

# Small-scale self-focusing of divergent beam in nonlinear media with loss

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Bespalov-Talanov theory on small-scale self-focusing is extended to include medium loss for a divergent beam. Gain spectrum of small-scale perturbation is presented in integral form, and based on the derived equations we find that the cutoff spatial frequency for perturbation keeps a constant value. The larger the medium loss is, the smaller the fastest growing frequency and the maximum gain of perturbation with defined propagation distance are. For a given medium loss the maximum gain of perturbation becomes larger, while the fastest growing frequency becomes smaller as the propagation distance becomes longer. Furthermore, physical explanations for the appearance of these features are given.

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In high power laser systems, small-scale self-focusing is a paramount process that draws considerable attention, for it predominantly limits the maximum power available from solid-state lasers. Sources of the small-scale self-focusing may be regular owing to diffraction by confined apertures or random ones owing to various inhomogeneities in laser amplifier. Perturbations exhibiting small-amplitude or phase ripples originating from the sources can grow exponentially with propagation distance, after sufficient growth these perturbations evolve into intense ‘filaments’, causing degradation and breakup of laser beams. The worse is that nonlinear materials may be damaged if intensities of the filaments exceed the damage threshold of the materials. At first Bespalov *et al.* proposed the basic mechanism involved in small-scale self-focusing<sup>[1]</sup>. They presented a simple interpretation of the phenomenon and obtained an analytic expression of the exponential growth of ripples’ spectral components by means of linear approximation. Afterwards many further theoretical and experimental discussions appeared<sup>[2–3]</sup>. Bespalov-Talanov (BT) theory was extended to include effects such as beam-confining<sup>[4]</sup>, beam nonparaxiality<sup>[5]</sup>, and medium gain<sup>[6]</sup>. Recently a divergent beam has been shown as a prospect for suppressing small-scale self-focusing<sup>[7–8]</sup>. In Ref. [9], the extended BT theory of divergent beams was studied detailedly in lossless model, whereas in real laser systems optical intensity changes with propagation distance due to inevitable loss of medium, thus the supplements and refinement of the BT theory including the medium loss are desirable. In this letter, we investigate small-scale self-focusing of a divergent beam in nonlinear media with loss. The most possible damage of medium is determined by the maximum gain, while the fastest frequency is a key parameter that should be circumvented by spatial filter in practical systems, so their variation features are introduced in detail.

In nonlinear medium with loss, there are three physical processes, beam diffraction, medium loss, and nonlinear effects. They should be considered synthetically accord-

ing to quasi-steady paraxial nonlinear wave equation:

$$\nabla_{\perp}^2 E + 2jk \frac{\partial E}{\partial z} = -2k^2 \left[ \frac{\Delta n}{n_0} \right] E, \quad (1)$$

where  $k = \frac{2\pi}{\lambda} n_0$  is the wave number ( $n_0$  is the linear refraction index of the medium),  $\Delta n$  is the index change concerning nonlinearity and loss, the effective index takes on the form of  $\frac{n_2}{2} |A|^2 + j \frac{n_0}{2k} \alpha$ , where  $n_2$  is the nonlinear refractive index,  $\alpha$  is loss coefficient. A divergent beam in loss medium can be assumed as

$$E = \frac{RA(x, y, z)}{z} \exp\left(\frac{jk(x^2 + y^2)}{2z}\right), \quad (2)$$

where  $R$  is the initial radius,  $A(x, y, z)$  is the amplitude of electric field,  $z$  is the propagation distance within the range of  $[R, \infty)$  and can be expressed as  $z = R + Z$ , where  $Z$  is the propagation distance in the nonlinear medium with loss. With the relation of the coordinate transformation in the new coordinate system ( $x' = \frac{R}{z}x$ ,  $y' = \frac{R}{z}y$ ,  $z' = R - \frac{R^2}{z}$ ),  $A(x, y, z)$  is converted into  $\tilde{A}'(x', y', z')$ . Substituting Eq. (2) into Eq. (1) and making use of effective index replacement and coordinate transformation, we deduce the wave propagation equation as

$$(\nabla')_{\perp} A' + 2jk \frac{\partial A'}{\partial z'} = -k^2 \frac{n_2 |A'|^2}{n_0} - \frac{R^2}{(R - z')^2} jk \alpha A', \quad (3)$$

where  $(\nabla')_{\perp} = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2}$ . As in the same way of the BT theory, the electric field can be expressed as the supposition of the background field and the small-scale perturbation field:

$$A'(x', y', z') = A'_s(z') [1 + u'(x', y', z') + j\nu(x', y', z')], \quad (4)$$

Where  $u'(x', y', z')$  and  $\nu(x', y', z')$  are the real and imaginary parts of the normalized perturbation fields, satisfying the condition  $|u'|^2 = 1$  and  $|\nu'|^2 = 1$ . Form Eq. (3), the background field is obtained as

$$A'_s(z) = A_0 \exp \left[ -\alpha \left( \frac{R^2}{R-z'} \right) + jk \int_0^{z'} \frac{n_2}{2n_0} |A_s|^2 dz' \right], \quad (5)$$

where  $A_0$  is the initial amplitude of the background field. Then, substituting Eqs. (5) and (4) into Eq. (3), following the reverse coordinate system transformation, and after a series of complicated derivation, we acquire two coupled linear differential propagation equations:

$$\begin{cases} \nabla_{\perp}^2 u - 2k \frac{\partial \nu}{\partial z} = -2k^2 \frac{n_2 R^2}{n_0 z^2} A_0^2 [\exp(R-z)\alpha] u \\ \nabla_{\perp}^2 \nu + 2k \frac{\partial u}{\partial z} = 0 \end{cases} \quad (6)$$

