Resonant characteristics of microcantilever by using self-mixing interferometer based on phase reconstruction method

Shaoqing Wang (王少卿)[1,2*] and Xiangzhao Wang (王向朝)[1]

[1] *Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China*

[2] *Graduate University of the Chinese Academy of Sciences, Beijing 100049, China*

**E-mail: wangsq@siom.ac.cn*

Received April 23, 2009

The self-mixing interferometer is used to investigate the characteristics of resonant frequency of the microresonator, which is excited by a sinusoidally driven loud-speaker. The detected self-mixing signal is processed by the phase reconstruction method. The 1st-order resonant frequency of the microresonator is measured to be 4.437 kHz with full-width at half-maximum (FWHM) of 0.13 kHz. The measurement results are verified by the sinusoidal phase modulating (SPM) interferometer.

OCIS codes: 120.0120, 220.4840, 230.5750.

doi: 10.3788/COL20100802.0177.

Microcantilever is a device with a beam vibrating at resonance frequency, and the resonant frequency changes as a function of physical or chemical parameters. Owing to the advantages such as long-distance signal transmission and no need of analog/digital (A/D) conversion, microcantilever can be effectively used in the measurement of temperature and pressure, femtogram mass detection, sensitive biological detection, etc[1−4].

There have been many publications on interferometric measurement in microresonators[5−8]. But all these methods use the conventional interferometer, and they are complex structure and sensitive to perturbation and collimation. Owing to the characteristics of simple structure, auto collimating, and high sensitivity, the self-mixing interferometer have attracted more and more attention[9−13]. The self-mixing interferometer can not only avoid the disadvantage of conventional laser-interference system, but also have many advantages which are especially helpful to study the characteristics of microcantilever, for example, no need for critical alignment, ability to operate on diffusive surface, only moving parts contribute to the self-mixing signal, sub-wavelength sensitivity up to several hundred kHz is easily achieved and fabricated with microcantilever on one chip[14].

In this letter, we report the measurement results of a microcantilever by using self-mixing interferometer based on the phase reconstruction method. The microcantilever is excited by a loud-speaker. The measurement results are compared with the experimental data obtained by the sinusoidal phase modulating (SPM) interferometer.

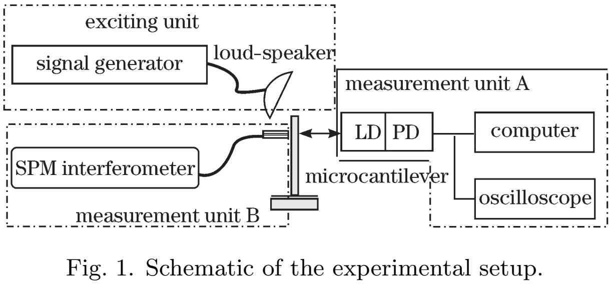

Fig. 1. Schematic of the experimental setup.

The experimental setup is shown schematically in Fig. 1. It is composed of an excitation unit and two measurement units. A microcantilever was placed in the middle of this system. In the excitation unit, a loud-speaker was driven by a signal generator. In the measurement unit A, a self-mixing interferometer was established by taking the surface of microcantilever as the external reflecting component. In the measurement unit B, a conventional SPM interferometer was used to measure the vibration of the microcantilever.

As we all know, the microcantilever has its mechanical resonant frequency, which can change with the environment conditions. The basic resolution of self-mixing interferometry is about $\lambda_0/2$ by using the common fringe counting method. Different methods have also been proposed to increase the basic resolution of $\lambda_0/2$ such as phase measurement[15], heterodyne interference[16], phase lock[17], and so on. However, these techniques require approximate photo diode (PD) signal linearization and external optical components, and are not fully compatible with real-time microcantilever vibration measurements. Without using external optical components, the direct data processing method of self-mixing signal is also researched and the principle method can be summarized as follows[18].

When the microcantilever is excited to resonant vibration, the self-mixing phenomenon can be observed, and then the angular frequency of laser with optical feedback ω_F is no longer the angular frequency of solitary laser ω_0 but a function of time t.

Since $\phi(t)$ equals $\omega\tau$, $\phi_\mathrm{F}(t)$, which denotes the frequency fluctuations of time t, can be obtained by solving the phase equation:

$$\begin{aligned}\phi_0(t) &= \phi_\mathrm{F}(t) + C\sin\left[\phi_\mathrm{F}(t) + \arctan(\alpha)\right]\\ &= F\left[\phi_\mathrm{F}(t);\alpha, C\right],\end{aligned} \tag{1}$$

where τ is the light round trip time in the external cavity, α is the linewidth enhancement factor of semiconductor laser, and C is the optical feedback factor.

