# Simulation of stabilizing process of dielectric nanoparticle in optical trap using counter-propagating pulsed laser beams 

Ho Quang Quy ${ }^{1 *}$, Mai Van Luu ${ }^{2}$, Hoang Dinh Hai ${ }^{3}$, and Donan Zhuang ${ }^{4}$<br>${ }^{1}$ Institute of Applied Physics, NEWTECHPRO, Vietnam Academy of Science and Technology, 8 Hoang Quoc Viet, Hanoi, Vietnam<br>${ }^{2}$ Faculty of Physics, Vinh University, Vinh, Vietnam<br>${ }^{3}$ Pedagogical College of Nghean, Nghe An, Vietnam<br>${ }^{4}$ Department of Physics, King Mingkut's University of Technology Thonburi, Bangkok, Thailand<br>*E-mail: hoquy1253@yahoo.com<br>Received July 1, 2009


#### Abstract

The stabilizing process of glass particle in water by optical trap using the pulsed counter-propagating Gaussian beams is investigated. The influence of the optical power and the particle dimension on the rate and time of the stabilizing process is simulated and discussed.

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In 1970, Ashkin demonstrated the optical trapping of particles using the radiation force produced by focused continuous-wave (CW) Gaussian beam ${ }^{[1]}$. Since then, the optical traps and tweezers have been the powerful tools for manipulating dielectric particles ${ }^{[2,3]}$. Usually, optical traps or tweezers in many experiments are conducted by using CW laser. It is well known that the CW laser with the power of a few milliwatts can only produce the radiation force with an order of a few piconewtons to manipulate the micro-sized particles. Recently, Ambardekar et al. used a pulsed laser to generate the large gradient force, up to 2500 pans within a short duration of several picoseconds ${ }^{[4-7]}$. Up to now, we have paid attention to optical traps using pulsed Gaussian beam (PGB) ${ }^{[7]}$ and counter-propagating pulsed PGBs ${ }^{[8]}$. In Refs. [811], the discussions about stability of the optical traps and tweezers as well as the effective control of dielectric particles like gold nanoparticles and live membrane, have been conducted taking into account of the Brown-
ian force. But the stabilizing process during the pulsing of the optical beam and the absolutely stable conditions of dielectric particles are not clear. Therefore, it is desired to advance the studies of the above questions for the pulsing optical trap.

In this letter, we introduce the gradient optical force acting on dielectric nanoparticles in the optical trap using two counter-propagating PGBs and the set of Langevin equations concerning thermal fluctuations of the probe. A simulation method is presented and used to present the radial variances of glass nanoparticles in water, which are trapped by picosecond PGB.

A PGB optical trap is considered to trap fluctuating dielectric nanoparticles in a water plate (Fig. 1). The gradient optical force is induced by two counter-propagating PGBs acting on a Rayleigh dielectric particle. The polarization direction of the electric field is assumed to be along the $x$ axis.

The expression for the electric field of the above PGB is defined by ${ }^{[6]}$

$$
\begin{align*}
\boldsymbol{E}_{\text {above }}(\rho, z, t)= & \mathbf{x} E_{0} \frac{\mathrm{i} k W_{0}^{2}}{\mathrm{i} k W_{0}^{2}+2 z} \exp \left\{-\mathrm{i}\left[k(z)-\omega_{0} t\right]\right\} \times \exp \left[-\mathrm{i} \frac{2 k z \rho^{2}}{\left(k W_{0}^{2}\right)+2 z^{2}}\right] \\
& \times \exp \left[-\frac{\left(k W_{0}^{2}\right)^{2} \rho^{2}}{\left(k W_{0}^{2}\right)^{2}+4 z^{2}}\right] \exp \left[-(t-z c)^{2} / \tau^{2}\right] \tag{1a}
\end{align*}
$$

and for the below PGB,

$$
\begin{align*}
\boldsymbol{E}_{\text {below }}(\rho, z, t)= & \mathbf{x} E_{0} \frac{\mathrm{i} k W_{0}^{2}}{\mathrm{i} k W_{0}^{2}+2 z} \exp \left\{-\mathrm{i}\left[k(z)-\omega_{0} t\right]\right\} \times \exp \left[-\mathrm{i} \frac{2 k z \rho^{2}}{\left(k W_{0}^{2}\right)+2 z^{2}}\right] \\
& \times \exp \left[-\frac{\left(k W_{0}^{2}\right)^{2} \rho^{2}}{\left(k W_{0}^{2}\right)^{2}+4 z^{2}}\right] \exp \left[-(t+z c)^{2} / \tau^{2}\right], \tag{1b}
\end{align*}
$$

