

# Predistortion of high speed optical OFDM signal for aliasing-free receiving in multiple low-bandwidth receiver system

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A cost-effective method of multi-receiver optical orthogonal frequency-division multiplexing (OOFDM) system is proposed to reduce the requirement for the speed of the receiver and the size of fast Fourier transform (FFT). The sampling rate of analog-to-digital converters (ADCs) can be reduced to  $1/N$  of the original signal bandwidth in an  $N$ -receiver system. Aided by signal predistortion at the transmitter, aliasing-free signal can be retrieved independently and directly at the low speed receivers. A back-to-back experimental result is given for a two-receiver system. The effect of the electrical filters added before the ADC is studied and the analysis for filters optimization is given.

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In comparison with the orthogonal frequency-division multiplexing (OFDM) in radio communication, optical OFDM (OOFDM) can be applied in a much wider band which makes it an excellent candidate for broadband optical transmission because of its superior performance attributed to its optical signal-to-noise ratio (OSNR) requirement, dispersion tolerance, and spectral efficiency<sup>[1]</sup>. With the bit rate growing to 10–100 Gb/s or beyond, however, the electrical part has become a limit for the whole system, in which the analog-to-digital (AD) and digital-to-analog (DA) devices are the main barrier to improve the speed. Some methods to combat the DA speed limit have been reported<sup>[2–4]</sup>, but rare work is reported regarding to analog-to-digital converter (ADC) solutions. A common method to gain a high speed ADC is to ensemble multiple low speed ADC chips with sampling time interleaved<sup>[5]</sup>, in which synchronization and precise timing control are required strictly<sup>[6]</sup>. Here, we present a novel method that cooperatively utilizes multiple receivers of ADC and fast Fourier transform (FFT) units with low speed and small size to obtain broadband signal. Different from the time-interleaved method, the ADCs cooperate in frequency domain. This can take the advantage of OFDM signal — the direct expression of information in frequency domain and the discrete spectrum make a convenient possibility to achieve any digital signal processing (DSP) in frequency domain. The crucial processing is the signal predistortion at transmitter according to the different transfer characteristics of the receivers, so that without any succeeding processing, each receiver can receive one section of the original sequence independently, which is suitable for multiple-access application.

When the sampling rate of ADC at the receiver is set to  $1/N$  of the bandwidth of transmitted OFDM signals, the aliasing of received signal spectrum will happen according to the Nyquist sampling theorem. The aliasing mixes the different parts of the spectrum by linear super-

position. Denote the subcarriers of an OFDM symbol as  $S(K)$  with a length of  $n_c$ , where  $1 \leq K \leq n_c$  is the index of the subcarriers. They cover a spectrum bandwidth BW. The subcarriers at the receiver after the sampling with a low sampling rate of  $BW/N$  will be the linear superposition of  $S(K)$  superposed into a length of  $n_c/N$ . Denote it as  $S'(k)$  ( $1 \leq k \leq n_c/N$ ),

$$S'(k) = \sum_{m=0}^{N-1} S(k + mn_c/N), \quad 1 \leq k \leq n_c/N. \quad (1)$$

In our method, as shown in Fig. 1, there are  $N$  such low speed receivers for receiving. The original subcarriers  $S(K)$  are predistorted into  $S_{pr}(K)$  in certain form before transmission. And before each receiver, the predistorted subcarriers  $S_{pr}(K)$  are multiplied by a unique weighting function  $A_i(K)$  ( $1 \leq K \leq n_c$ ,  $1 \leq i \leq N$ ) before the low speed sampling. Then, after the linear superposition of spectrum by aliasing, the final received subcarriers  $S'_i(k)$  ( $1 \leq k \leq n_c/N$ ) at receiver  $i$  ( $1 \leq i \leq N$ ) can be just a section of the original subcarriers  $S(K)$ :

$$\begin{aligned} S'_i(k) &= \sum_{m=0}^{N-1} A_i(k + mn_c/N) S_{pr}(k + mn_c/N) \\ &= S[k + (i-1)n_c/N], \quad 1 \leq k \leq n_c/N. \end{aligned} \quad (2)$$

Then, to know how to implement the predistortion, we get an equation set with  $N$  independent equations corresponding to the  $N$  different receivers. The weighting function  $A_i(K)$  ( $1 \leq K \leq n_c$ ,  $1 \leq i \leq N$ ) are defined before, and  $S'_i(k)$  ( $1 \leq k \leq n_c/N$ ,  $1 \leq i \leq N$ ) are known as the received OFDM subcarriers we expect the receivers to recover, which are sections of the original subcarriers  $S(K)$ . Thus, the predistorted OFDM subcarriers  $S_{pr}(K)$  ( $1 \leq K \leq n_c$ ) to be sent can be computed by solving the equation set above. Then, the entire data sequence can be transported to one receiver for one section.

