Sudden death of entanglement in the two-mode cavity field

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The entanglement dynamics involving the so-called entanglement sudden death of atoms in two-photon Tavis-Cummings model is investigated. Various initial conditions that may have influences on the entanglement evolution of atoms, especially on the appearance of atomic entanglement sudden death, are studied. The appearance of entanglement sudden death is sensitive to the initial conditions of the whole system, i.e., the concrete type of atomic initial state, the photon number in the cavity field, and the dipole-dipole interaction between atoms. It is shown that the strong dipole-dipole interaction between atoms can weaken the atomic entanglement sudden death.

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Quantum entanglement is not only a remarkable feature of quantum mechanics but also an indispensable resource for realization of quantum information processing (QIP), such as quantum cryptography[1], quantum teleportation, and quantum computation[2,3]. However, in the process of entanglement distribution and the manipulation of particles, the particles would have unavoidable interactions with external environment as well as have possible interactions between themselves. This leads to decoherences which will sooner or later spoil the necessary entanglement of the shared states. Therefore, it is important to understand the entanglement from its role and the manifestation in realistic systems[4−9]. Recently, Yu *et al*. discovered the evolution of entanglement of a bipartite system in which two particles coupled with two independent environment can vanish abruptly in finite time[10]. Such a surprising phenomenon was termed as entanglement sudden death (ESD)[10]. The ESD of a composite system exhibits clear contrast with decoherence of an individual quantum system that decays only asymptotically. Because of its intrinsic and practical interests, ESD has attracted much attention from both theoretical and experimental points of view[11].

In some cases, the entanglement of a composite system would decay asymptotically, the same as decoherence of a single quantum system. However, a composite system experiencing whether the finite-time or the infinite-time disentanglement would depend on the initial conditions of the external environment as well as the initial entanglement degree of the system. In previous work, ESD in the Jaynes-Cummings (JC) model has been intensively studied and acquired significant achievements[12−17]. In Ref. [15], the authors dealt with a double JC model in which two initially entangled two-level atoms were independently coupled with separate cavity fields. Focusing on the atomic subsystem, they found out that the entanglement might exhibit ESD, depending on the type of atomic initial state and the degree of initial entanglement, or not. In other words, the entanglement could decay asymptotically or abruptly in the same model dependent on the initial entanglement type and degree of atoms. Since the disentanglement in the asymptotical domain is intuitive and well understood, most studies pay attention to the finite-time disentanglement, i.e., ESD. In addition to the JC model, ESD in the Tavis-Cummings (TC) model[18] has also been investigated[19−25]. In Ref. [19], the authors explored the conditions for appearance of atomic ESD in cavity quantum electrodynamics (QED) by considering the effects of the single-mode cavity photon number without touching on the dipole-dipole interaction between atoms. In Ref. [20], the authors discussed the entanglement qualities involving the occurrence of ESD between atoms in the non-degenerate two-photon TC model. They considered dipole-dipole interaction between atoms in the absence of the impact of the cavity photon number. In this letter, we study the conditions for ESD between atoms embedded in a two-mode cavity field in the two-photon TC (TPTC) model, and the collective influences of dipole-dipole interaction between atoms and the two-mode cavity photon number on the entanglement dynamics of atoms. It is shown that ESD is sensitive to the initial atomic state when we assume initial cavity fields to be in the vacuum. The interval of entanglement vanishing is strictly related to the strength of dipole-dipole interaction between atoms and the number of photons in the two-mode cavity field.

In two-qubit domains, there exist a number of good measures of entanglement such as concurrence[26] and negativity[27]. Although these entanglement measures may be different quantitatively, all of them are equal to zero for separable states. Here we adopt Wootters′ concurrence[24] because of its convenience in definition, normalization, and calculation. The concurrence $C(\rho)$ for any reduced density matrix ρ of two qubits is defined as

$$C(\rho)=\max(0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}),\qquad(1)$$

where $\lambda_i(\lambda_1\geq\lambda_2\geq\lambda_3\geq\lambda_4)$ are the eigenvalues of the matrix $\zeta=\rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y)$, with a Pauli matrix σ_y and the complex conjugation of ρ in the standard basis ρ^*. For separate states $C(\rho)=0$, whereas for maximally entangled states $C(\rho)=1$.

