Design of dispersive multilayer with particle swarm optimization method

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We present a new and efficient method for the design of dispersive multilayer by employing a particle swarm optimization (PSO) technique. Its mathematical background is given and an adaptive PSO is realized with computer code. Two practical designing tasks are solved with this method, and the obtained results are competitive compared with other published structures. The adaptive PSO method demonstrates its merits of fast convergence and powerful global search ability, and could be used as a valuable tool for the optical thin film design.

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In ultrashort laser technology, the main limitation of short pulse generation is the pulse broadened effect caused by material dispersion. Multilayer dispersive mirrors, especially the chirped mirror (CM)[1], could offer precise dispersion compensation over a certain spectral range, and thus become a key element for the ultrashort laser system.

Since CM was proposed in 1994[1], tremendous progress has been achieved, and its developing trends in recent years are mainly focused on two distinct directions: one is to develop the high-dispersion mirror (HDM) to replace the prisms and gratings in conventional chirped pulse amplification (CPA) systems with the added benefit of providing high-order dispersion control[2], which requires a significant large group delay dispersion (GDD) compensation amount (>1000 fs^2) within a narrow wavelength band (20–30 nm); the other is to design the broadband-chirped mirror (BCM) to control dispersion over an octave (e.g., 600–1200 nm) with reduced GDD ripples[3]. Such a BCM is extremely important for the ultrashort laser systems[4].

Although the optimization of common thin film filters has been well developed, design of the dispersive mirror is still a great challenge. Needle optimization and gradual evolution[5] have been proved successful in solving this problem, but it is too difficult for a common thin film researcher to understand and realize them in computer code. Other global optimization techniques such as genetic algorithm (GA) and stimulated annealing (SA)[6] are also introduced. However, their refinement results are both strongly related to some pre-defined parameters. Particle swarm optimization (PSO) method, developed by Kennedy *et al.*[7−9], has recently been utilized in design of long-period grating and guided mode resonant filter[10], and demonstrates its advantage over other algorithms for fast convergence speed and less dependence on the initial parameters. To our knowledge, PSO has been rarely used in optimization of multilayer thin film stack.

In this letter, an introduction to the basis of PSO and its mathematical realization is given. Then, two practical design problems are solved by PSO to test its global search ability and convergence speed. This novel technique exhibits great potential application for the design of dispersive multilayer.

PSO technique roots in the social behavior of large number of birds or fish, with a simple but effective working schedule. Similar to other evolutionary computation, there are a number of particles in the swarm. Each represents a potential solution and is considered as a point in an N-dimensional search space, represented by an N-parameter vector given by $\mathbf{X}_m = \{\mathbf{X}_{1m}, \mathbf{X}_{2m}, \cdots, \mathbf{X}_{Nm}\}$. The excellence of each point is determined by the fitness function value at this position. For every iteration, each particle would adjust its movement according to its own experience as well as the experiences of other particles. The modification of the mth particle's position in the kth iteration can be modeled as

$$\begin{aligned}\mathbf{V}_m^{k+1} &= \phi\mathbf{V}_m^k \\ &\quad + \alpha_1\mathrm{rand}_1()(\mathbf{P}_m - \mathbf{X}_m^k) + \alpha_2\mathrm{rand}_2()(\mathbf{G} - \mathbf{X}_m^k), \\ \mathbf{X}_m^{k+1} &= \mathbf{X}_m^k + \mathbf{V}_m^k, \end{aligned} \tag{1}$$

where ϕ indicates the inertia weight function, α_1 and α_2 are acceleration functions for cognitive and social rates, respectively, and rand() means random operation. These two rates control the relative influence of the memory of the neighborhood to the memory of the individual. $\mathbf{P}_m$ and $\mathbf{G}$ represent the best position of the mth particle itself and the total swarm, respectively, and both would be altered when a better position appears. From this model, it is clear that the new velocity has memory of previous velocity, and would be adjusted in each iteration according to both its own best position and the best position of the total swarm. The underlying rules of cooperation and competition within social swarms give it good capability for global optimization with the help of memory rather than a simple random search. Therefore, it has a large possibility to fly into a better solution with a faster speed and finally generate better results.

Improvement of the original PSO could be made in two distinct aspects. The first one is to place constrains on the search area ($\mathbf{X}_{\max}$ and $\mathbf{X}_{\min}$) as well as the maximum velocity $\mathbf{V}_{\max}$. By this way, the generation of illogical solution could be eliminated. At the first sight, this can be simply done by adding boundaries for the search space. However, this method may cause some undesirable explosive feedback effects[7]. Instead, we choose a better modification provided by Carlisle *et al.*[11]: deriving a constriction factor K to avoid the velocity exceeding $\mathbf{V}_{\max}$, computed as

$$K = \frac{2}{|2-\gamma-\sqrt{\gamma^2-4\gamma}|}, \quad (2)$$

where $\gamma = \alpha_1+\alpha_2$.

