Focusing contribution of individual pinholes of a photon sieve: dependence on the order of local ring of underlying traditional Fresnel zone plate

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A photon sieve can be composed of a large number of circular pinholes. Each circular pinhole has a contribution to the focusing. For the case of point-to-point imaging, the focusing contribution of an individual circular pinhole can be analytically given. We investigate the dependence of the focusing contribution on the order m of local ring of underlying traditional Fresnel zone plate. In particular, we find that the focusing contribution is simply inversely proportional to the order m. We also present an intuitive explanation. These results are helpful for better understanding of the focusing property of photon sieves.

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The focusing and imaging of soft X-ray and extreme ultraviolet (EUV) radiation have applications in many fields. However, refractive lenses are prevented from imaging and focusing in these spectral regions because of strong absorption. Traditional Fresnel zone plates can be used for this kind of focusing[1−3], but its resolution is approximately the order of the width of the outermost zone[4−6]. In 2001, Kipp *et al.* suggested the novel concept of photon sieve, which consists of a great number of pinholes properly distributed over the Fresnel zones[7]. Because photon sieves make it possible to focus soft X-rays to spot sizes smaller than the diameter of the smallest pinhole, it may be used in soft X-ray microscopy, lithography, and spectroscopy[7−12]. Besides the focusing and imaging of soft X-rays, photon sieves also offer a new approach for the construction of ultra-large (>20 m) space telescope primaries[13−15] and for laser free-space communication system[16]. In addition, fractal photon sieve, multi-wavelength and phase photon sieve have also been suggested[17−21].

As an important theoretical work, an individual far-field model for the imaging and focusing of photon sieve was previously presented[8,9]. For the important case of point-to-point imaging, the dependence of focusing contribution on the ratio of the diameter of the circular pinhole to the width of the local Fresnel ring of underlying traditional Fresnel zone plate was investigated in detail[8,9].

In this letter, we further investigate the important case of point-to-point imaging. In particular, we discuss the dependence of focusing contribution on the order m of local rings of underlying traditional Fresnel zone plate. An analytical expression for this kind of dependence is derived and the related explanation is given.

As shown in Fig. 1, for the case of point-to-point imaging, the object point O and the focal point (or image point) S are both on the optical axis. The distance from the object point to the photon sieve plane is p and the distance from the photon sieve plane to the focal point is q. We denote the photon sieve plane by the $x-y$ plane and the focal plane (or image plane) by the $X-Y$ plane. The coordinates of the center of the nth circular pinhole are denoted by (x_n,y_n). Similarly, the distance from the center of the photon sieve to the center of the nth circular pinhole is denoted by r_n. Obviously, $r_n^2 = x_n^2+y_n^2$. The distance between the object point and the center of the nth circular pinhole is P_n. The distance between the center of the nth circular pinhole and the focal point is Q_n. $P_n = (p^2+r_n^2)^{1/2}$, $Q_n = (q^2+r_n^2)^{1/2}$.

Based on the scalar diffraction theory and the small-size property of the individual circular pinholes, an individual far-field model for the imaging and focusing of photon sieves was developed[8,9]. According to the individual far-field model, each circular pinhole has a diffracted field $U_n(X,Y)$ at the focal plane and this field has reached its own far-field, where n denotes the nth pinhole. According to the linear superposition principle, the total diffracted field $U(X,Y)$ is the simple sum of those individual far-fields $U_n(X,Y)$. As a result of the individual far-field model, the field value $U_n(0,0)$ at the focal point was explicitly presented[8,9]. In particular, for the important case of point-to-point imaging, the following formula was derived in[8,9]

$$U_n(0,0) \propto \frac{d}{w} J_1\left(\frac{\pi}{2}\frac{d}{w}\right), \tag{1}$$

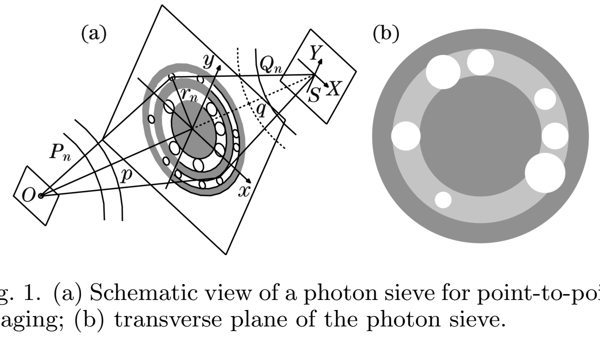

Fig. 1. (a) Schematic view of a photon sieve for point-to-point imaging; (b) transverse plane of the photon sieve.