As a proof-of-concept experiment, we demonstrate the transmitter and receiver ends of the 2-receiver OOFDM system in Fig. 2. The predistorted OFDM signal with 250-MHz bandwidth is generated by an Agilent N8241A and sampled by two ADCs (Agilent DSA80000 at 4-GS/s sampling rate and then downsampling in MATLAB) both operating at 125 MS/s, and the FFT size at receivers is 128 in comparison with that of 256 at the transmitter. Different electrical filters are placed in front of the ADCs supplying distinctive channel characteristics for receivers. Here we use two 2-GHz bandwidth lowpass

filters with different phase characteristics. The option of the types of filters will be discussed later. Before the transmission, we test  $H_1(\omega_K)$  and  $H_2(\omega_K)$  as the characteristics of the channels from transmitters to receivers 1 and 2, respectively, at frequency points of subcarriers, as shown in Figs. 3(a) and (b).  $H_i(\omega_K)$  processes the incoming signal just like the weighting functions  $A_i(K)$  does.

Given the original data  $S(K)$  ( $1 \leq K \leq n_c$ ) (Fig. 4(a)), the predistorted signal  $S_{pr}(K)$  ( $1 \leq K \leq n_c$ ) to be launched is subjected to

$$\begin{cases} S'_1(k) = S_{pr}(k)H_1(\omega_k) + S_{pr}(k + n_c/2)H_1(\omega_{k+n_c/2}) = S(k), \\ S'_2(k) = S_{pr}(k)H_2(\omega_k) + S_{pr}(k + n_c/2)H_2(\omega_{k+n_c/2}) = S(k + n_c/2), \end{cases} \quad 1 \leq k \leq n_c/2, \quad (3)$$

where the number of subcarriers  $n_c$  is 256 in our experiment. The predistorted signal  $S_{pr}(K)$  ( $1 \leq K \leq n_c$ ) is computed (Fig. 4(b)) by solving Eq. (3) and loaded to the inverse FFT (IFFT) inputs. Before the receivers, the detected electrical signal is firstly filtered by different electrical filters to generate the subcarriers  $S_{pr}(K)H_1(\omega_K)$  (Fig. 4(c)) and  $S_{pr}(K)H_2(\omega_K)$  ( $1 \leq K \leq n_c$ ). The aliasing induced by the low sampling rate of the ADCs cancels the predistortion process and the filtering process by overlaying the subcarriers into  $S(k)$  ( $1 \leq k \leq n_c/2$ ) at receiver 1

(Fig. 4(d)) and  $S(k + n_c/2)$  ( $1 \leq k \leq n_c/2$ ) at receiver 2, so that the upper and lower sections of the original subcarriers are retrieved at the FFT outputs directly. A combination is optional to assemble them. In the experiment, parameters as follows are employed; transmitter FFT size 256, cycle prefix 1/16, bit rate 500-Mb/s, encoding with four quadrature amplitude modulators (QAMs).

The solved constellation of the upper subcarriers, i.e.,  $S(k)$  ( $1 \leq k \leq n_c/2$ ) at receiver 1, is shown in Fig. 5(b). The constellation for conventionally receiving the same amount of data points by full bandwidth sampling without multiple receivers is given in Fig. 5(a) for comparison. The well-remained structures of the constellations and error-free bits of 2-Mb data demonstrate that the proposed scheme is feasible and effective.

This method can be extended to more receivers to reduce more bandwidth. In an  $N$ -receiver system, in the DSP at transmitter, for each OFDM signal, the solving of the equation set can be written as a matrix equation:

$$\mathbf{A}\mathbf{S}_{pr} = \mathbf{S}, \quad (4)$$

$$\mathbf{A}^{i,j} = \begin{cases} H_{\lceil iN/n_c \rceil}(\omega_j), & i = j \text{ mod } n_c/N \\ 0, & i \neq j \text{ mod } n_c/N \end{cases}, \quad (5)$$