The second aspect is to tune global and local searching capability throughout this refinement process. This can be achieved by altering the inertia weight in each iteration following $\phi^k = 0.9 - 0.5\ k/N$, where N is the maximal times of iterations. From the formula, ϕ^k would decrease linearly from 0.9 to 0.4 throughout a run. Since a bigger inertia weight means stronger global searching ability, the refinement would mainly perform the global search at the beginning of the iterations, and then gradually take more consideration on finding the local optimum. This can guarantee good convergence and maintain considerable global search ability.

We introduce an adaptive PSO (APSO) technique with a new formula of velocity:

$$\begin{aligned}\mathbf{V}_m^{k+1} = & K[\phi^k \mathbf{V}_m^k + \alpha_1 \text{rand}_1()(\mathbf{P}_m - \mathbf{X}_m^k) \\ & + \alpha_2 \text{rand}_2()(\mathbf{G} - \mathbf{X}_m^k)]. \end{aligned} \quad (3)$$

Its learning factor would be self-modified in each iteration. In what follows, we would implement the PSO and APSO to design two types of dispersive mirror and compare their refinement abilities.

We designed a HDM to meet a specific target very similar to Ref. [2]. That is, a reflective mirror with nearly constant amount of GDD = –2400 fs^2 between 1020–1040 nm, while maintaining a reflectivity as high as possible. The merit function is defined as

$$\begin{aligned} M = & \sum_\lambda W_{\mathrm{R}}(\lambda)\left[\frac{R(\lambda)-R_{\text{target}}(\lambda)}{R(\lambda)}\right]^4 \\ & + \sum_\lambda W_{\mathrm{GDD}}(\lambda)\left[\frac{\mathrm{GDD}(\lambda)-\mathrm{GDD}_{\text{target}}(\lambda)}{\mathrm{GDD}(\lambda)}\right]^2, \end{aligned} \quad (4)$$

where λ is the wavelength, R and R_{target} are the designed and desired reflectivities, and W_{R} and W_{GDD} are the weighting functions. A multi-conjugate cavity structure $(0.94\mathrm{H}0.94\mathrm{L})^{15}(\mathrm{HL})^5(0.5\mathrm{HL}0.5\mathrm{H})^2 2\mathrm{H}(0.5\mathrm{LH}0.5\mathrm{L})^2$ is chosen as the initial design[12], where H and L represent quarter-wavelength Ta_2O_5 (n = 2.1) and SiO_2(n = 1.431) at 1040 nm, respectively.

For comparison, we implement a common PSO with constant learning factor $\{\phi, \alpha_1, \alpha_2\} = \{0.6, 1.7, 1.7\}$, and the APSO, both with particle number of 30 and iteration number of 800. Before the starting, a simplex optimization method is employed to find a local optimum solution, and the initial seeds are randomly generated around this solution. Figure 1 shows the variation of merit function during the iteration procedure for PSO and APSO. It is clear that both types of PSO could jump out the local minimum found by simplex and approach to a much better solution. However, the decrease of the merit function for APSO is monotonic, while that for the common PSO is oscillatory and much slower. Moreover, the APSO could find a good solution in less than 250 iterations, and generate a much smaller merit function value than that of the normal PSO. This proves that APSO outperforms normal PSO with better search ability and faster convergence. The GDD and reflectivity characteristics for the results obtained with the two methods are displayed in Fig. 2 for comparison. It could be observed that HDM designed by APSO has significantly less GDD oscillations than the one designed by PSO. For the foregoing refinement process, the weighting function is constant. By carefully changing the weight factors for different wavelengths, and taking the design results generated in the previous refinement as the initial guess for the new optimization process, a much better result can be achieved. As shown in Fig. 3, it meets the GDD target well with very small deviations, and with a reflectivity higher than 99.8%. The final structure has a total physical thickness of 7.14 μm. Both the physical thickness and the number of layers of the structure are less than the design result

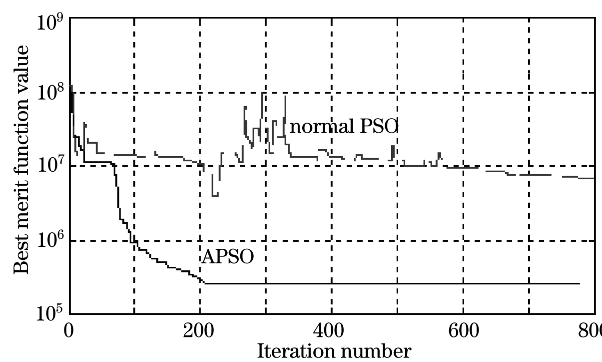

Fig. 1. Merit function value of the best particle versus iteration number.

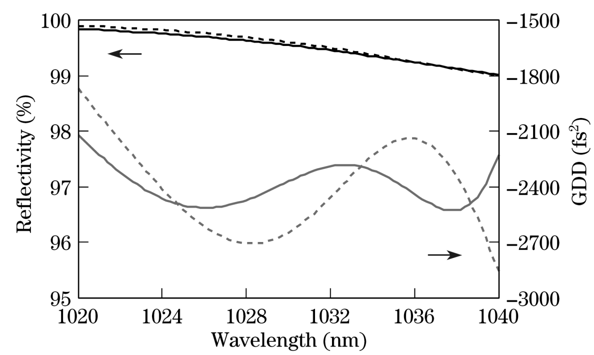

Fig. 2. Reflectivity and GDD characteristics for the design results. Solid: designed by APSO; dashed: designed by normal PSO.

