Monte-Carlo simulation of effective stiffness of time-sharing optical tweezers

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The Brownian motion of a polystyrene bead trapped in a time-sharing optical tweezers (TSOT) is numerically simulated by adopting Monte-Carlo technique. By analyzing the Brownian motion signal, the effective stiffness of a TSOT is acquired at different switching frequencies. Simulation results confirm that for a specific laser power and duty ratio, the effective stiffness varies with the frequency at low frequency range, while at high frequency range it keeps constant. Our results reveal that the switching frequency can be used to control the stability of time-sharing optical tweezers in a range.

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Time-sharing optical tweezers (TSOT) is a very effective technique that produces multiple optical tweezers using a single laser beam to stretch bio-molecules and human red blood cells[1−3]. Traditionally, TSOT adopts acousto-optic deflector[4] (AOD) or a piezoelectric scanning mirror[5,6] to translate light slightly at different frequencies to form quasi-stationary multiple optical traps[7] or oscillatory traps[8]. With the aid of two orthogonally mounted AODs, array optical tweezers is generated by time sharing the laser beam among several positions, and the geometry of the multi-spot pattern is controlled via the deflector system. Recently, Wu *et al.* developed a novel method to construct time-sharing optical tweezers based on tilt rotating glass plate[2]. For both the traditional and the novel techniques forming TSOT, it is a great challenge to study the stability property because of the small bandwidth of detector compared with Brownian motion signals of trapped beads. Meanwhile, the stability is of great importance to control the force at each trap independently from others by feedback[9,10].

To our knowledge, there is little quantitative report about the stability of TSOT at higher switching frequencies since the bandwidth of detectors restricts the simultaneous measurement of stiffness of multiple traps. Quantitatively, the use of fast beam deflectors is of crucial importance as the time when the trap is "off", servicing another position, has to be shorter than the time the particle needs to diffuse away from its trapping position[4]. The more time the trap is "on", the stiffer the trap is. To better understand the stability property of time-sharing multiple optical traps, we use Monte-Carlo technique to simulate the motion of a bead in a time-sharing optical trap in a large frequency domain, and numerically calculate the effective stiffness of TSOT according to equipartition theorem.

TSOT generates multiple optical tweezers by sharing a single laser beam with different trap positions. The laser beam serves a certain trap at a time interval and immediately switches to another position to form a new one. For a certain position, the laser switches on and off periodically. Sequential diagram of a trap formed through time-sharing technique is shown in Fig. 1. The ratios of durations with laser "on" and "off" is defined as duty ratio of a TSOT in the following form:

$$D = \frac{a}{b}, \tag{1}$$

where a and b are durations with laser "on" and laser "off" correspondingly, and the sum of them is trap switching periodicity T. Accordingly, the trap switching frequency $f_{\rm sw}$ can be written as

$$f_{\rm sw} = \frac{1}{T} = \frac{1}{a+b}. \tag{2}$$

Because the fast beam deflection is very well achieved by the use of AOD and the rising time to produce different trap positions is of the order of μs, which is smaller than the trap switching periodicity of several orders, the rising time is neglected in our simulation model as a proper assumption.

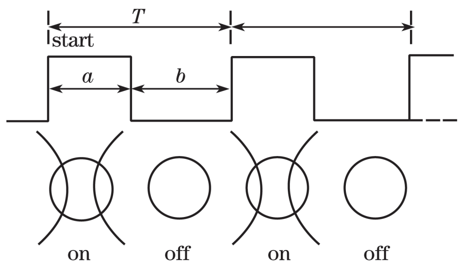

Fig. 1. Sequential diagram of TSOT.