1671-7694/2010/020177-04

The output light intensity $I_{\mathrm{F}}(t)$ can be written as

$$I_{\mathrm{F}} = I_0 \left(1 + m \cos\left(\phi_{\mathrm{F}}(t)\right)\right). \tag{2}$$

Equation (1) defines a nonlinear function of $F\left[\phi_{\mathrm{F}}(t); \alpha, C\right]$, which plays an important role in the phase reconstruction method used in this letter. Actually, $\phi_0(t)$ can be derived from Eq. (1) if an estimation of $\phi_{\mathrm{F}}(t)$ can be obtained.

The principle of the phase reconstruction method is shown in Fig. 2. The phase reconstruction method consists of two main signal process steps. The first step deals with the rough estimation of phase $\phi_{\mathrm{F}}(t)$ based on the nonlinear characteristic of Eq. (2), and then the estimation phase $\widehat{\phi}_{\mathrm{F}}(t)$ is obtained. Actually the self-mixing signal exhibits many discontinuities owing to the nonlinear behavior of Eq. (1), and the phase jump corresponding to these light intensity discontinuities equals $\Delta\phi_{\mathrm{F}} = 2\pi C/1 + C$. This expression of phase jump indicates that when C increases, $\Delta\phi_{\mathrm{F}}$ is close to but lower than 2π. So when the light intensity discontinuity occurs, the rough estimation of $\phi_{\mathrm{F}}(t)$ consists in adding 2π to or subtracting 2π from $\phi_{\mathrm{F}}(t)$, depending on the sign of the discontinuity. After the gain control (GC) of light intensity $I_{\mathrm{F}}(t)$, we have $\widehat{I}_{\mathrm{F}}(t) = (I_{\mathrm{F}}/I_0 - 1)/m$, and then $\phi_{\mathrm{F}}(t) \bmod \pi$ is obtained by using an arcos function to $\widehat{I}_{\mathrm{F}}(t)$. In order to get the estimation phase $\widehat{\phi}_{\mathrm{F}}(t)$, we also need to detect the transition of $I_{\mathrm{F}}(t)$ by detecting the peaks of the derivation of $\cos^{-1}\widehat{I}_{\mathrm{F}}(t)$. Generally speaking, a simple method can be used to detect these peaks, when the self-mixing signal is clean enough. However, the self-mixing signal generated by vibration of the microcantilever is always noisy owing to the low-frequency mechanical coupling. Simple method will lead to wrong estimation of the phase. In this letter, the wavelet transform mothed is employed to detect the transition of the self-mixing signal, then the self-mixing signals are broken into the shifted and scaled versions of the mother wavelet with an approximate part and several detailed parts. So the self-mixing signals are decomposed into coefficients, which are related to the scales of these parts mentioned above. The maximum of the coefficients of the detailed parts of the decomposed signal, which is corresponding to the transition of the self-mixing signal, can be easily obtained. After each transition is detected in order to add 2π to or subtract 2π from the phase $\phi_{\mathrm{F}}(t)$, the rough estimation of phase $\widehat{\phi}_{\mathrm{F}}(t)$ is finally obtained. It should be noticed that this phase estimation process is an approximate process because the real phase jump depends on C. So we need choose proper values of C and α to correct the estimation of phase $\widehat{\phi}_{\mathrm{F}}(t)$.

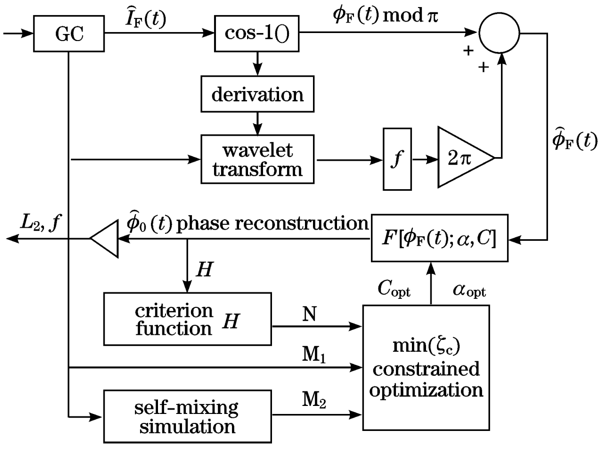

Fig. 2. Principle of phase reconstruction of the self-mixing signal.

