Equal-amplitude optical comb generation using multi-frequency phase modulation in optical fibers

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We theoretically analyze and experimentally demonstrate a method of generating equal-amplitude optical comb exploiting multi-frequency phase modulation. The theoretical analysis shows that $4n$–1 equal-amplitude spectral lines can be obtained when the modulation signal comprises n frequency components including the fundamental frequency and the odd harmonic frequencies, and $2n$+1 equal-amplitude spectral lines can be obtained when the modulation signal comprises n frequency components including the fundamental frequency and the even harmonic frequencies. Then, we numerically simulate the spectra of 5, 7, 9, and 11 equal-amplitude spectral lines, respectively, which are also obtained in experiments with frequency separation of 30 MHz and flatness of better than 0.3 dB.

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In recent years, optical comb generation has gathered great interests in a number of areas in photonics technology. For example, in microwave photonics technology, optical comb has been used for optical frequency standards in absolute frequency measurement systems, local-oscillator remoting in radio-on-fiber systems, and control of phased array antennas in radio astronomy systems[1]; in optical communications, superdense wavelength division multiplexing (SD-WDM) uses tightly spaced optical channels (<50 GHz spacing) generated from optical comb to transmit data in both access and long-haul networks[2]. Recently, a novel optical transmission format known as coherent wavelength division multiplexing (CoWDM) has been proposed, which relies on a stable phase relationship between adjacent spectral lines[3]. Of all the applications mentioned above, a flattened spectrum profile of optical comb is an important essential.

Conventionally, mode-locked laser[4] and supercontinuum[2] based on optical pulse compression through a nonlinear fiber are popular candidates for such optical comb generation. However, the mode-locked laser has difficulties in starting and maintaining mode locking, and supercontinuum needs a complicated design and an adjustment to obtain flattened profile of the output spectrum. With the development of waveguide electro-optic modulators (EDMs), optical comb generated with a waveguide EDM has gathered great attentions, which provides phase-locked optical comb with compact devices. In Ref. [5] tandem Mach-Zehnder (MZ) amplitude modulator and phase modulator are used to generate 9 spectral lines with flatness of 2 dB, where the modulated voltages of amplitude modulator and phase modulator are set to π V and 2.4π V, respectively. In Ref. [3], two MZ amplitude modulators are used to generate 11 spectral lines with flatness of 1.97 dB, where the modulated voltage is set to 3.5π V. A phase modulator is also used in optoelectronic oscillator to generate optical comb with a larger modulated voltage[6]. These technologies, however, cannot obtain equal-amplitude optical comb. Previously it was considered that equal-amplitude optical comb could not be achieved by only phase modulation[5,7], since just single frequency modulation signal was considered. More recently, Ozharar *et al.* has demonstrated dual-sine-wave phase modulation, and obtained 9 and 11 comb lines with flatness of 0.8 and 1.9 dB, respectively[8].

In this letter, we propose a method to generate equal-amplitude optical comb by multi-frequency phase modulation, which features several advantages. Firstly, just a single phase modulator is needed, so that the system is simple and cheap. Secondly, the driving voltages are relatively low (i.e., about one half-wave voltage), so the needed radio-frequency (RF) power level is small. Thirdly, compared with amplitude modulator, phase modulator needs no precise adjustment on direct current (DC) bias. In experiment, we obtained 5, 7, 9, and 11 spectral lines with very good flatness of 0.13, 0.17, 0.24, 0.29, and 0.29 dB, respectively (the case of 7 spectral lines corresponds to two values of 0.17 and 0.24 dB).

In the case of single-frequency phase modulation, 3 equal-amplitude spectral lines can be obtained at most[9]. In order to obtain more equal-amplitude spectral lines, multi-frequency phase modulation should be applied to the incident lightwave. As for an arbitrary modulation signal $m(t)$ with a period of $f_{\rm m}$, its Fourier expansion is given by

$$m(t) = \sum\nolimits_{k=0}^{+\infty} \gamma_k \sin(2\pi k f_{\rm m} t + \phi_k). \quad (1)$$