where  $\mathbf{S}_{pr}$  and  $\mathbf{S}$  are  $n_c \times 1$  vectors whose elements are  $S_{pr}(K)$  and  $S(K)$ , respectively;  $\mathbf{A}$  is a fixed  $n_c \times n_c$  matrix containing the information of channel characteristics. The elements  $H_i(\omega_K)$  ( $1 \leq K \leq n_c$ ) in  $\mathbf{A}$  describe the frequency characteristic of the entire channel from the transmitter to receiver  $i$  including the filter before the receiver. Before the transmission,  $\mathbf{A}$  is obtained by sending test symbols and a similar calculation as Eq. (4), and then saved in the DSP module at the transmitter. As its elements described in Eq. (5), matrix  $\mathbf{A}$  can be divided into  $N \times N$  blocks and block-diagonal. The inverse of matrix  $\mathbf{A}$  is needed when computing  $\mathbf{S}_{pr} = \mathbf{A}^{-1}\mathbf{S}$ . Attributed to the above properties of the matrix, its inverse retains block-diagonal and easy to be calculated. Therefore,  $N$  times of complex multiplications and  $N-1$  complex additions are calculated for every QAM symbol, hence  $n_c \times N$  complex multiplications and  $n_c \times (N-1)$  complex additions are required for every OFDM symbol. This DSP cost is negligible because of  $N$  being a small integer and the significant speed of modern DSP devices.

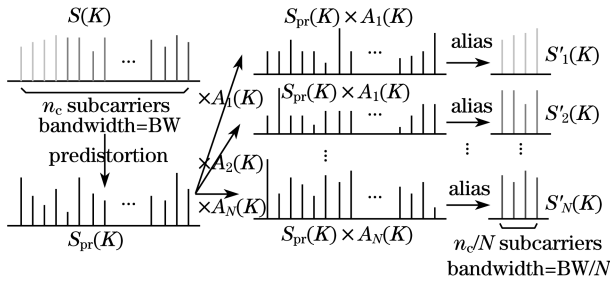


Fig. 1. Subcarrier evolution in an  $N$ -receiver OOFDM system.

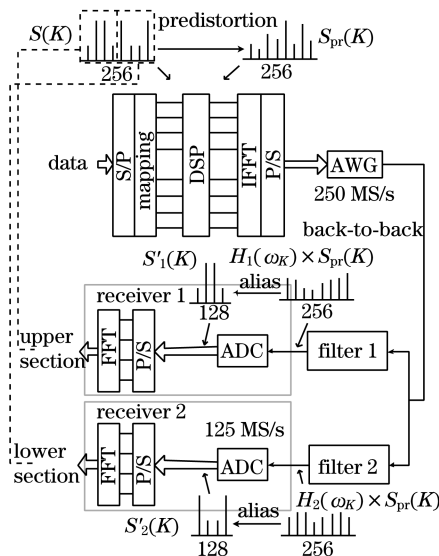


Fig. 2. Transmitter and receiver ends of a 2-receiver OOFDM system. S/P and P/S represent serial/parallel and parallel/serial conversion, respectively.

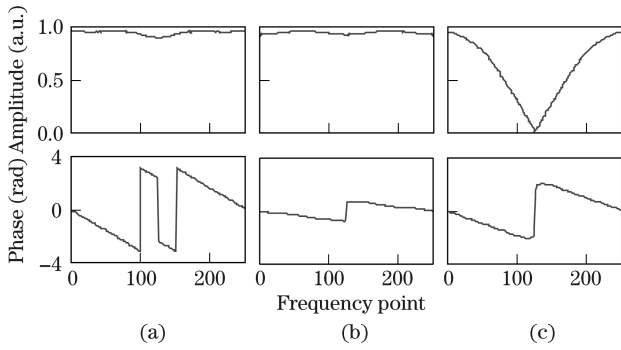


Fig. 3. Frequency characteristics of filters at 256 digital frequency points.

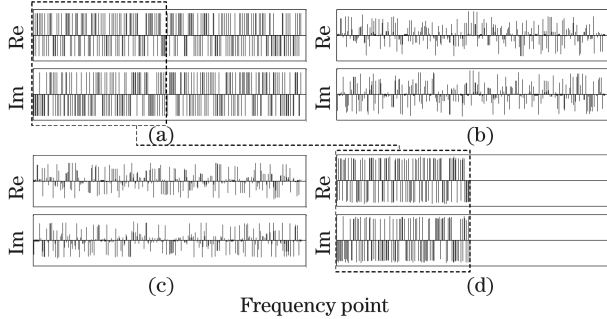


Fig. 4. Real and imaginary parts of subcarriers of (a) original OFDM symbol, predistorted symbols (b) before and (c) after transmission and filtering, and (d) the directly recovered upper section of subcarriers by the low speed sampling.