Here, we consider that, in the TPTC model, two identical two-level atoms interact with the two-mode cavity resonantly in the presence of dipole-dipole interaction of the two atoms. In the rotation wave approximation, this system can be described in the interaction picture by the following Hamiltonian ($\hbar = 1$)

$$H = \sum_{i=\mathrm{a}}^{\mathrm{b}} \omega_i a_i^+ a_i + \frac{1}{2}\omega_0 \sum_{j=\mathrm{A}}^{\mathrm{B}} \sigma_j^z + g \sum_{j=\mathrm{A}}^{\mathrm{B}} (a_\mathrm{a} a_\mathrm{b} \sigma_j^+ + a_\mathrm{a}^+ a_\mathrm{b}^+ \sigma_j^-)$$
$$+ \varOmega(\sigma_\mathrm{A}^+ \sigma_\mathrm{B}^- + \sigma_\mathrm{B}^+ \sigma_\mathrm{A}^-), \quad (2)$$

where $a_i(a_i^+)$ (i=a,b) denotes the annihilation (creation) operator of the quantization field with frequency ω_i, ω_0 is the resonant transition frequency of the two-level atom, $\sigma_j^z = |e\rangle_{jj}\langle e| - |g\rangle_{jj}\langle g|$, $\sigma_j^+ = |e\rangle_{jj}\langle g|$, and $\sigma_j^- = |g\rangle_{jj}\langle e|$ are the atomic operators with $|e\rangle_j$ and $|g\rangle_j$ being the excited and ground states of the jth atom (j = A, B), g is the coupling constant between atom and field, and $\varOmega$ is dipole-dipole coupling strength between atoms. For simplicity, we concentrate our analysis only on the resonant case with $\omega_0 = \sum_{i=\mathrm{a}}^{\mathrm{b}} \omega_i$.

We first choose that the two atoms are prepared initially in the Bell-like state denoted as $|\psi_\mathrm{atom}\rangle = \cos\theta|e_\mathrm{A}, g_\mathrm{B}\rangle + \sin\theta|g_\mathrm{A}, e_\mathrm{B}\rangle$, and both cavity modes are prepared in the state $|n_\mathrm{a}\rangle \otimes |m_\mathrm{b}\rangle$, thus the initial state for the total system is given by

$$|\psi(0)\rangle = \cos\theta|e_\mathrm{A}, g_\mathrm{B}, n, m\rangle + \sin\theta|g_\mathrm{A}, e_\mathrm{B}, n, m\rangle. \quad (3)$$

Then we can obtain the corresponding time evolution of the total system at time t according to Eqs. (2) and (3) as

$$\begin{aligned} |\psi(t)\rangle = {} & x_1|e_\mathrm{A}, g_\mathrm{B}, n, m\rangle + x_2|g_\mathrm{A}, e_\mathrm{B}, n, m\rangle \\ & + x_3|g_\mathrm{A}, g_\mathrm{B}, n+1, m+1\rangle \\ & + x_4|e_\mathrm{A}, e_\mathrm{B}, n-1, m-1\rangle, \end{aligned} \quad (4)$$

where the coefficients are expressed as

$$\begin{aligned} x_1 &= \Xi_+ \mathrm{e}^{-i\varepsilon\lambda}(\cos\alpha\lambda - \frac{i\varepsilon}{2\alpha}\sin\alpha\lambda) - \Xi_- \mathrm{e}^{2i\varepsilon\lambda}, \\ x_2 &= \Xi_+ \mathrm{e}^{-i\varepsilon\lambda}(\cos\alpha\lambda - \frac{i\varepsilon}{2\alpha}\sin\alpha\lambda) + \Xi_- \mathrm{e}^{2i\varepsilon\lambda}, \\ x_3 &= -\frac{2i\sqrt{(n+1)(m+1)}\Xi_+}{\alpha}\mathrm{e}^{-i\varepsilon\lambda}\sin\alpha\lambda, \\ x_4 &= -\frac{2i\sqrt{nm}\Xi_+}{\alpha}\mathrm{e}^{-i\varepsilon\lambda}\sin\alpha\lambda, \end{aligned} \quad (5)$$

