = 2m\pi \quad (m = 1, 2, 3, \cdots). \quad (3)$$

Considering the fact that $\Omega + \Omega^{-1} > 2$, the condition $|\mathrm{Tr}(T)| > 2$ implies that $\sin(\alpha d_1)$ and $\sin(\beta d_2)$ are of opposite sign. Especially, when $d_1 = d_2 = d$, we derive $0 < \alpha d_1 < \pi < \beta d_2 < 2\pi$ (setting $m = 1$ and $\varepsilon_\parallel > 1$), and thus $\sin(\alpha d_1)$ is positive but $\sin(\beta d_2)$ is negative. While for the general case ($d_1 \neq d_2$), there must be an additional requirement (i.e., $\alpha d_1 \neq \beta d_2$).

To sum up, when α and β are both real, the conditions for realization of ANM gaps are Eq. (3) and

$$\sin(\alpha d_1)\sin(\beta d_2) < 0. \quad (4)$$

The second case is $\omega < \omega_\mathrm{c}$. In this case, α is imaginary but β is real. We suppose $\alpha = \mathrm{i}\gamma$($\gamma$ is positive real), then Eq. (1) becomes

$$\mathrm{Tr}(T) = -\left(\mathrm{e}^{-\gamma d_1} + \mathrm{e}^{\gamma d_1}\right) - 2\cosh(\gamma d_1)\left[\frac{1}{2}\cdot\left(\frac{\beta}{\varepsilon_\parallel \gamma} - \frac{\varepsilon_\parallel \gamma}{\beta}\right)\sin(\beta d_2)\tanh(\gamma d_1) - 2\cos^2\left(\frac{\beta d_2}{2}\right)\right]. \quad (5)$$

We assume $A = -\left(\mathrm{e}^{-\gamma d_1} + \mathrm{e}^{\gamma d_1}\right)$, $B = -2\cosh(\gamma d_1)$, $C = \frac{1}{2}\cdot\left(\frac{\beta}{\varepsilon_\parallel\gamma} - \frac{\varepsilon_\parallel\gamma}{\beta}\right)\sin(\beta d_2)\tanh(\gamma d_1)$, and $D = -2\cos^2(\beta d_2/2)$, Eq. (5) becomes $\mathrm{Tr}(T) = A + B(C + D)$.

As $A <$–2 and $B <$–2, if we can insure $C + D > 0$, $\mathrm{Tr}(T) < -2$ is satisfied. We assume that there is a special equilibrium position where $\beta_\mathrm{e} d_2 = \varepsilon_\parallel \gamma_\mathrm{e} d_2 = \pi$. According to the conclusion in Ref. [11] that C increases more quickly than $|D|$ when leaving the equilibrium position, $C + D > 0$ is derived. Then the requirement becomes to ensure $C > 0$. By examining the dispersion relation of γ and β, we can easily find that if $\left(k_x^2 + k_y^2\right)$ is increasing (decreasing), γ and β are larger (smaller) than γ_e and β_e, respectively, and correspondently, $\sin(\beta d_2) < 0 (> 0)$. Only if $\left|\frac{\varepsilon_\parallel}{\varepsilon_\perp}\right| < \varepsilon_\parallel^2$ is the absolute value of βvariation smaller than that of $\varepsilon_\parallel\gamma$ variation, so $\left(\frac{\beta}{\varepsilon_\parallel\gamma} - \frac{\varepsilon_\parallel\gamma}{\beta}\right) < 0 (> 0)$, and in this way C is positive.

Simple analysis shows that the equivalent position requires that

$$k_0^2 \cdot \frac{\varepsilon_\perp\left(\varepsilon_\parallel + 1\right)}{\varepsilon_\parallel\varepsilon_\perp + 1} = \frac{\left(\pi/d_2\right)^2 + \varepsilon_\parallel^2 k_0^2}{\varepsilon_\parallel^2}. \quad (6)$$

We emphasize that all the requirements discussed above are sufficient conditions to guarantee $|\mathrm{Tr}(T)| > 2$, but they are not necessary conditions. Nevertheless, they are still helpful to search for suitable parameters when PBG is needed.