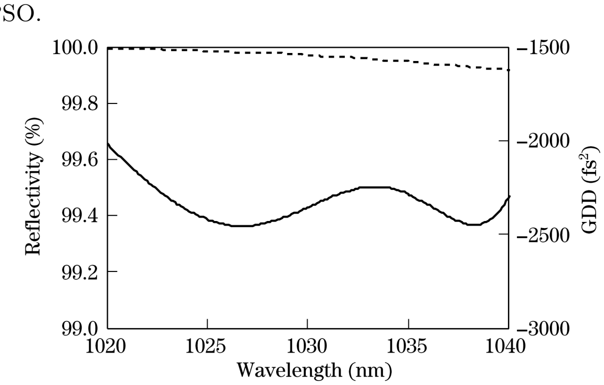

Fig. 3. Reflectivity and GDD characteristics for the optimized structure.

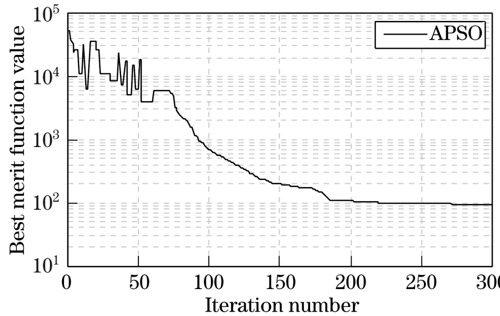

Fig. 4. Merit function value of the best particle versus iteration number.

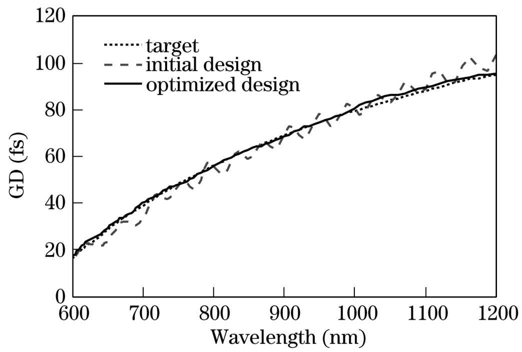

Fig. 5. Comparison of the GD curves of the target, initial design, and the optimized results.

presented in Ref. [2].

The second example is to design a BCM which could compensate dispersion over a large spectrum (~1 octave). The main challenge for this design is to eliminate unwanted GDD ripples. Steinmeyer invented the Brewster-angled CMs[13] as one effective approach, and we would apply his method to set our initial structure. The design target is set to offer –50 fs^2 GDD compensation in the wavelength range of 600–1200 nm for p-polarized light incident at the Brewster angle of 55.22°.

For the refinement of the BCM, it is more effective to set the group delay (GD) properties as the design target firstly, and then to optimize the GDD characteristics. Since the variable number in this task is relatively large (a layer number of 90), the normal PSO fails to generate reasonable results. We would apply APSO for this task only, with a particle number of 180. The variation of merit function is plotted in Fig. 4. The trace for APSO experiences some oscillation when performing the global search at the beginning of iterations, and then decreases monotonically to the optimum solution. Figure 5 shows the GD curves. The initial design suffers from the GD ripple as a result of impedance mismatch. After the optimization with our APSO algorithm, those ripples are significantly eliminated. With the obtained GD curves, the next step is to refine the structure directly for the GDD properties. It can be seen from Fig. 6 that the magnitude of GDD oscillation is also inhibited after this refinement, and the GDD, for most wavelength ranges are centered around –50 fs^2 with a residual ripple less than 15 fs^2. Compared with the structure published in Ref. [13], our design exhibits similar GDD and reflectivity properties, but with reduced layer number (90 layers) and total physical thickness (11.8 μm).

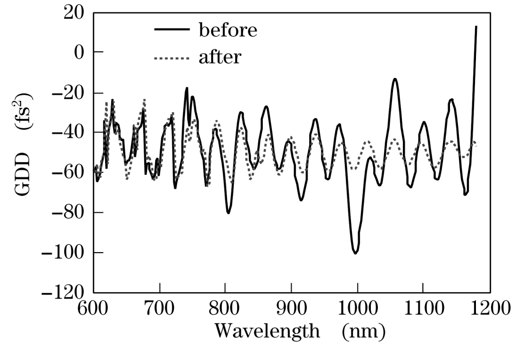

Fig. 6. GDD curves before and after GDD optimization.

In conclusion, we have introduced a novel type of PSO for dispersive multilayer design. A HDM mirror and a Brewster-angled BCM are designed by this method and the obtained results are competitive compared with those published. We demonstrate that the APSO outperforms the normal PSO for its fast convergence and powerful global search ability, and could be used as a valuable tool for optical thin film design. Moreover, much more potential improvements of the PSO method are in prospect. For example, we can combine it with the robust design technique[14] to lower the structure's sensitivity to manufacturing errors, or use it simultaneously with time domain optimization method[15] for achieving better results. Works towards these two aspects are still in process.

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