with $\Xi_\pm = (\sin\theta \pm \cos\theta)/2, \varepsilon = \varOmega/2g, \lambda = gt, \alpha = \sqrt{2(1+n+m+2nm)+\varepsilon^2}$. The information about the entanglement of two atoms is contained in the reduced density matrix $\rho_\mathrm{AB}^\psi(t)$ for the two atoms which can be obtained from Eq. (4) by tracing out the degree of freedom of cavity modes. In the standard basis$\{|ee\rangle, |gg\rangle, |ge\rangle, |gg\rangle\}$, the density matrix $\rho_\mathrm{AB}^\psi(t)$ is given by

$$\rho_\mathrm{AB}^\psi(t) = \begin{pmatrix} |x_4|^2 & 0 & 0 & 0 \\ 0 & |x_1|^2 & x_1 x_2^* & 0 \\ 0 & x_1^* x_2 & |x_2|^2 & 0 \\ 0 & 0 & 0 & |x_3|^2 \end{pmatrix}, \quad (6)$$

where x^* stands for the complex conjugate of x. The time-dependent matrix elements are given by Eq. (5). By virtue of Eq. (1), the corresponding concurrence of the density matrix Eq. (6) is given by

$$C[\rho_\mathrm{AB}^\psi(t)] = 2\max(0, |x_1 x_2^*| - |x_3||x_4|). \quad (7)$$

Figure 1 is a plot for the entanglement evolution of two atoms for $n = m = 0$ and $\varepsilon = 0$ with the initial atomic state $|\psi_\mathrm{atom}\rangle$. Obviously, there is no entanglement sudden death phenomenon when the cavity is in the vacuum state and the strength of dipole-dipole interaction between atoms is zero. The concurrence for $\rho_\mathrm{AB}^\psi(t)$ fluctuates periodically for different degrees of initial entanglement in terms of parameter θ.

In Fig. 2, we plot the entanglement evolution of two atoms when the two-mode cavity is not in the vacuum state, where the ESD of atoms occur periodically only under the condition $n > 0$ and $m > 0$. Moreover, we can see from Fig. 2 that the larger the photon number in the two-mode cavity field is, the shorter the time interval for the zero entanglement (of ESD between atoms AB) is, and the faster the entanglement vanishes.

Figure 3 shows the entanglement evolution of two atoms with different dipole-dipole interaction between atoms. It can be observed from Fig. 3 that the entanglement between two atoms can be increased by introducing the dipole-dipole interaction. The appearing time of atomic ESD becomes shorter with the increase of parameter $\varepsilon(\varOmega/g)$. Another important point is that the ESD phenomenon can be completely eliminated through appropriately modulating the dipole-dipole interaction. In contrast to the condition $\varepsilon = 1$, the cases $\varepsilon = 5$ and $\varepsilon = 10$ show that atomic entanglement can be preserved highly by strong dipole-dipole interaction. This can be also understood from the physics that the independence of atoms will be weakened by the dipole-dipole interaction, thus the initial state will be more entangled.

As a comparison, we consider another type of Bell-like state, i.e.$|\phi_\mathrm{atom}\rangle = \cos\theta|e_\mathrm{A}, e_\mathrm{B}\rangle + \sin\theta|g_\mathrm{A}, g_\mathrm{B}\rangle$, as the initial state of atoms, while both cavity modes are prepared still in the state $|n_\mathrm{a}\rangle \otimes |m_\mathrm{b}\rangle$. Therefore the initial state for the total system is given by

$$|\psi(0)\rangle = \cos\theta|e_\mathrm{A}, e_\mathrm{B}, n, m\rangle + \sin\theta|g_\mathrm{A}, g_\mathrm{B}, n, m\rangle, \quad (8)$$