As it is mentioned above, the second step of the phase reconstruction method deals with the constrained optimization of parameters C and α. This optimization is based on the target displacement discontinuities, which are far less frequent than the discontinuities of $\phi_{\mathrm{F}}(t)$ and $I_{\mathrm{F}}(t)$ caused by the nonlinear behavior of Eqs. (1) and (2), respectively. The reconstruction phase $\widehat{\phi}_0(t)$ can be obtained from the estimation phase $\widehat{\phi}_{\mathrm{F}}(t)$. We use a criterion function H to detect the discontinuities of $\widehat{\phi}_0(t)$ and denote the number of discontinuitie of $\widehat{\phi}_0(t)$ as N. Meanwhile we define the function $\zeta_1 = N$ and denote the number of discontinuities of $I(t)$ as M_1. We also calculate the simulated self-mixing signal $I_2(\mathrm{t})$ by using the reconstruction phase $\phi_{\mathrm{F}}(t)$, and denote the number of discontinuities of $I_2(\mathrm{t})$ as M_2. Define the function $\zeta_2 = |M_1 - M_2|$ and the evaluation function $\zeta_{\mathrm{c}} = k_1\,\zeta_1 + k_2\zeta_2$, where k_1 and k_2 are the weight factors, and then the optimum parameters C_{opt} and α_{opt}, which lead to a discontinuity minimization of the reconstructed phase $\widehat{\phi}_0(t)$, can be obtained by solving Min(ζ_{c}). By correcting the reconstruction phase $\widehat{\phi}_0(t)$ based on the optimum paramters C_{opt} and α_{opt}, the resonant characteristics of microcantilever can be easily calculated from $\widehat{\phi}_0(t)$.

In the experiment, the microcantilever (Si/Si02/Al) had a length of 700 μm, width of 100 μm, and thickness of 50 μm. The surface of the microcantilever beam was coated by an aluminum film. A Hitachi HL7851G 785-nm multiple quantum well (MQW) semiconductor laser with maximum output power of 30 mW, which was driven by a direct current (DC), was used in the self-mixing interferometer in measurement unit A. The loud-speaker, which was sinusoidally driven by a signal generator, was placed near the microcantilever. The distance between the microcantilever and the semiconductor laser was about 5 cm, which could be considered as the short cavity regime. This external cavity length depended on the focusing length of the microscope objective. The voltage of output DC of the signal generator was 10 V. The self-mixing signal was detected by the PD packaged in the semiconductor laser, and the signal was output to a computer and an oscilloscope.

Since the microcantilever has a small size, one important step in the experiment is to obtain a small light spot. In the measurement unit A, the elliptical light beam of the semiconductor laser was collimated to a parallel light beam by using two convert lenses. After focusing the parallel light beam by a 40 × microscope objective with a focal length of about 5 cm, a light spot with diameter of near 30 μm was obtained. The main factor affecting the size of the focus light spot is the parallel light beam quality. Using the gradient-index lens instead of the conventional optical collimating system, smaller light spot will be obtained.

It is well know that exciting a linear system by a white-noise is equivalent to sweeping the frequency of a sinusoidal signal. But in the experiment using white-noise to excite the microcantilever is not the suitable method to obtain the resonant frequency. The microcantilever is fabricated in the center of a silicon substrate, and the silicon substrate is only 5 mm long and 3 mm wide. Compared with the microcantilever, the size of the substrate is large. But compared with the loud-speaker, both the sizes of the substrate and of the microcantilever are quite small. So the white-noise can easily excite the substrate to resonant vibration state, even the high-order resonant state of the substrate can also be achieved owing to the high efficiency of acoustic excitation. The vibrating amplitude of the substrate is much greater than that of the microcantilever. So in the frequency domain of the self-mixing signal, the resonant frequency of the microcantilever is submerged in the noise and very difficult to be detected. So in the experiment, we still used the method of sweeping frequency to excite the microcantilever. We also used a SPM interferometer to monitor the vibration of the substrate by placing the probe of the SPM interferometer close to the substrate. We firstly tuned the modulation frequency of DC of the loud-speaker from 10 Hz to 10 MHz, and then estimated the possible resonant frequency of the microcantilever by monitoring the self-mixing signal displayed in the oscilloscope. Meanwhile, the resonant frequency of the substrate could be found and filtered by using SPM interferometer. Here we used the 1st-order resonant frequency of the microcantilever as the modulation frequency of the loud-speaker because the amplitude of the 1st-order resonant vibration was the maximum compared with those of the 2nd-order and 3rd-order resonant vibration. After evaluating the 1st-order resonant frequency, we adjusted the modulation frequency of the loud-speaker. Since the vibration amplitude of the microcantilever was very sensitive to the vibration frequency of the loud-speaker, the resonant vibration state of the microcantilever could be easily achieved and maintained. After the signal processing by means of phase reconstruction method, we finally measured the resonant vibration amplitude and frequency of the microcantilever. In measurement unit B, the SPM interferometer permitted a precise and calibrated measurement of the microcantilever vibration.

Figure 3 shows a self-mixing signal detected by measurement unit A when the microcantilever was excited in its 1st-order resonant vibration. With the phase reconstruction method, we calculated that the mean resonant vibration amplitude and frequency of the microcantilever, being 540 nm and 4.437 kHz, respectively. The full-width at half-maximum (FWHM) of the microcantilever's resonant response curve was 0.13 kHz, indicating a Q value of 64.6. The measurement resolution of resonant frequency can reach 1 Hz. The resonant vibration was also measured by the SPM inteferometer, and the mean measurement values of the vibration amplitude and frequency were 543 nm and 4.43 kHz, respectively. We found that the measurement results of unit A agreed well with those of unit B.