With this multi-frequency modulation signal, the modulated field $E_{\rm m}$ is given by

$$E_{\rm m}(t) = \cos\left[2\pi f_{\rm c} t + \sum\nolimits_{k=1}^{+\infty} \gamma_k \sin(2\pi k f_{\rm m} t + \phi_k)\right], \quad (2)$$

where $f_{\rm c}$ is the carrier wave, $f_{\rm m}$ is the fundamental frequency of modulation signal, $\gamma_k = \pi \cdot V_k/V_\pi$ represents the modulated indices, V_k and V_π are the amplitudes of the modulated voltage and the half-wave voltage of phase modulator, respectively, ϕ_k is the initial phase of harmonic wave. Here we assume that there is no continuous-wave (CW) component in the modulation signal, i.e., $\gamma_0 = 0$, and the initial phase of the fundamental frequency is zero, i.e., $\phi_1 = 0$. Obviously, these assumptions will not change the spectrum of the modulated field. Equation (2) can also be expressed as

$$E_{\mathrm{m}} = \mathrm{Re}\left\{\exp\left[\mathrm{j}2\pi f_{\mathrm{c}}t + \mathrm{j}\sum_{k=1}^{+\infty}\gamma_k \sin\left(2\pi k f_{\mathrm{m}} t + \phi_k\right)\right]\right\}. \quad (3)$$

The Fourier expansion of the above equation is given by

$$S(t) = \sum_{n_1=-\infty}^{+\infty} \cdot \sum_{n_2=-\infty}^{+\infty} \cdots \sum_{n_k=-\infty}^{+\infty} \cdots \left\{\prod_{k=1}^{+\infty} J_{n_k}\left(\gamma_k, \phi_k\right) \cdot \cos\left[2\pi\left(f_{\mathrm{c}} + \sum_{k=1}^{+\infty} n_k k f_{\mathrm{m}}\right)\right]\right\}, \quad (4)$$

where

$$J_n(\gamma, \phi) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\left[\mathrm{j}\left(\gamma \sin\left(x + \phi\right) - nx\right)\right] \cdot \mathrm{d}x \quad (5)$$

is a kind of transformative Bessel function with the symmetrical characteristic of $J_n(\gamma, \phi) = (-1)^n J_{-n}^*(\gamma, \phi)$. In order to obtain equal-amplitude optical comb, the upper and lower side frequencies of the same order should have the same intensity. With the symmetrical characteristic of the transformative Bessel function mentioned above, it can be proved that the spectra of the modulated lightwave exhibit the same intensity of the upper and lower side frequencies of the same order when harmonic waves and their phases meet either of the two requirements below.

1) The modulation signal comprises the fundamental frequency f_{m} and odd harmonic frequencies $3f_{\mathrm{m}}$, $5f_{\mathrm{m}}$, $7f_{\mathrm{m}}, \cdots$, in which the odd harmonic frequencies could have the arbitrary phases;

2) The modulation signal comprises the fundamental frequency f_{m}, odd harmonic frequencies $3f_{\mathrm{m}}$, $5f_{\mathrm{m}}$, $7f_{\mathrm{m}}$, $\cdots$, and even harmonic frequencies $2f_{\mathrm{m}}$, $4f_{\mathrm{m}}$, $6f_{\mathrm{m}}, \cdots$, in which the phases of odd and even harmonic frequencies should be set at 0 and $\pi/2$, respectively.

If the modulation signal in case 1), modulation indices γ_1, γ_3, $\gamma_5, \cdots$ and phases ϕ_3, $\phi_5, \cdots$ are variables; in case 2), with all phases being fixed, only the modulation indices γ_1, γ_2, $\gamma_3, \cdots$ are variables. If the number of variables in the modulation signal is n, $2n+1$ equal-amplitude spectral lines can be obtained. As for case 1), there are $2n-1$ variables when the modulation signal comprises n frequency components, so that $4n-1$ equal-amplitude spectral lines can be obtained; while for case 2), there are n variables when the modulation signal comprises n frequency components, so that $2n+1$ equal-amplitude spectral lines can be obtained.

With multi-frequency phase modulation, the complex amplitudes of the fundamental frequency and side frequencies are given by

$$\begin{cases} A_0 = \sum\limits_{\sum_{k=1}^{+\infty} k\cdot n_k = 0} \prod_{k=1}^{+\infty} J_{n_k}\left(\gamma_k, \phi_k\right) \\ A_i = \sum\limits_{\sum_{k=1}^{+\infty} k\cdot n_k = i} \prod_{k=1}^{+\infty} J_{n_k}\left(\gamma_k, \phi_k\right) \quad (i = 1, 2, 3\cdots), \\ A_{-i} = \sum\limits_{\sum_{k=1}^{+\infty} k\cdot n_k = -i} \prod_{k=1}^{+\infty} J_{n_k}\left(\gamma_k, \phi_k\right) \end{cases} \quad (6)$$

where A_0 is the complex amplitude of the fundamental frequency, A_i and A_{-i} are the complex amplitudes of the upper and lower ith side frequencies, respectively. If the modulation signal meets either of the two requirements mentioned above, there will be $|A_i| = |A_{-i}|$. There are infinite terms in all of the right parts of Eq. (6), which makes numerical simulation impossible. Considering that the practical voltage applied in phase modulator should not be large, we choose the modulation indices smaller than 4 in the numerical simulation. With a fixed modulation index, the absolute values of the transformative Bessel function of high order are small, so we neglect the high order transformative Bessel function in the right parts, resulting in the possibility of numerical simulation.