The evolved state of the total system at time t can be expressed in the standard basis as

$$\begin{aligned} |\psi(t)\rangle = {} & x_1|e_\mathrm{A}, e_\mathrm{B}, n, m\rangle + x_2|g_\mathrm{A}, g_\mathrm{B}, n, m\rangle \\ & + x_3|g_\mathrm{A}, e_\mathrm{B}, n+1, m+1\rangle + x_4|g_\mathrm{A}, e_\mathrm{B}, n-1, m-1\rangle \\ & + x_5|e_\mathrm{A}, g_\mathrm{B}, n-1, m-1\rangle + x_6|e_\mathrm{A}, g_\mathrm{B}, n+1, m+1\rangle \\ & + x_7|e_\mathrm{A}, e_\mathrm{B}, n-2, m-2\rangle + x_8|g_\mathrm{A}, g_\mathrm{B}, n+2, m+2\rangle, \end{aligned} \quad (9)$$

where the coefficients are given as

$$x_1 = \Lambda_1 \cos\theta[1 + \frac{\Lambda_3^2 e^{-i\varepsilon\lambda}}{\Lambda_1^2}(\cos\beta\lambda + \frac{i\varepsilon}{\beta}\sin\beta\lambda)],$$
$$x_2 = \Lambda_2 \sin\theta[1 + \frac{\Lambda_4^2 e^{-i\varepsilon\lambda}}{\Lambda_2^2}(\cos\alpha\lambda + \frac{i\varepsilon}{\alpha}\sin\alpha\lambda)],$$
$$x_3 = x_6 = -\frac{i\sqrt{\Xi_2}\cos\theta}{\beta}e^{-i\varepsilon\lambda}\sin\beta\lambda,$$
$$x_4 = x_5 = -\frac{i\sqrt{\Xi_3}\sin\theta}{\alpha}e^{-i\varepsilon\lambda}\sin\alpha\lambda,$$
$$x_7 = -\Lambda_4\sin\theta e^{-i\varepsilon\lambda}(1 - \frac{i\varepsilon}{\alpha}\sin\alpha\lambda - \cos\alpha\lambda),$$
$$x_8 = -\Lambda_3\cos\theta e^{-i\varepsilon\lambda}(1 - \frac{i\varepsilon}{\beta}\sin\beta\lambda - \cos\beta\lambda), \quad (10)$$

with $\Lambda_1 = \Xi_1/(\Xi_1 + \Xi_2)$, $\Lambda_2 = \Xi_4/(\Xi_3 + \Xi_4)$, $\Lambda_3 = \sqrt{\Xi_1\Xi_2}/(\Xi_1 + \Xi_2)$, $\Lambda_4 = \sqrt{\Xi_4\Xi_3}/(\Xi_3 + \Xi_4)$, $\lambda = gt, \varepsilon = \Omega/2g$, $\Xi_1 = (m+2)(n+2), \Xi_2 = (m+1)(n+1), \Xi_3 = mn, \Xi_4 = (m-1)(n-1)$, $\alpha = \sqrt{2(1-m-n+2mn)+\varepsilon^2}$, $\beta = \sqrt{2(5+3m+3n+2mn)+\varepsilon^2}$. In the basis of $\{|ee\rangle, |gg\rangle, |ge\rangle, |gg\rangle\}$, we can write the corresponding reduced density matrix

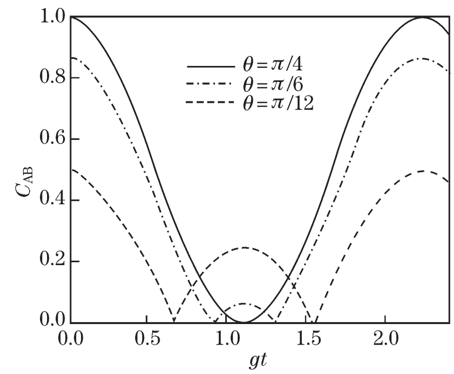

Fig. 1. The concurrence for $\rho_{AB}^{\psi}(t)$ versus the rescaled time gt and the mixing θ for $n = m = 0$ and $\varepsilon = 0$with the initial atomic state $|\psi_{atom}\rangle = \cos\theta|e_A, g_B\rangle + \sin\theta|g_A, e_B\rangle$.

$$\rho_{AB}^{\phi}(t) = \begin{pmatrix} |x_1|^2 + |x_7|^2 & 0 & 0 & x_1x_2^* \\ 0 & |x_5|^2 + |x_6|^2 & x_5x_4^* + x_6x_3^* & 0 \\ 0 & x_4x_5^* + x_3x_6^* & |x_3|^2 + |x_4|^2 & 0 \\ x_2x_1^* & 0 & 0 & |x_2|^2 + |x_8|^2 \end{pmatrix}. \quad (11)$$