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Generally, the Brownian motion of a polystyrene bead in aqueous solution is classified into two categories. The first one is that the bead does confine Brownian motion with trapping laser "on", while the second is that the bead does free Brownian motion without the exposure of trapping laser. The motion equation of a bead is

$$m\ddot{x} + \gamma\dot{x} + k_x x - F_{\mathrm{rand}}(t) = 0, \tag{3}$$

where $\gamma = 6\pi\eta R$, R is the radius of the bead, η is the viscosity coefficient of aqueous solution, $F_{\mathrm{rand}}(t)$ is the random force, the time average of which is zero, and k_x is the nominal stiffness of an optical trap without modulation of the power. In low Reynolds number case, the polystyrene bead can be considered as an over-damping oscillator, and thus the inertia force term can be ignored. Therefore, Eq. (3) can be simplified to

$$\gamma\dot{x} + k_x x = F_{\mathrm{rand}}(t). \tag{4}$$

When the laser switches on one of the multiple trapping positions, the confined Brownian motion of a trapped bead is simulated using Monte-Carlo technique[11]. The motion equation can be described as[12−14]

$$x_n = x_{n-1} + v_{n-1} \triangle t, \tag{5}$$

$$v_n = v_{n-1} - k_x x_{n-1} \triangle t/m + \sqrt{12\pi k_B T \eta R \triangle t/m^2} \times \sqrt{-2\log(u)}\cos(2\pi\nu) - v_{n-1}(6\pi\eta R/m) \triangle t, \tag{6}$$

where n indicates the ordinal number of a time step, x_n and v_n are the bead position and velocity correspondingly, u and v are uniformly distributed random numbers ranging in (0,1). When it comes to the state, the laser is off, the algorithm performs well just by eliminating the nominal stiffness term in Eq. (6).

Throughout the simulation, the temperature is set at T = 298 K with drag coefficient of aqueous solution $\eta = 0.894 \times 10^{-3}$ kg/m · s. The mass density of polystyrene bead is $\rho = 1.05 \times 10^{-3}$ kg/m^3. Initially, the simulated bead is at the equilibrium position with velocity of zero. When the laser is switched to be "on", the stiffness k_x in Eq. (6) equals 18 pN/μm, and while the laser is "off", k_x is set to be zero during simulation.

For continuous-wave laser tweezers, the stability property of a trap is characterized by a stiffness adopting equipartition theorem. Similarly, TSOT is characterized by effective stiffness, which has the form of

$$k_{\mathrm{eff}} = \frac{k_B T}{\langle x^2 \rangle}. \tag{7}$$

In the simulation, the time step is 10 ns, integration time of detector adopts 0.1 ms, and the total time for measurement is 1 s. Therefore, 10000 hits of Brownian motion are collected to generate effective stiffness according to Eq. (7).

The simulation shows that for 2-μm polystyrene bead diffused in distilled water, the effective stiffness increases with trap switching frequency under different duty ratios in a certain range, as shown in Fig. 2. Similar results for 3-μm polystyrene bead are shown in Fig. 3. The results for beads with diameter of both 2 and 3 μm indicate a general trend for the relationship between effective stiffness and switching frequency.

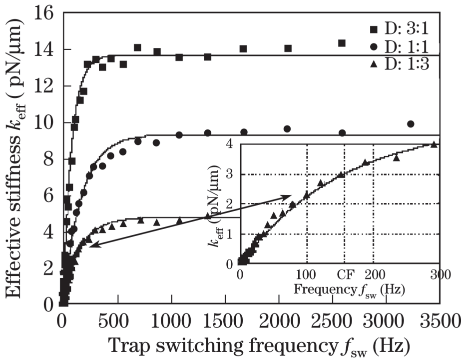

Fig. 2. Monte-Carlo simulated effective stiffness as a function of trap switching frequency for 2-μm polystyrene bead with duty ratios of 3:1, 1:1, and 1:3.