From the right side of the first formula of Eq. (6), the dependence of  $z$  results from two physical processes, divergent wavefront and medium loss, moreover it can be found that the two effects are mutual independent. When  $R \rightarrow \infty$ , Eq. (6) becomes consistent with Eq. (4) in Ref. [10]. We further let  $\beta = 0$ , and then the result is the same as that in the BT theory. According to the conventional approach, the small-scale perturbation can be expanded into a series of plane waves with different directions, so by Fourier transform of Eq. (6), evolution of each spectral component can be obtain as

$$\begin{cases} \frac{dU}{dz} = \left[ \left( \frac{2k^2 n_2 A_0^2 R^2}{n_0 z^2} \exp[\alpha(R-z)] - q_{\perp}^2 \right) / (2k) \right] V \\ \frac{dV}{dz} = \frac{q_{\perp}^2}{2k} U \end{cases}, \quad (7)$$

where  $q_{\perp}^2 = q_x^2 + q_y^2$  is spatial frequency inversely proportional to the transverse size of the perturbation,  $U(q_x, q_y, q_z)$  and  $V(q_x, q_y, q_z)$  are respectively Fourier spectrums of  $u$  and  $v$ . Making  $q_c = \frac{q_{c0} R}{z} \exp[\frac{\beta(R-z)}{2}]$ , ( $q_{c0} = \sqrt{\frac{2k^2 n_2 A_0^2 R^2}{n_0}}$ ) and supposing the complex amplitude having character of exponential growth:

$$\begin{cases} U = U_0 \exp[\eta(z)] \\ V = V_0 \exp[\eta(z)] \end{cases}, \quad (8)$$

where  $\eta(z)$  is the growth parameter or called gain which is taken to be a real quantity, and then inserting Eq. (8) into Eq. (7), we obtain

$$\frac{d\eta}{dz} = \frac{q_{\perp}^2}{2k} (q_c^2 - q_{\perp}^2)^{1/2}. \quad (9)$$

The equation denotes that different plane wave spectra have different growths, and the maximum gain corresponds to the fastest growing frequency. For a given propagation distance  $Z_p$ , the gain is given by

$$\eta(Z_p) = \begin{cases} \int_0^{Z_p} \frac{q_{\perp}}{2k} \sqrt{q_c^2 - q_{\perp}^2} dZ & 0 < q_{\perp} < q_c(Z_p) \\ \int_0^{Z_n} \frac{q_{\perp}}{2k} \sqrt{q_c^2 - q_{\perp}^2} dZ & q_c(Z_p) < q_{\perp} < q_{c0} \end{cases}, \quad (10)$$

where  $Z_n$  is the root of equation  $q_{\perp} = q_c(Z_n)$  within  $[0, Z_p]$ . It can be seen that Eq. (10) cannot be solved in a closed form, so we resort to numerical calculation.

The initial intensity for the calculation is  $7.8 \text{ GW/cm}^2$ , then  $q_{c0} = 154 \text{ cm}$  and the gain spectrum of perturbation with different medium loss with  $R = 1000 \text{ cm}$  at  $Z_p = 20 \text{ cm}$  is given in Fig. 1(a). It can be seen that the maximum gain and the fastest growing frequency in lossy medium are all smaller than that in a lossless one. As medium loss become larger, the fastest growing frequency and the maximum gain of the perturbation become correspondingly smaller. Figure 1(b) clearly shows that the maximum gain decreases monotonously and the fastest growing frequency basically decreases linearly with the increase of medium loss. We try to fit the curve and obtain that the slope is  $-509.47$ . Initial radius denotes the degree of diffraction divergence, changing the initial radius to be  $200 \text{ cm}$  at  $Z_p = 20 \text{ cm}$ , and the gain spectrum is given in Fig. 2(a), besides, the variations of the maximum gain and the fastest growing frequency versus medium loss are also demonstrated in Fig. 2(b). The slope of the line fitted of the fastest growing frequency is  $-503.53$ . Comparing Figs. 1(a) and (b) with Figs. 2(a) and (b), we find that cut-off frequency keeps a constant value of  $q_{c0}$ . Smaller initial radius induces smaller gain, which can be easily understood that smaller initial radius means larger divergence and smaller local background intensity which is decisively important for suppression of growth of each spectral component. In the same way, medium loss decreases background intensity to a certain degree, so spectral component growth is attenuated accordingly. It can be concluded that the larger the medium loss is, the smaller the maximum gain and the fastest growing frequency are. Divergent wavefront and medium loss both afford to reduce background beam intensity essentially.