The parameters of different equal-amplitude spectral lines are shown in Table 1, where N is the number of equal-amplitude spectral lines and V_{m} is the amplitude of modulated voltage. As for a specific combination of the fundamental frequency and harmonic frequencies, the solutions are not single, and the solutions in Table 1 are the ones with the smallest V_{m}. From Table 1, we can see that the amplitudes of modulated voltage are generally small, about one half-wave voltage and even smaller.

As for the combination of the fundamental frequency and the 2nd harmonic frequency, the phase of the second harmonic frequency should be set at 0.5π, and 5 equal-amplitude spectral lines can be obtained, as shown in Fig. 1(a). When the modulation signal comprises the fundamental frequency and the 3rd harmonic frequency, 7 equal-amplitude spectral lines can be obtained, as shown in Fig. 2(a); 7 equal-amplitude spectral lines can also be obtained under the combination of the fundamental frequency, the 2nd and 3rd harmonic frequencies, as shown in Fig. 2(c), where the phases of the 2nd and 3rd harmonic frequencies should be set at 0.5π and 0, respectively. When the modulation signal comprises the fundamental frequency, the 3rd and

Table 1. Parameters of Modulation Signals for Different Equal-Amplitude Spectral Lines and Flatness Measured in Experiments

γ_1	γ_2	ϕ_2	γ_3	ϕ_3	γ_5	ϕ_5	N	V_{m}(V)	Flatness (dB)
1.240	1.531	0.5π					5	0.883π	0.13
1.386			1.432	0.506π			7	0.845π	0.17
1.092	1.073	0.5π	0.962	0			7	0.662π	0.24
1.459			1.323	0.576π	0.290	0	9	0.833π	0.29
1.650			0.724	0.995π	1.404	0.48π	11	1.108π	0.29

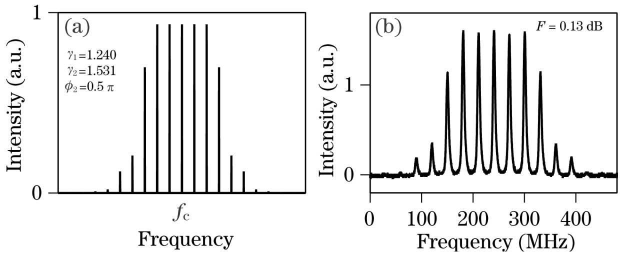

Fig. 1. Spectra of five equal-amplitude spectral lines. (a) Simulation; (b) experiment, where F is the flatness.

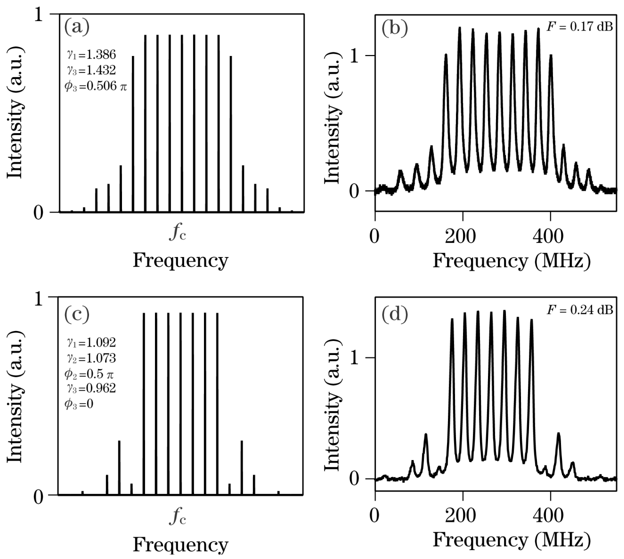

Fig. 2. Spectra of 7 equal-amplitude spectral lines. (a) Simulation and (b) experiment under the modulation signal comprising the fundamental frequency and the 3rd harmonic frequency; (c) simulation and (d) experiment under the modulation signal comprising the fundamental frequency, the 2nd and the 3rd harmonic frequencies.