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where d is the diameter of the nth circular pinhole, w is the width of the local ring of underlying traditional Fresnel zone plate, and $J_1(\cdot)$ is the first-order Bessel function of the first kind. Equation (1) reveals the dependence of focusing contribution of the nth circular pinhole on the ratio d/w.

We investigate the influence on the focusing contribution from the order m of the local ring at which the nth circular pinhole is located. For simplicity, we assume that the nth pinhole is centered on a circle whose radius is r_m. It should be noted that $r_m - w/2$ and $r_m + w/2$ are the lower and upper edges of the mth local ring (i.e., the mth white zone in Ref. [7]) of the underlying traditional Fresnel zone plate, respectively. We start from Eq. (16) of Ref. [9], which is the complete expression for the nth diffracted field value $U_n(0,0)$ at the focal point:

$$U_n(0,0) = \frac{kqA_n a_n^2}{Q_n^2} \exp\left[jk(P_n + Q_n)\right] \text{Jinc}\left(\frac{ka_n}{f_n} r_n\right), \quad (2)$$

where $\text{Jinc}(\cdot)=J_1(\cdot)/(\cdot)$, a_n is the radius of the nth circular pinhole, and A_n is the field value of the incident spherical wave at the nth circular pinhole. As an approximation, the parameter A_n can be regarded as a constant for all the pinholes. For this reason, we shall ignore the parameter A_n. The center of the nth circular pinhole is located at the center of the mth local ring of underlying traditional Fresnel zone plate. So in this case, $r_n = r_m$, $P_n = P_m$, $Q_n = Q_m$, and $f_n = f_m$. Substituting these relations into Eq. (2), we get

$$U_n(0,0) = \frac{qa_n f_m}{r_m Q_m^2} \exp\left[jk\left(P_m + Q_m\right)\right] J_1\left(\frac{ka_n r_m}{f_m}\right). \quad (3)$$

We know that the phase factor changes very rapidly while the amplitude changes very slowly with the coordinate on the focal plane. So we can deal with the phase factor on the non-paraxial condition while the amplitude part on the paraxial condition. According to the theory of Fresnel zone plate, the optical path difference of the lower (or upper) edges of the two adjacent white zones to the focal point is a wavelength. In this way, we know $P_m + Q_m - p - q = m\lambda$. By use of this relation in Eq. (3), one can get

$$U_n(0,0) = \frac{q}{Q_m^2} \frac{f_m}{r_m} a_n J_1\left(\frac{ka_n r_m}{f_m}\right). \quad (4)$$

Using the relation $a_n = d/2$ and $w \approx \lambda f_m/(2r_m)$[9] in Eq. (4), we can further get

$$U_n(0,0) \approx g(r_m) \frac{d}{w} J_1\left(\frac{\pi}{2} \frac{d}{w}\right), \quad (5)$$

where

$$g(r_m) = \frac{\lambda q}{4} \frac{1}{r_m^2} \left(\frac{f_m}{Q_m}\right)^2. \quad (6)$$

Using the relation $1/f_m = 1/P_m + 1/Q_m$ in Eq. (6), we get

$$\begin{aligned} g(r_m) &= \frac{\lambda q}{4} \frac{1}{r_m^2} \left(\frac{1}{1 + Q_m/P_m}\right)^2 \\ &= \frac{\lambda q}{4} \frac{1}{r_m^2} \left(1 + \frac{Q_m}{P_m}\right)^{-2}. \end{aligned} \quad (7)$$

Substituting the two approximations $P_m = (p^2 + r_m^2)^{1/2} \approx p$ and $Q_m = (q^2 + r_m^2)^{1/2} \approx q$ into Eq. (7), we can further get

$$g(r_m) \approx \frac{\lambda q}{4} \frac{1}{r_m^2} \left(1 + \frac{q}{p}\right)^{-2}. \quad (8)$$