The concurrence of this mixed state Eq. (11) can be obtained in the same way

$$C[\rho_{AB}^{\phi}(t)] = 2\max[0, |x_1x_2^*| - \sqrt{(|x_5|^2 + |x_6|^2)(|x_3|^2 + |x_4|^2)}, \\ |x_5x_4^* + x_6x_3^*| - \sqrt{(|x_1|^2 + |x_7|^2)(|x_2|^2 + |x_8|^2)}]. \quad (12)$$

In contrast to Fig. 1, we find that the entanglement can abruptly vanish and remain zero for a period of time before entanglement reviving when the parameter θ is small, as shown in Fig. 4. Hence, we can obtain the same conclusion as the previous studies, i.e., the ESD phenomenon is sensitive to the atomic initial state and the vacuum cavity field.

Since the ESD only occurs in the restricted range of small θ, we shall take $\theta = \pi/30$ when study the influence of the dipole-dipole interaction and the photon number in the cavity on the evolution of entanglement between atoms. Different from the previous case, Fig. 5 shows that the influence of the photon number in the cavity on the time interval of disentanglement is nonlinear. The appearing time of ESD of atoms AB is also related to the photon number in the cavity, i.e., the lager the photon number is, the faster the ESD appears. From Fig. 6, we can see the same results as those in Fig. 3, i.e., the ESD effect is weakened by introducing the dipole-dipole interaction and can be prevented completely by choosing appropriate dipole-dipole interaction. However, the degree of influence on the ESD phenomenon is different as choosing different initial states of the system. It is obviously to require stronger strength of the dipole-dipole interaction to completely eliminate ESD effect for the initial atomic state $|\phi_{atom}\rangle$.

From the previous studies, we know that there is no ESD effect between atoms when the whole system is described by Eq. (3). The reason is that only one atom is in the excited state and the other one with ground state does not interact with the vacuum cavity field. We also find that by introducing dipole-dipole interaction, the initial entanglement between atoms can be strengthened and maintains the average value at the relatively large value of unity. Therefore, this method may be a significant way for avoiding the system suffering from ESD. As for the influence on the time interval for the zero entanglement of the photon number, this is very easily understood since it is governed by $\alpha = \sqrt{8(1+n+m+2nm)+\varepsilon^2}/2$ in

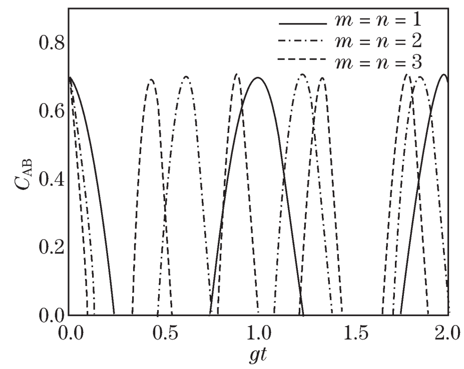

Fig. 2. The concurrence for $\rho_{AB}^{\psi}(t)$ versus the rescaled gt with different photon numbers for $\theta = \pi/8, \varepsilon = 0$.

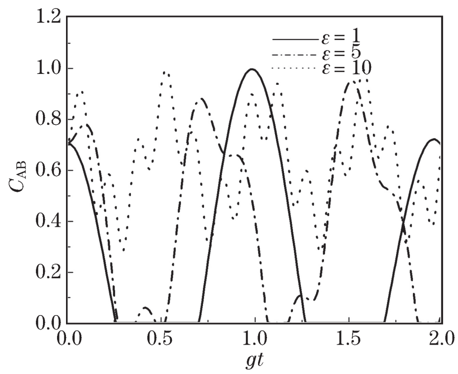

Fig. 3. The concurrence for $\rho_{\rm AB}^{\psi}(t)$ versus the rescaled gt with different dipole-dipole interactions for $\theta = \pi/8, m = n = 1$.

