Non-classical properties of the single-mode cavity field after interacting with a Ξ-type three-level atom

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We study the non-classical properties of the single-mode electro-magnetic field resulting from the interaction of a Ξ-type three-level atom initially in the coherent state, such as squeezing properties and sub-Poisson statistics. We show that if there are more photons in the cavity, the squeezing will appear earlier and be stronger under the same state, but the sub-Poisson statistics will be weaker, while sub-Poisson statistics and squeezing are more pronounced after the selective atomic measurement.

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Non-classical properties of light, such as squeezing and sub-Poisson statistics, play a significant role in quantum optics[1], and have attracted more and more attention over the past score years. It has been confirmed that there are great applications with them in optical communication[2] and high-precision optical measurements. It is noted that light squeezing exists in two-level or three-level Jaynes-Cummings models when cavity fields are in coherent fields[3−10]. In 1989, Hillery showed that large photon numbers should be required for significant squeezing of a single-mode cavity field, interacting with a two-level atom system[11]. Recently, Wu *et al.* showed that the squeezing might be greatly enhanced in the two-mode cavity field after interacting with a Λ-type atom, even for a low photon number, after the selective atomic measurement[12]. Besides, squeezing enhancement has also been found in other models, such as the two-photon Jaynes-Cummings model[13]. In this letter, we show that the squeezing and sub-Poisson statistics exist in the resonant interaction of a Ξ-type three-level atom with a single-mode field initially in the coherent state, and both of them are enhanced after the selective atomic measurement. We also find that squeezing will appear earlier and be stronger as the cavity photons increases while the sub-Poisson statistics become weaker, although it also appears earlier.

We consider a Ξ-type three-level atom interacting resonantly with a single-mode cavity field initially in the coherent state. Let the upper atomic level be denoted by $|e\rangle$, the middle and the lower one by $|g\rangle$ and $|i\rangle$, respectively. In the interaction picture, the corresponding Hamiltonian in rotating-wave approximation is[14]

$$H_I = \hbar[g_1(a^+|g\rangle\langle e| + a|e\rangle\langle g|) + g_2(a^+|i\rangle\langle g| + a|g\rangle\langle i|)], \quad (1)$$

where a and a^+ are the annihilation (creation) operators of cavity field, g_1 and g_2 are the atom-cavity coupling constants between $|e\rangle$ and $|g\rangle$, $|g\rangle$ and $|i\rangle$, respectively.

In the following, we set $g_1 = g_2 = g$ and $\hbar = 1$. Then, we introduce the criterion of judging the existence of non-classical properties. The quadrature operators for the two directions are defined as

$$X = \frac{a + a^+}{2}, \quad Y = \frac{a - a^+}{2i}. \quad (2)$$

The variances satisfy the uncertainty relation:

$$\left\langle (\triangle X)^2 \right\rangle \left\langle (\triangle Y)^2 \right\rangle \geq \frac{1}{16}, \quad (3)$$

when one of the quadrature components is squeezed, i.e., $\langle(\triangle X)^2\rangle < \frac{1}{4}$ or $\langle(\triangle Y)^2\rangle < \frac{1}{4}$. To describe the magnitude of squeezing, we defined the parameters as[15]

$$q_x = \frac{\left\langle (\triangle X)^2 \right\rangle - 0.25}{0.25}, \quad q_y = \frac{\left\langle (\triangle Y)^2 \right\rangle - 0.25}{0.25}. \quad (4)$$

If $-1 \leq q_x < 0$ or $-1 \leq q_y < 0$, it implies that squeezing occurs in the corresponding quadrature.

We calculate the Mandel Q-parameters, which is defined as

$$Q = \frac{\left\langle (a^+a)^2 \right\rangle - \langle a^+a \rangle^2 - \langle a^+a \rangle}{\langle a^+a \rangle}. \quad (5)$$

When $-1 \leq Q < 0$, sub-Poisson statistics exists.

Now, we assume that the atom is initially on the upper level $|e\rangle$ and the cavity field is initially in coherent state $|\psi_f(0)\rangle = \sum_{n=0}^{\infty} C_n|n\rangle$,

$$C_n = e^{-\overline{n}/2} \frac{\alpha^n}{\sqrt{n!}}, \quad (6)$$

where $\overline{n} = |\alpha|^2$ is the averaging photon number. We simply assume that α is a real value for convenient to calculate. Then the initial state of the system is given by

$$|\psi(0)\rangle = \sum_{n=0}^{\infty} C_n|e, n\rangle. \quad (7)$$

After an interaction time t, the system evolves to

$$|\Psi(t)\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \frac{1}{2n+3} \cdot \{C_1|e, n\rangle + C_2|g, n+1\rangle + C_3|i, n+2\rangle\}, \quad (8)$$

where

$$C_1 = (n+1)\cos(gt\sqrt{2n+3}) + n + 2, \tag{9}$$

$$C_2 = -i\sqrt{(n+1)(2n+3)}\sin(gt\sqrt{2n+3}), \tag{10}$$

$$C_3 = \sqrt{(n+1)(2n+3)}\cos(gt\sqrt{2n+3}) - \sqrt{(n+1)(2n+3)}. \tag{11}$$

Then the atom is selectively measured. If the atom is found in the state $|e\rangle$, the field state collapses to

$$|\Psi'(t)\rangle = \frac{1}{N}\mathrm{e}^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\frac{1}{2n+3}C_1|n\rangle, \tag{12}$$

where N is a normalization factor.