We consider a stack with 16 unit cells to illustrate the ANM gap effects discussed in the first case, and discuss transmission property of the structure by 4×4 transfer-matrix method[13]. We choose the semiconductor LHM[14], which is an example of nonmagnetic metamaterial proposed in Ref. [12], as the LHM used in the periodic structure. The semiconductor LHM is created by a Princeton-led research team in 2007 as the first 3D metamaterial at infrared (IR) or optical frequencies constructed entirely from semiconductors, the principal ingredient of microchips and optoelectronics. To realize partially negative permittivity, the semiconductor LHM is fabricated with n$^+$-GaInAs/i-AlInAs heterostructures with appropriately thick layers that have alternating positive and negative dielectric constants. When the frequency is much higher than ω_p (the plasma frequency of the InG aAs layers), $\varepsilon_\perp$ is positive and approximately equals $\varepsilon_\parallel$. As the frequency decreases and approaches ω_p, $\varepsilon_\perp$ decreases and eventually becomes negative. Then partially negative permittivity is realized in the material. Otherwise, by choosing proper thickness of the layers, the quantization of the energy levels in the system is irrelevant and the effective medium approximation is applicable. In this way, the multilayer composite material can be regarded as a homogeneous structure, and the effective permittivity components of $\varepsilon_\perp$ and $\varepsilon_\parallel$ for the layered system are related to the relative permittivity of AlInAs (ε_1) and InGaAs (ε_2) as[15]

$$\varepsilon_\perp = \frac{2\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2}, \quad \varepsilon_\parallel = \frac{\varepsilon_1 + \varepsilon_2}{2}, \quad (7)$$

where $\varepsilon_1 = 10.23$ and $\varepsilon_2 = 12.15(1 - \omega_\mathrm{p}^2/\omega^2)$. The dispersion curve of semiconductor LHM with the above parameters is shown in Fig. 2.

The dispersion relationship of the PBG structure with 16 unit cells of air and semiconductor metamaterial with the above parameters and its corresponding transmittance spectra are shown in Figs. 3 and 4. As an example, we find the set of parameters $\omega_\mathrm{p} = 2 \times 10^{14}$ rad/s and $k_x = k_y = 0.26$ rad/μm. In Fig. 3, we consider three cases of $d_1/d_2 = 0.5$ including $d_1 = 4$ μm and $d_2 = 8$ μm (solid line) and they are scaled by 3/4 (dashed line)

Fig. 1. Schematic of the PBG structure with a double-layer unit cell of an air layer and an anisotropic LHM layer.

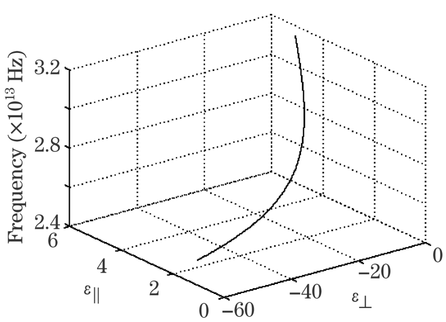

Fig. 2. Dispersion curve of semiconductor metamaterial with parameters shown in Eq. (7).

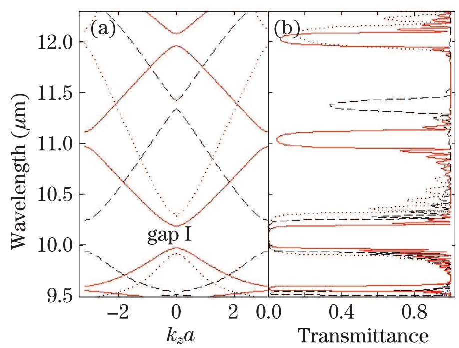

Fig. 3. (a) Dispersion relationship of a PBG structure with 16 unit cells of air and semiconductor metamaterial with parameters shown in Eq. (7). Solid line: $d_1 = 4$ μm and $d_2 = 8$ μm; dashed line: d_1 and d_2 are scaled by 3/4; dotted line: d_1 and d_2 are scaled by 1/2. (b) Transmittance spectra through the 16 unit cells corresponding to the band gaps in (a).