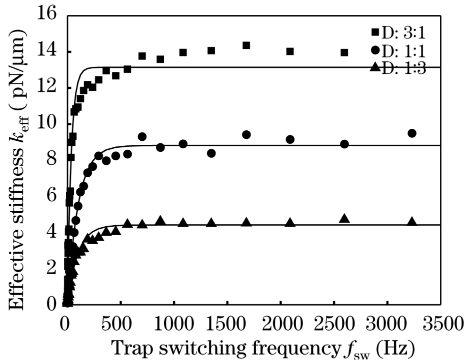

Fig. 3. Monte-Carlo simulation results of effective stiffness for 3-μm polystyrene bead with duty ratios of 3:1, 1:1, and 1:3.

To better understand the dependence of stiffness on the trap switching frequency, several models are tried to conclude that Box Lucas model[15], which was firstly introduced to describe the yield of intermediate product of a consecutive chemical reaction, fits well with our simulation dependence of effective stiffness on trap switching frequency. Accordingly, the relation is qualitatively described by $k_{\mathrm{eff}} = k_0 \cdot (1 - \exp(-f_{\mathrm{sw}}/f_{\mathrm{ch}}))$, where k_0 is transient-free stiffness and f_{ch} represents characteristic frequency. For 2-μm polystyrene bead trapped in TSOT with duty ratio 1:3, the values of the two parameters are $k_0 = 4.74 \pm 0.05$ pN/μm and $f_{\mathrm{ch}} = 156 \pm 6$ Hz. The coefficient of determination R^2 is 0.9839 which means the Box Lucas model fits well to the dependence of effective stiffness on switching frequency in a large frequency domain. The "CF" in the inset of Fig. 2 indicates where characteristic frequency locates for 2-μm bead trpped in TSOT with duty ratio of 1:3. Following the same procedure, the parameters k_0 and f_{ch} for beads with different diameters in TSOT with different duty ratios are listed in Table 1.

Actually, for higher frequency ranges, such as the case of femtosecond laser tweezers[16], increase of modulation frequency does not cause the variation of effective stiffness. The trap is stable subjected to the change of repetition rate of the femtosecond laser, and the effective stiffness varies with the average power of high repetition rate laser.

In lower frequency range, the model can be approxi-

Table 1. Fitted Parameters for 2- and 3-μm Polystyrene Beads

Duty Ratio	2-μm	
	k_0 (pN/μm)	f_{ch} (Hz)
3:1	13.65±0.12	78±3
1:1	9.30±0.10	169±7
1:3	4.74±0.05	156±6
	3-μm	
3:1	13.14±0.19	36±2
1:1	8.82±0.09	102±4
3:1	4.41±0.06	95±5

mated by a linear regression model with high accuracy. When the switching frequency is smaller than the characteristic frequency, namely $f_{\mathrm{sw}} < f_{\mathrm{ch}}$, simulation results show that it performs well even using linear regression model, which is of great importance when using unstable TSOT with low switching frequencies such as the case of studying the colloidal collision frequency.

Our simulation also show that for a certain bead trapped in TSOT with different duty ratios, the transient-free stiffness k_0 increases with duty ratio, which determines the average power of a certain trap. As for the same bead, the characteristic frequency varies with the duty ratio, and according to our simulation, the characteristic frequency with duty ratio of 3:1 is smaller than those with other two duty ratios both for beads with diameters of 2 and 3 μm. A proper explanation is that the effective stiffness transits to a stable value quicker than that with small duty ratio when increasing the trap switching frequency. Meanwhile, the characteristic frequencies with duty ratios of 1:1 and 1:3 are larger and closer to each other.

In conclusion, the effective stiffness of TSOT is studied by numerically simulating the Brownian motion signals of polystyrene beads through Monte-Carlo technique. Simulation results show that the effective stiffness varies with trap switching frequency in low frequency ranges, and keeps a constant value at high frequency ranges, which is determined by duty ratio and input laser power. The dependence of effective stiffness on trap switching frequency is well fitted by a Box Lucas model, the result of which indicates potential application to vary the effective stiffness by feedback control and to study the colloid stability using TSOT with tunable effective stiffness.

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