where $W_{0}$ is the beam waist at the plane $z=0, \rho$ is the radial coordinate, $\mathbf{x}$ is the unit vector of the polarization along the $x$ direction, $k=2 \pi / \lambda$ is the wave number, $\omega_{0}$ is the carrier frequency, and $\tau$ is the pulse duration. For the
fixed input energy $U$ of a single pulsed beam, the constant $E_{0}$ is determined by $E_{0}^{2}=4 \sqrt{2} U /\left[n_{2} \varepsilon_{0} c W_{0}^{2}(\pi)^{3 / 2} \tau\right]$, where $n_{2}$ is the refractive index of the surrounding medium (water). The corresponding magnetic field under


Fig. 1. (a) Schematic of optical trap; (b) motion with radial variance of glass particle in the water plate.
paraxial approximation can be given by

$$
\begin{equation*}
\boldsymbol{H}(\rho, z, t) \cong \mathbf{y} n_{2} \varepsilon_{0} c \boldsymbol{E}(\rho, z, t) \tag{2}
\end{equation*}
$$

where $c=1 /\left(\varepsilon_{0} \mu_{0}\right)^{1 / 2}$ is the light speed in vacuum, and $\varepsilon_{0}$ and $\mu_{0}$ are the dielectric constant and permeability in vacuum, respectively.
From the definition of the Pointing vector, we can readily obtain the intensity distributions for the above and the below PGBs as follows:

$$
\begin{align*}
& I_{\text {above }}(\rho, z, t)=\left\langle\boldsymbol{E}_{\text {above }}(\rho, z, t)\right\rangle=\frac{P}{1+4 \tilde{z}^{2}} \\
& \quad \times \exp \left(-\frac{2 \tilde{\rho}^{2}}{1+4 \tilde{z}^{2}}\right) \exp \left(-2\left(\tilde{t}-\frac{\tilde{z} k W_{0}^{2}}{c \tau}\right)^{2}\right),  \tag{3a}\\
& I_{\text {below }}(\rho, z, t)=\left\langle\boldsymbol{E}_{\text {below }}(\rho, z, t)\right\rangle=\frac{P}{1+4 \tilde{z}^{2}} \\
& \quad \times \exp \left(-\frac{2 \tilde{\rho}^{2}}{1+4 \tilde{z}^{2}}\right) \exp \left[-2\left(\tilde{t}+\frac{\tilde{z} k W_{0}^{2}}{c \tau}\right)^{2}\right] \tag{3b}
\end{align*}
$$

where $P=2 \sqrt{2} U /\left[(\pi)^{3 / 2} W_{0}^{2} \tau\right], \tilde{z}=z / k W_{0}^{2}, \quad \tilde{\rho}=$ $\rho / W_{0}$, and $\tilde{t}=t / \tau$.

For simplicity, we assume that the radius $a$ of the particle is much smaller than the wavelength of the laser (i.e., $a \ll \lambda)$. In this case, we can treat the dielectric particle as a point dipole. We also assume that the refractive index of the glass particle is $n_{1}$ and $n_{1} \gg n_{2}$.

By argument similar to that shown in Ref. [6] for one PGB, the gradient optical force acting on dielectric particle of two counter-propagating PGBs is given by

$$
\begin{equation*}
\boldsymbol{F}_{\text {grad }, \rho}(\rho, z, t)=\frac{-\boldsymbol{\rho} 2 \beta\left[I_{\text {above }}(\rho, z, t)+I_{\text {below }}(\rho, z, t)\right] \tilde{\rho}}{\left[c n_{2} \varepsilon_{0} W_{0}\left(1+4(\tilde{z})^{2}\right)\right]}, \tag{4}
\end{equation*}
$$