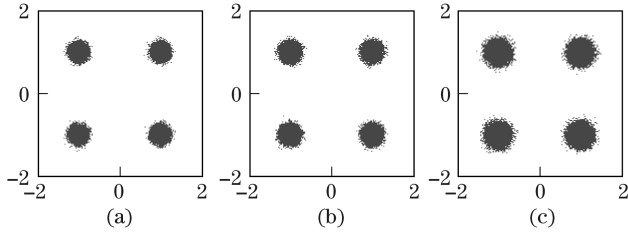


Fig. 5. (a) Received constellations of conventional single receiver with sampling rate of 250 MS/s and 256 FFT, and (b), (c) upper data received by one receiver in 2-receiver systems with sampling rate of 125 MS/s and 128 FFT, installed with different filter groups.

This new receiving method is cost-efficient thanks to the least requirement of the filters placed before the receivers. We indicate that the choice of filters is arbitrary. The only mathematical demand of these filters is  $\det(\mathbf{A}) \neq 0$  to guarantee the independence of the equation set. However, a small condition number of matrix  $\mathbf{A}$  can provide a superior tolerance toward the noise added on the predistorted subcarriers and a satisfying result of the original signal recovery. Therefore, a group of filters that make the  $2N$ -dimensional planes described by the normalized equation set orthogonal are preferred. For example, in the 2-receiver system, the pair of filters with flat amplitude-frequency characteristics and subjected to

$$H_1(\omega_K)H_1^*(\omega_{K+n_c/2}) + H_2(\omega_K)H_2^*(\omega_{K+n_c/2}) = 0, \quad (6)$$

$$1 \leq K \leq n_c/2,$$

can provide a normalization and an orthogonality, and thus minimize the condition number of matrix  $\mathbf{A}$  to 1.

For an  $N$ -receiver system, the orthogonality term is

$$\sum_{m=1}^N [\mathbf{H}_K^{\text{ac}}(m, i)]^* \mathbf{H}_K^{\text{ac}}(m, j) = 0 \quad \forall 1 \leq i, j \leq N, \quad (7)$$

where  $\mathbf{H}_K^{\text{ac}}(m, i)$  stands for the algebraic cofactor of element  $\mathbf{H}_K^{m, i}$  in matrix  $\mathbf{H}$ . And the elements in matrix  $\mathbf{H}_K$  can be expressed as

$$\mathbf{H}_K^{i, j} = H_i(\omega_{(j-1)n_c/N+K}), \quad 1 \leq K \leq n_c/N, \quad (8)$$

whose rows contain same-frequency interfering points of the filters during the aliasing.

Here we illustrate the effect on the constellations from noise with constant OSNR of different filter groups. The filter group described in Figs. 3(a) and (b) used in the experiment above brings matrix  $\mathbf{A}$  a condition number of 1.06, while the group described in Figs. 3(a) and (c) brings a condition number of 2.74. The constellation received by the receiver after the filter given in Fig. 3(a) of the latter group is shown in Fig. 5(c), and a deterioration can be observed in comparison with the former one's in Fig. 5(b). Attributed to the approximation to Eq. (6), the former group performs a better tolerance to noise.

Attributed to the linearities of the predistortion and the evolution of the subcarriers in this method, the conventional operations to compensate various linear impairments in optical communications such as chromatic dispersion<sup>[7,8]</sup>, common phase noise<sup>[9]</sup>, etc., can be retained. Some compensations against time-invariant impairments can be merged into matrix  $\mathbf{A}$  to save the DSP consuming. Since additive noises during the transmission are amplified by  $\mathbf{A}$ , the option of filters subjected to Eqs. (6) and (7) can minimize the errors. The effect from nonlinearity<sup>[10]</sup> is transferred among the subcarriers but not deteriorated.

In conclusion, we propose a method for high bit rate OOFDM receiving. The cooperation of the low bandwidth receivers is cost-effective to reduce the bandwidth requirement. Aided with a deliberate predistortion according to the different channel characteristics before the receivers, the aliasing free signal can be retrieved directly at the receivers. A back-to-back experiment shows that the OFDM signal with bandwidth of 250 MHz can be retrieved by a 2-receiver system with ADCs of sampling rate of 125 MS/s. And the FFT size is also halved from 256 to 128. An optimal group of filters can also minimize the noise effect of the aliasing recovery. The independence and disconnection of the receivers offer the possibility of receiving and processing data simultaneously and parallelly. This receiving method could give a way to break the bottleneck of high speed OOFDM receiving, and its property is also suitable for constructing a multiple-access OOFDM system by dividing the IFFT channels at the transmitter.

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