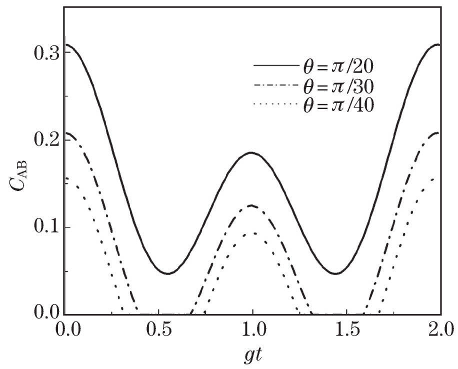

Fig. 4. The concurrence for $\rho_{\rm AB}^{\phi}(t)$ versus the rescaled gt and the mixing θ for $n = m = 0$ and $\varepsilon = 0$ with the initial atomic state $|\phi_{\rm atom}\rangle = \cos\theta|e_{\rm A}, e_{\rm B}\rangle + \sin\theta|g_{\rm A}, g_{\rm B}\rangle$.

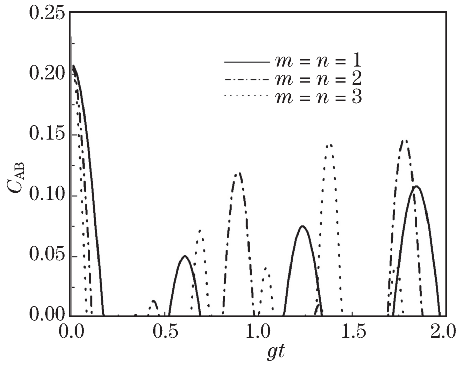

Fig. 5. The concurrence for $\rho_{\rm AB}^{\phi}(t)$ versus the rescaled gt with different photon numbers for $\theta = \pi/30, \varepsilon = 1$.

the first case and by $\alpha = \sqrt{2(1 - m - n + 2mn) + \varepsilon^2}$, $\beta = \sqrt{8(5 + 3m + 3n + 2mn) + \varepsilon^2}$ in the second case.

Recently, a great deal of works is devoted to understanding the ESD phenomenon from the energy transfer of the system[28] interested and the corresponding cavity field. In this letter, we point out that the energy transfer between atoms and the cavity may be related to the ESD. Let us review the Hamilton Eq. (1), when $n = m = 0$ and $\varepsilon = 0$, the dynamics of entanglement between atoms is determined by the interaction terms, which is the charge of the energy transfer between the system and field. In the first case, the energy transfer is

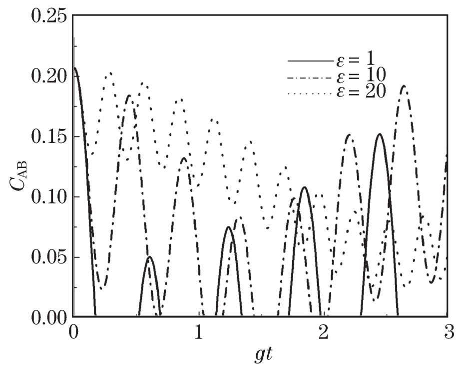

Fig. 6. The concurrence for $\rho_{\rm AB}^{\phi}(t)$ versus the rescaled gt with different dipole-dipole interactions for $\theta = \pi/30$ and $m = n = 1$.

described by $\langle H_0\rangle_{\rho_{\rm AB}^{\psi}} = \langle \frac{\omega_0}{2}(\sigma_{\rm A}^z + \sigma_{\rm B}^z)\rangle_{\rho_{\rm AB}^{\psi}} = |x_4|^2 - |x_3|^2$; in the second case, the energy transfer is described by $\langle H_0\rangle_{\rho_{\rm AB}^{\phi}} = \langle \frac{\omega_0}{2}(\sigma_A^z + \sigma_B^z)\rangle_{\rho_{\rm AB}^{\phi}} = |x_1|^2 + |x_7|^2 - |x_2|^2 - |x_8|^2$. Obviously, it is the difference between the above two equations that gives rise to the different dynamics of entanglement under the different initial states.

In conclusion, we have investigated the entanglement dynamics between atoms for different forms of Bell-like states in the TPTC model. The results demonstrate that for the two different initial states of the system, the entanglement evolution exhibits dramatic difference and the appearance of ESD effect is sensitive to the initial conditions. The time interval for the entanglement vanishing is dependent on not only the photon number in the cavity field but also the dipole-dipole interaction between atoms. Besides, the influence of the photon number in the cavity field on the atomic entanglement is nonlinear and the strength of the dipole-dipole interaction between atoms can weaken the ESD.

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