where $\alpha=\left(128 \pi^{5} a^{6} / 3 \lambda^{4}\right)\left[\left(m^{2}-1\right) /\left(m^{2}+2\right)\right]^{2}$ is the scattering cross section, $\beta=4 \pi n_{2}^{2} \varepsilon_{0} a^{3}\left[\left(m^{2}-1\right) /\left(m^{2}+2\right)\right]$ is the polarizability, and $m=n_{1} / n_{2}^{[6,7]}$.
Assuming a low Reynold's number regime ${ }^{[12]}$, the Brownian motion of the dielectric in the optical force field (in the optical trap) is described by a Langevin equation as

$$
\begin{equation*}
\gamma \boldsymbol{\rho}(t)+\boldsymbol{F}_{\operatorname{grad}, \rho} \boldsymbol{\rho}(t)=\sqrt{2 D} \gamma \boldsymbol{h}(t), \tag{5}
\end{equation*}
$$

where $\boldsymbol{\rho}(t)=[x(t), y(t)]$ is the dielectric particle's position in the water plate, $\gamma=6 \pi a \eta$ is its friction coefficient, $\eta$ is the medium viscosity, $\sqrt{2 D} \gamma \boldsymbol{h}(t)=$
$\sqrt{2 D} \gamma\left[h_{x}(t), h_{y}(t)\right]$ is a vector of independent white Gaussian random processes describing the Brownian forces, $D=k_{\mathrm{B}} T / \gamma$ is the diffusion coefficient, $T$ is the absolute temperature, and $k_{\mathrm{B}}$ is the Boltzmann constant.

We compute the two-dimensional (2D) motion and the radial variance (position) of a glass particle in water using the Brownian dynamic simulation method. A particle/bead-spring model is employed to represent the glass particle, and the following motion equation is computed for each particle:

$$
\begin{align*}
\boldsymbol{\rho}(t+\delta t)-\boldsymbol{\rho}(t)= & -\frac{\boldsymbol{F}_{\mathrm{grad}, \rho}[\boldsymbol{\rho}(t)]}{\gamma} \times \boldsymbol{\rho}(t) \times \delta t \\
& +\sqrt{2 D} \times \delta t \times \boldsymbol{h}(t), \tag{6}
\end{align*}
$$

where $\delta t$ is the time increment of the simulation, $\boldsymbol{h}(t)$ is a random vector whose components are chosen from the range $[-1,1]$ in each time step. $\boldsymbol{F}_{\text {grad }, \rho}[\boldsymbol{\rho}(t)]$ in Eq. (6) describes the gradient optical force acting on the particle located at position $\rho$ at time $t$. For example, at the beginning time $t=0$, the glass particle is assumed to be located at the position $|\rho(t=0)|=W_{0}$, then we understand that the gradient optical force $\boldsymbol{F}_{\text {grad, } \rho}\left(W_{0}, z, 0\right)$ acts on the particle, which will be located at position $W+\Delta \rho$ after a time increment $\delta t$.
We are only interested in the radial variance of glass particle in the pulsing time (this parameter describes the stability of particle), so the simulation will be computed from the beginning moment $t=-3 \tau$ (or $t=0$ ) to the ending moment $t=3 \tau$ (or $t=6 \tau$ ) of the optical pulse. In the following numerical simulation, we choose the parameters as follows: $\lambda=1.064 \mu \mathrm{~m}, m=n_{1} / n_{2}=$ $1.592 / 1.332, \eta=7.797 \times 10^{-1} \mathrm{~Pa} \cdot \mathrm{~s}$ (the small glass particle and water, for instance $)^{[6]}, W_{0}=1 \mu \mathrm{~m}$ (or $W_{0}=$ $2 \mu \mathrm{~m}$ ), $a=10 \mathrm{~nm}$ (or $a=1 \mathrm{~nm}$ ), $\tau=1 \mathrm{ps}$, and the input power $U$ is changed from 0.1 to $1 \mu \mathrm{~J}^{[7]}, T=25^{\circ} \mathrm{C}$. The gradient optical force $\boldsymbol{F}_{\text {grad, } \rho}$ is calculated by Eq. (4) with $t$ ranging from $-3 \tau$ to $3 \tau$, and $\rho$ ranging from $-2 \mathrm{~W}_{0}$ to $2 \mathrm{~W}_{0}$ at $z=0 \mu \mathrm{~m}$ (considering the beam waist of pulsed Gaussian beam located in the traping plane $z=0$ ).