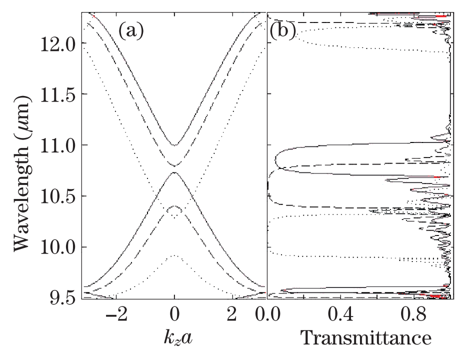

Fig. 4. (a) Dispersion relationship of a PBG structure with 16 unit cells of air and semiconductor metamaterial with parameters shown in Eq. (7). Solid line: $d_1 = 5$ μm and $d_2 = 4$ μm; dashed line: $d_1 = d_2 = 4$ μm; dotted line: $d_1 = 2$ μm and $d_2 = 4$ μm. (b) Transmittance spectra through the 16 unit cells corresponding to the band gaps in (a).

and 1/2 (dotted line), respectively. We find that as d_1 and d_2 decreases, the width of gap I (one of the ANM gaps) enlarges, but the middle of each gap is almost unchanged. This property distinguishes the gap I from a Bragg gap, because the middle of the Bragg gap will shift noticeably while the width of the scale changes a little. It also shows that the width of gap I can be controlled by scaling while zero-Φ_{eff} gap never changes with the same d_1/d_2 ratio. In Fig. 4, we consider three cases of $d_1 = 5$ μm, $d_1 = 4$ μm, and $d_1 = 2$ μm when $d_2 = 4$ μm. We find that as the d_1/d_2 ratio decreases, the width of

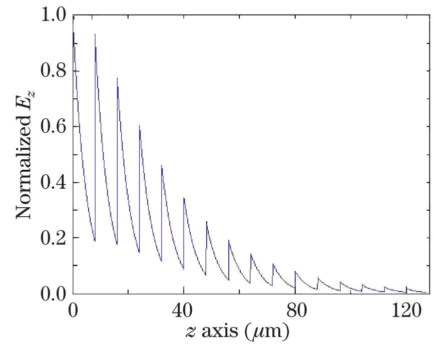

Fig. 5. Distribution of E_z inside the PBG structure with 16 unit cells of air and semiconductor metamaterial with parameters as the dashed line in Fig. 4(a) and $\lambda = 10.5$ μm.

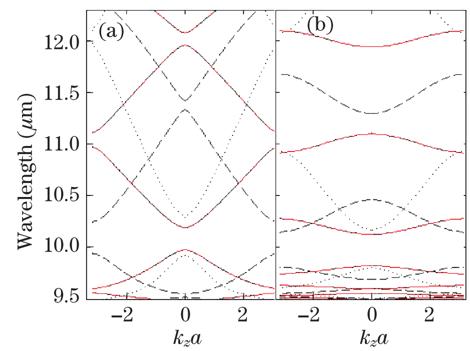

Fig. 6. Dispersion relationship of a PBG structure with 16 unit cells of air and semiconductor metamaterial with parameters shown in Eq. (7). (a) α and β are both real and with the same parameters as those in Fig. 3(a); (b) α is imaginary but β is real. Solid line: $d_1 = 5$ μm and $d_2 = 4$ μm; dashed line: $d_1 = d_2 = 4$ μm; dotted line: $d_1 = 2$ μm and $d_2 = 4$ μm.

ANM gap enlarges and the middle frequency of the gap has upward shifts. In addition, the distribution of E_z inside the PBG structure with parameters as the dashed line in Fig. 4(a) is shown in Fig. 5, where we choose $\lambda = 10.5$ μm which is at the middle of the ANM gap. Obviously, the evanescent envelope of E_z is shown to be coincident with the forbidden gaps.

We can easily identify that the above ANM gaps along the periodic direction are induced by the interference of propagating waves, and it is an intrinsic consequence of periodicity in which PBG frequency range is tied with the thickness of every layer in a unit cell. Then in Fig. 6(b), we show the ANM gaps when α is imaginary, which is a result of the interaction of evanescent waves and propagating waves. Given the same parameters in Fig. 3, we find that evanescent modes help to enlarge the ANM gaps noticeably but never change their intrinsic properties by simple comparison with Fig. 3(a) (we represent it in Fig. 6(a)).

In conclusion, we propose and demonstrate that 1D periodic structures created by a double-layer unit cell with an air layer and an anisotropic nonmagnetic LHM layer whose permittivity elements are partially negative possess an ANM gap, which is different from the traditional Bragg gap or zero-Φ_{eff} gap. Not only can the designed structure generate more than one band gaps within a same range of frequency, which is obviously superior to only one PBG in Ref. [11] when multiple band gaps are needed, but also it can achieve PBG without magnetic resonance which is necessary in common PBG with LHM. It can also be used as a filter or a polarizer because it is just effective with TM polarized waves in contrast to ordinary PBG structures[16−18]. The LHM with partially negative permittivity elements brings us a new opportunity to realize PBG at special frequencies or for different polarized waves.

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