It should be emphasized that for a photon sieve or a Fresnel zone plate working in the soft X-ray region, a numerical aperture (NA) value of ~0.05 can be regarded as high and a NA value of ~0.2 can be regarded as rather high. Of course, the absolute value of NA is still small. For a NA not larger than 0.2, we can use $P_m \approx p[1 + r_m^2/(2p^2)]$ and $Q_m \approx q[1 + r_m^2/(2q^2)]$. Comparing Eqs. (5) and (8) with Eq. (1), one can see that the factor $g(r_m)$, which does not appear in Eq. (1), is inversely proportional to the square of the parameter r_m. For the Fresnel zone plate, we know that $(p^2 + r_m^2)^{1/2} + (q^2 + r_m^2)^{1/2} - p - q = m\lambda$, $m = 0,1,2,\cdots$. In terms of binomial expansion, one can obtain $r_m \approx [2m\lambda pq/(p+q)]^{1/2}$. Inserting this relation into Eq. (8), we can further get

$$g(m) \approx \frac{1}{8m} \frac{p}{p+q}. \quad (9)$$

It should be pointed out that the relation $r_m \approx [2m\lambda pq/(p+q)]^{1/2}$ is only an approximate expression. This approximate expression can be used to get the dependence relation $g(m)$. However, for the exact calculation of the parameter r_m, one should still use the exact relation $(p^2 + r_m^2)^{1/2} + (q^2 + r_m^2)^{1/2} - p - q = m\lambda$. The exact expression for r_m can be analytically given in a somewhat complicated form. Here we do not further discuss this issue.

For a certain photon sieve, the parameters p and q are given. These two parameters are the same for all the circular pinholes of a photon sieve and can be regarded as two constants. After ignoring the constant factor $p/[8(p+q)]$, one can further obtain

$$g(m) \approx \frac{1}{m}. \quad (10)$$

Equation (10) explicitly reveals that $g(m)$ is simply inversely proportional to the order m.

To understand the above results better, we now present an intuitive explanation. For simplicity, consider the special case of a plane wave illumination for which $p = \infty$. We also suppose that, as shown in Fig. 2, the pinholes are located one by one without interval. Thus, the perimeter of the mth ring of underlying Fresnel zone plate is approximately equal to the sum of the diameter of all the pinholes on the same ring. That is to say,

$$2\pi r_m = N_m d, \quad (11)$$

where N_m is the number of pinholes on the mth ring. Note that N_m can be a non-integer in theory.

We suppose that the ratio d/w is a constant η. Then

$$N_m = \frac{2\pi r_m}{\eta w}. \quad (12)$$

It is known that the width w of the mth local ring of underlying traditional Fresnel zone plate can be approx-

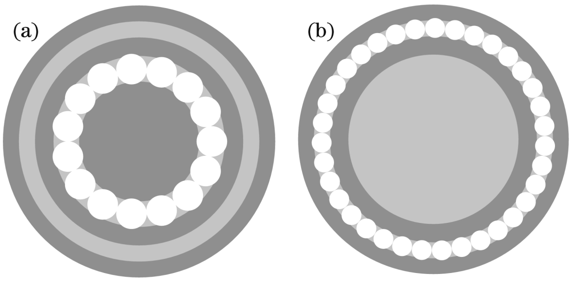

Fig. 2. Pinholes in different local rings of underlying traditional Fresnel zone plate when the ratio d/w is fixed. (a) Pinholes in a lower-order ring; (b) pinholes in a higher-order ring.

imately given by $w \approx \lambda f_m/(2r_m)$, where f_m can be determined by $1/f_m = 1/P_m + 1/Q_m$. Substituting this relation into Eq. (12), we can get

$$N_m \approx \frac{4\pi}{\lambda\eta}\frac{r_m^2}{f_m}. \tag{13}$$

Because the photon sieve is illuminated by a plane wave for which $p = \infty$, the parameter P_m is infinite. Then f_m is approximately given by

$$f_m = Q_m = \left(q^2 + r_m^2\right)^{1/2} \approx q, \tag{14}$$

where we have used the property of $q \gg r_m$. By use of another approximation of $r_m^2 \approx 2m\lambda q$, we can further obtain

$$N_m \approx \frac{4\pi}{\lambda\eta q} r_m^2 \propto m. \tag{15}$$

Equation (15) shows that, just as we have found, there are more pinholes on a higher-order local ring than those on a lower-order local ring if d/w is constant. It can be proved that all the individual focusing contributions (i.e., the individual diffracted field values at the focal point) of all the open rings of a traditional Fresnel zone plate are approximately the same[22,23]. Then, we find that the focusing contribution of an individual pinhole is inversely proportional to the order m of the local ring of underlying Fresnel zone plate:

$$g\,(m) \propto N_m^{-1} \propto m^{-1}. \tag{16}$$

In conclusion, we have investigated the dependence of focusing contribution of an individual pinhole on the order m of the local ring of underlying traditional Fresnel zone plate. It is found that the normalized dependence relation $g(m)$ is simply inversely proportional to the order m. We also present an intuitive explanation for the dependence relation. The results obtained in this letter are helpful for better understanding of the focusing property of photon sieves.

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