The simulations show that the gradient optical forces



Fig. 2. Distribution of optical force (a) in phase plane $(\rho, t)$, (b) in direction $\rho$, and (c) in pulsing time.


Fig. 3. Radial variance of glass particle during time for $U=$ $0.1 \mu \mathrm{~J}, a=1 \mathrm{~nm}$, with beginning positions (a) $\rho_{0}=1 \mu \mathrm{~m}$, (b) $\rho_{0}=2 \mu \mathrm{~m}$.


Fig. 4. Radial variance of glass particle during time for $U=1 \mu \mathrm{~J}, a=1 \mathrm{~nm}$, with beginning positions (a) $\rho_{0}=1$ $\mu \mathrm{m}$, (b) $\rho_{0}=2 \mu \mathrm{~m}$.


Fig. 5. Radial variance of glass particle during time for $U=$ $0.1 \mu \mathrm{~J}, a=10 \mathrm{~nm}$, with beginning positions (a) $\rho_{0}=1 \mu \mathrm{~m}$, (b) $\rho_{0}=2 \mu \mathrm{~m}$.


Fig. 6. Radial variance of glass particle during time for $a$ $=10 \mathrm{~nm}, \rho_{0}=2 \mu \mathrm{~m}$ with some values of $\mathrm{U}:(\mathrm{a}) 0.1 \mu \mathrm{~J}$, (b) $0.12 \mu \mathrm{~J}$, (c) $0.13 \mu \mathrm{~J}$, (d) $0.14 \mu \mathrm{~J}$, (e) $0.15 \mu \mathrm{~J}$, and (f) $0.19 \mu \mathrm{~J}$.
acting on the glass particle in water are divided into two parts whose directions are opposite to each other (Fig. 2(a)) and magnitudes are distributed as Gaussian functions of radial distance (Fig. 2(b)) and pulsing time (Fig. 2(c)). This distribution of gradient optical force is in good agreement with that presented in Ref. [6].

The radial variances of the glass particle with radius of 1 nm as a function of pulsing time (a function of pulse power or optical force, which is illustrated in Fig. 2(c)
at two initial positions are shown in Fig. 3. With the increase of the gradient optical force from zero to peak (depending on time), the glass particle is pulled into the center of the trap, where $\rho=0$, and fluctuates in the stable region defined as a circle with radius $\rho_{\mathrm{st}}=100 \mathrm{~nm}$. This fluctuation is in good agreement with experimental and theoretical results for phagocytic membrane ${ }^{[9]}$ and gold nanoparticles ${ }^{[10]}$, respectively.

When the optical force decreases at the end of the pulse, the thermal fluctuation of glass particle is stronger till it comes out of the stable region. If the beginning position $\rho_{0}=2 W_{0}=2 \mu \mathrm{~m}$ (the position is more far away from the trap center), the pulling rate is slower.
For the case that the power of PBG is higher $(U=1 \mu \mathrm{~J}$ for example in Fig. 4), the stable region is reduced, i.e., the glass particle fluctuates in a more narrow region. Moreover, the pulling rate is more speedy.

For a bigger particle ( $a=10 \mathrm{~nm}$ for example in Fig. 5), the fluctuation amplitude decreases and the pulling rate decreases significantly. Especially, when the beginning position $\rho_{0}=2 W_{0}=2 \mu \mathrm{~m}$, the glass particle will not be pulled into the stable region. But this situation can be overcome by increasing the pulse power. As shown in Fig. 6, we can see that a glass particle with its radius $a=10 \mathrm{~nm}$ will be pulled into stable region when $U \geq 0.15 \mu \mathrm{~J}$ (Figs. 6(e) and (f)).

In conclusion, we find that the Brownian force has influences on the stabilizing process of glass particles in water by the optical trap using PGBs. The influence of Brownian force decreases when the dimension of particle increases. The stability of glass particle in trap depends on the beginning position, the pulse power, and the dimension of particle. It is possible to choose a collection of parameters so that the radius of stable region could be reduced to the dimension of particle, i.e., the particle is stable absolutely.

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