We investigate the magnitude of squeezing of the cavity field in Eqs. (8) and (12) by numerically calculating the q_x and q_y, as shown in Figs. 1 and 2. The squeezing parameter q_x (q_y never shows squeezing) is plotted as a function of gt. We find that more photons respectively lead to a earlier and stronger squeezing in states shown in Eqs. (8) and (12). We also find that for the same average number of photons, the selective atomic measurement could make sharper peaks and smoother wave hollows, and accelerate the oscillation of q_x, especially when $\overline{n}$ is larger. Then in the case of $\overline{n} = 1$, when no direct atomic measurement is performed, no squeezing can be find, as shown in Fig. 1, but when the atom is detected in the state $|e\rangle$, the squeezing is emerged, which can be inferred from Fig. 2. For example, the squeezing could nearly reach up to 37.9% belows the vacuum noise level at around $gt\sim2$. We can also infer from Fig. 1 that when the average number of photons is 16, the squeezing can only approximate 22.6% for no state selective measurement is made on the atom. But when the atom is directly detected and found in the state $|e\rangle$, peaks are sharper, wave hollows are flatter, and the maximum squeezing will rise to nearly 59.3%, which show that the selective atomic measurement can bring about a stronger squeezing (see Fig. 2).

In Figs. 3 and 4, we plot the Mandel Q-parameters versus the scaled interaction time $T = gt$ (here we consider the condition of $0 \le gt \le 30$). Q-parameters show intermittent oscillation during the interaction time. We find that more photons respectively lead to a earlier but weaker sub-Poisson statistics in the same state, and the amplitude of the oscillation reduces to negative value in intermittent timeslices. In other words, sub-Poisson

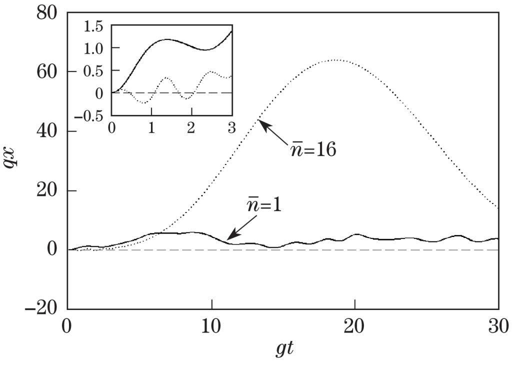

Fig. 1. Squeezing for single-mode cavity when no selective atomic measurement is made.

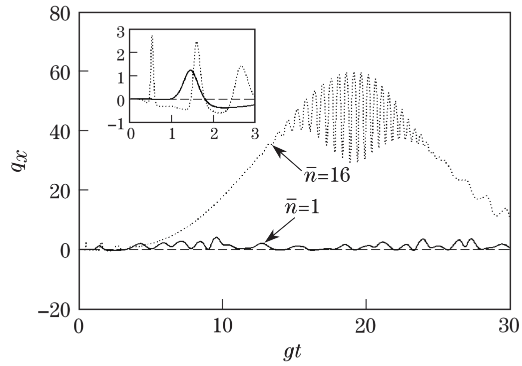

Fig. 2. Squeezing for single-mode cavity when the atom is detected in state $|e\rangle$.

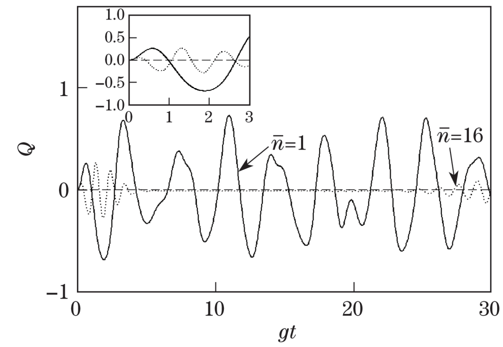

Fig. 3. Mandel Q-parameter for single-mode cavity when no selective atomic measurement is made.

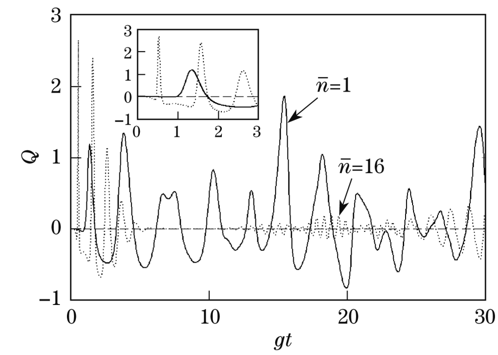

Fig. 4. Mandel Q-parameter for single-mode cavity when the atom is detected in state $|e\rangle$.

statistics could only exist in these timeslices. But when the atom is detected in state $|e\rangle$, we find the oscillates of Q become more apparent. Then in the case of $\overline{n} = 1$, the minimum Q reduces from –0.689 (before measurement) to –0.829 (after measurement), while in the case $\overline{n} = 16$, the minimum Q reduces from –0.283 (before measurement) to –0.68 (after measurement).

In conclusion, we study the non-classical properties exist in the single-mode coherent cavity field after resonantly interacting with a Ξ-type three-level atom. By comparing different cases of the system state, we find that the sub-Poisson statistics exists in the two field states of Eqs. (8) and (12), respectively. We also find that in the condition of more photons in the cavity, the squeezing will appear earlier and will be stronger under the same state. But the sub-Poisson statistics will be weaker, while sub-Poisson statistics and squeezing are more pronounced after the selective atomic measurement.

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