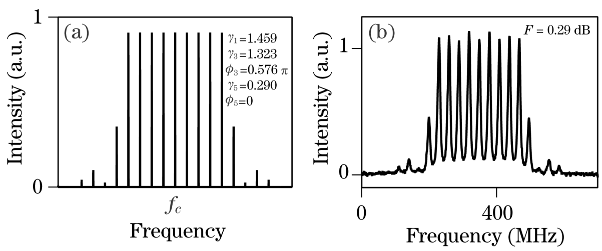

Fig. 3. Spectra of 9 equal-amplitude spectral lines. (a) Simulation; (b) experiment.

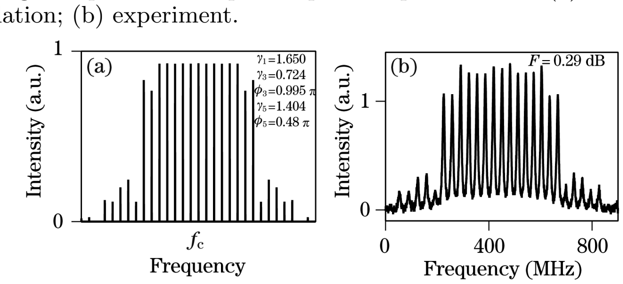

Fig. 4. Spectra of 11 equal-amplitude spectral lines. (a) Simulation; (b) experiment.

5th harmonic frequencies, 9 equal-amplitude spectral lines can be obtained without considering the phase of the 5th harmonics, as shown in Fig. 3(a); 11 equal-amplitude spectral lines can be obtained with considering the phase of the 5 harmonics, as shown in Fig. 4(a).

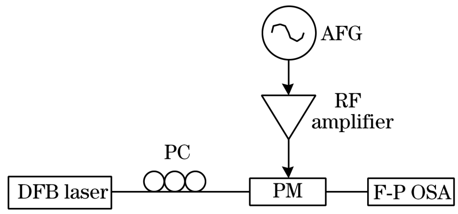

Fig. 5. Experimental setup. PC: polarization controller; PM: phase modulator.

To verify the validity of theoretical analysis and simulation results, we performed the experiment with the setup shown in Fig. 5. A narrow linewidth distributed feedback (DFB) laser at 1550.12 nm was used as the optical source. An arbitrary function generator (AFG) was used to generate the required modulation signal, which was then amplified to the required amplitude before being applied to the phase modulator. Because of the available bandwidth of AFG[10], we set the fundamental frequency at 30 MHz. The spectra of modulation lightwave were measured by a Fabry-Perot (F-P) optical spectrum analyzer (OSA) with a free spectrum range of 2 GHz and a resolution of 7 MHz. The measured half-wave voltage of the phase modulator was 8.3 V.

We experimentally verified the two cases mentioned above. When the modulation signal just comprises the fundamental frequency and odd harmonic frequencies, the experimental spectra show that the upper and lower side frequencies of the same order always have the same intensity; when there is an even harmonic frequency in the modulation signal and its phase is not set at 0.5π, and the phases of odd harmonic frequencies are not set at 0, the symmetry of the spectrum will be destroyed.

The experimental spectrum of 5 spectral lines is shown in Fig. 1(b), and the flatness is 0.13 dB. The experimental spectra of 7 spectral lines are shown in Figs. 2(b) and (d). Figure 2(b) is under the modulation signal comprising the fundamental frequency and the 3rd harmonic frequency, and the flatness is 0.24 dB. Figure 2(d) is under the modulation signal comprising the fundamental frequency, the 2nd and 3rd harmonic frequencies, and the flatness is 0.17 dB. Figure 3(b) is the experimental spectrum of 9 spectral lines, and the flatness is 0.29 dB. Figure 4(b) is the experimental spectrum of 11 spectral lines, and the flatness is 0.29 dB. The fluctuation of flatness is due to the instability of the RF amplifier, so that equal-amplitude spectral lines with smaller flatness are expected by using a more stable RF amplifier.

It can be observed that the spectra obtained in experiments agree with the results of simulations very well. It is noted that the modulation signal just includes odd harmonic frequencies, which will generate the same number of equal-amplitude spectral lines with that including all harmonic frequencies. For convenience, just the fundamental frequency and odd harmonic frequencies are required in practice.

In conclusion, we have proposed a method for generating equal-amplitude optical comb exploiting multi-frequency phase modulation. The combination of the fundamental frequency and odd harmonic frequencies can generate the maximum number of equal-amplitude spectral lines. In experiment, we have obtained 5, 7, 9, and 11 spectral lines with good flatness. With adding more high order harmonics frequencies, more equal-amplitude spectral lines can be obtained. In practice, however, large fundamental frequency may be limited by the available RF signal bandwidth as well as the bandwidth of the phase modulator.

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