At  $Z_p = 30 \text{ cm}$ , the gain spectrum is depicted in Fig. 3(a). Compared with Fig. 1(a), we can see that overall spectral components have varying degrees of growth

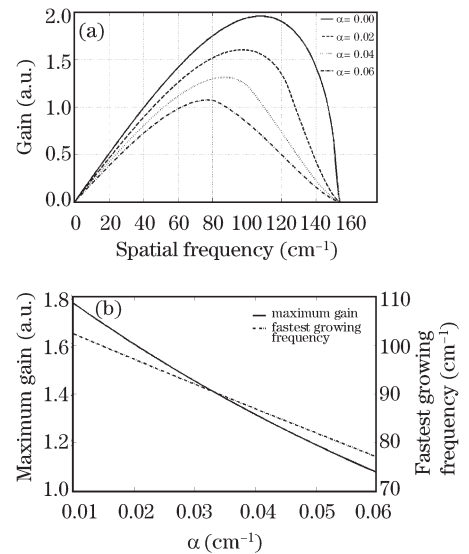


Fig. 1. (a) Gain spectrum of small-scale modulation with different loss coefficients. (b) Fastest growing frequency and maximum perturbation gain of small-scale modulation with loss coefficients.  $R = 1000 \text{ cm}$ ,  $Z_p = 20 \text{ cm}$ .

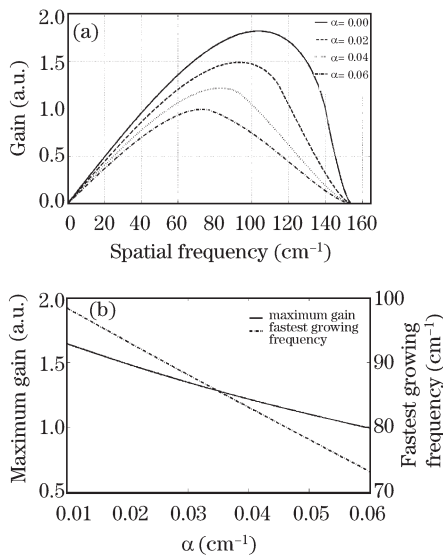


Fig. 2. (a) Gain spectrum of small-scale modulation with different loss coefficients. (b) Fastest growing frequency and maximum perturbation gain of small-scale modulation with loss coefficients.  $R = 200$  cm,  $Z_p = 20$  cm.

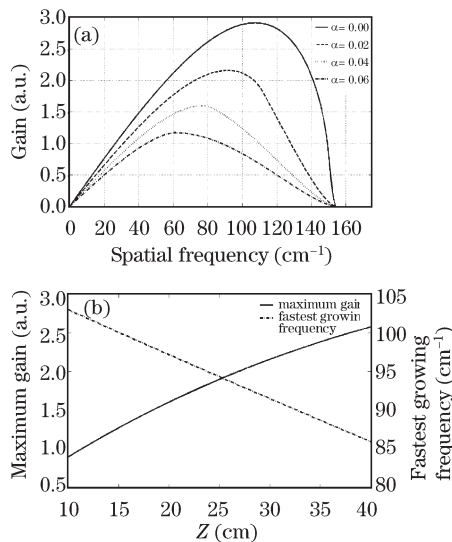


Fig. 3. (a) Gain spectrum of small-scale modulation with different loss coefficients.  $R = 1000$  cm,  $Z_p = 30$  cm. (b) Fastest growing frequency and maximum perturbation gain of small-scale modulation with propagation distance.  $R = 1000$  cm,  $\alpha = 0.02$  cm<sup>-1</sup>.

with longer propagation distance. The maximum gain increases greatly and the fastest growing frequency shifts to a smaller frequency component. Figure 3(b) gives variations of the maximum gain and the fastest growing frequency versus propagation distance. It can be found

that the fastest growing frequency decreases linearly with the increase of propagation distance, and the slope of the line fitted is  $-0.569$ . In fact, the maximum gain increases unsurprisingly with the increase of propagation distance, and the growth of each spectrum component accumulates gradually as the propagation distance becomes longer, resulting in the increase of the maximum gain. As is known to all, background intensity evolves to be smaller following the longer propagation distance. In this condition, smaller spectral components grow faster than larger ones, so the fastest growing frequency moves to a smaller spectrum component. That is to say, the shift direction of the fastest growing frequency can be determined by changing the trend of background intensity with the propagation distance.

In conclusion, a small-scale self-focusing of divergent beam in nonlinear medium with loss is studied based on the extended BT theory. The fastest growing frequency and the maximum gain of perturbation become smaller with the increase of the medium loss for the defined propagation distance. For a given medium loss, the maximum gain of perturbation becomes larger, while the fastest growing frequency becomes smaller as the propagation distance becomes longer, so in practical systems we should select proper parameters to purposefully suppress the growth of perturbation when using the spatial filter. For example, when we know the input intensity of the beam and the length and loss of nonlinear media, we can obtain the fastest growing frequency, and then the pinhole size in spatial filters should be designed optimally to filter this frequency for beam smoothing.

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