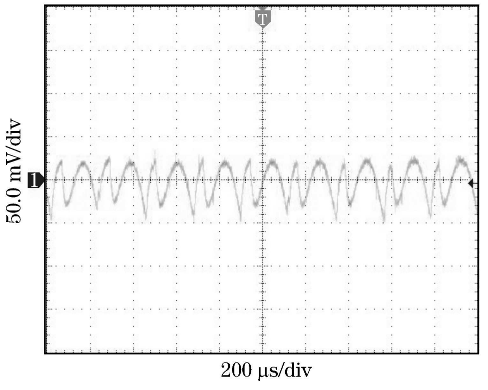

Fig. 3. Self-mixing signal detected by PD packaged in the LD.

Although the self-mixing interferometer's measurement resolution of the vibration amplitude is lower than that of the SPM interferometer. In the research of microcantilever, the most important parameter of microcantilever is its resonant frequency, because the key sensing mechanism of microcantilever is measuring variation of the resonant frequency caused by temperature, press, mass, and so on. In the experiment, we found that the measurement resolution was better than that of the conventional SPM interferometer. It is also worthy to mention that the structure of self-mixing interferometer is much simpler than that of SPM interferometer, and its cost is also lower than that of SPM interferometer.

In conclusion, vibration characteristics of microcantilever are investigated by using self-mixing interferometer, and the measurement results prove that self-mixing interferometer is a valuable tool for microresonator characterization, since it directly measures the microcantilever vibration, and it can work on a diffusive surface and be implemented with a very simple optical setup. A phase reconstruction method is used in this letter. The experimental results show that the measurement resolution of resonant frequency can reach 1 Hz. Our measurement results agree well with those obtained by SPM interferometer. By embedding the phase reconstruction method into the field programmable gate array (FPGA), the microcantilever, the self-mixing interferometer, and the FPGA can be fabricated on one chip, which means a novel resonant sensor with very small size, low cost, and high sensitivity. Research on this system has a wide range of potential applications.

This work was partially supported by the National Natural Science Foundation of China (No. 60578051) and the Key Basic Research Program of Science and Technology Commission of Shanghai Municipality (No. 07JC14056).

References

1. U. Gysin, S. Rast, P. Ruff, E. Meyer, D. W. Lee, P. Vettiger, and C. Gerber, Phys. Rev. B **69,** 045403 (2004).
2. C. J. Welham, J. W. Gardner, and J. Greenwood, Sens. Actuat. A **52,** 86 (1996).
3. N. V. Lavrik and P. G. Datskos, Appl. Phys. Lett. **82,** 2697 (2003).
4. Göran Stemme, J. Micromech. Microeng. **1,** 113 (1996).
5. D. Ramos, J. Tamayo, J. Mertens, and M. Calleja, J. Appl. Phys. **99,** 124904 (2006).
6. R. Puers, D. De Bruyker, and A. Cozma, Sens. Actuat.

A **60,** 68 (1997).

7. Y. Liu, X. Wang, and X. Wang, Chin. Opt. Lett. **4,** 1 (2006).
8. Y. Liu and X. Wang, Chinese J. Lasers (in Chinese) **33,** 1661 (2006).
9. W. M. Wang, K. T. V. Grattan, A. W. Palmer, and W. J. O. Boyle, J. Lightwave Technol. **12,** 1577 (1994).
10. T. Bosch and N. Servagent, Opt. Eng. **40,** 20 (2001).
11. C. Zakian and M. Dickinson, J. Opt. Pure Appl. Opt. **8,** 555 (2006).
12. G. Plantier, C. Bès, and T. Bosch, IEEE J. Quantum Electron. **41,** 1157 (2005).
13. N. Servagent, T. Bosch, and M. Lescure, IEEE Trans. Instrum. Meas. **46,** 847 (1997).
14. V. Annovazzi-Lodi, S. Merlo, and M. Norgia, J. Microelectromech. Syst. **10,** 327 (2001).
15. T. Yoshino, M. Nara, S. Mnatzakanian, B. S. Lee, and T. C. Strand, Appl. Opt. **26,** 892 (1987).
16. N. Takahashi, S. Kakuma, and R. Ohba, Opt. Eng. **35,** 802 (1996).
17. T. Suzuki, S. Hirabayashi, O. Sasaki, and T. Maruyama, Appl. Opt. **38,** 543 (1999).
18. G. Plantier, C. Bes, T. Bosch, and F. Bony, in *Proceedings of IMTC* 1013 (2005).