Pupil design based on Fisher information optimization to extend field depth in practical optical system

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Wavefront coding (WFC) is used to extend the field depth of an incoherent optical system by employing a phase mask on the pupil. We uses a Fisher information (FI) metric based optimization method to design a phase mask by taking the modulation transfer function (MTF) of the practical optical system into consideration. This method can modulate the wavefront so that the point spread function and optical transfer function are insensitive to the object distance. The simulation results show that the optimized phase mask based on the proposed method can further improve the defocusing image quality while maintaining the focusing image quality.

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Wavefront coding (WFC) is a novel method to extend the field depth of an incoherent optical system[1−3]. No modification but only an aspherical pupil phase mask is employed on the stop of the standard optical system as shown in Fig. 1. A simple air-spaced doublet is used here as an example. Focusing or defocusing images are blurred because the mask modulates the system's point spread function (PSF) and optical transfer function (OTF). The sharp image can be restored by one digital filter if PSF and OTF are insensitive to the object distance by employing a proper phase mask. Compared with the traditional method, which employs an optical power absorbing apodizer, WFC has advantages of maintaining the optical power and image resolution. Consequently, it has a prosperous application prospects in various optical systems[4,5], such as microscope, iris recognition system, and infrared (IR) system.

Traditional optimization method only considers the optimization of the cubic phase mask (CPM). But in order to use the WFC technology for the practical optical system, the system's characteristic must be taken into consideration. In this letter we proposed a new optimization method based on the system's Fisher information (FI) by considering the system's modulation transfer function (MTF) to find the optimized CPM. It is an effective method to design the pupil of the practical optical system.

Dowski *et al.* proved that the CPM can modulate the PSF and OTF to be insensitive to the object distances[1]. It is an interesting method to extend the field of the optical system without decreasing the aperture. The modified CPM used in this letter is shown in

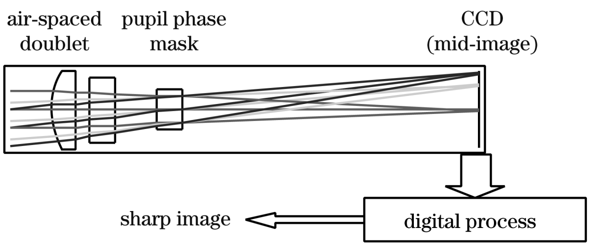


Fig. 1. WFC used in air-spaced doublet.

$$\theta(x, y, \alpha, \beta) = \alpha(x^3 + y^3) + \beta(x^2y + xy^2), \quad (1)$$

where (x, y) is a point in the pupil plane, α, β are the parameters of the pupil phase mask. The pupil function of a WFC system (a standard optical system employing the modified CPM in this letter) is of the form:

$$P(x, y, \alpha, \beta, \tau) = p(x, y) \exp\{\mathrm{i}[f(x, y) + \tau(x^2 + y^2) + \alpha(x^3 + y^3) + \beta(x^2y + xy^2)]\}, \quad (2)$$

where $p(x, y)$ equals 1 for all (x, y) inside the pupil and 0 outside, $f(x, y)$ is the phase function of the standard optical system, and τ is the defocus parameter. The PSF is proportion to the squared modulus of Fourier transform of the pupil function:

$$h(u, v, \alpha, \beta, \tau) = \kappa\{F[P(x, y, \alpha, \beta, \tau)](u, v)\} \times \{F^*[P(x, y, \alpha, \beta, \tau)](u, v)\}, \quad (3)$$

where (u, v) is the point in image plane; κ is the constant whose effect can be omitted; F denotes the operator of Fourier transform; $*$ stands for the complex conjugate.

FI has been introduced to estimate the similarity of the PSF with different object distance[6,7]. PSF can be considered as a joint probability density function, which is the same as likelihood function in this case. The score function[8] $S(u, v, \tau)$ is used to represent the sensitivity to the object distance:

$$S(u, v, \alpha, \beta, \tau) = \frac{\partial}{\partial \tau} \ln h(u, v, \alpha, \beta, \tau). \quad (4)$$

As the PSF become more sensitive to the object distance, the score function will be larger.

Then with $[]^{\mathrm{T}}$ denoting the transpose matrix, the FI metric can be expressed as

$$J(\alpha,\beta,\tau)=\iint h(u,v,\alpha,\beta,\tau)\left[\frac{\partial}{\partial\tau}\ln h(u,v,\alpha,\beta,\tau)\right]\left[\frac{\partial}{\partial\tau}\ln h(u,v,\alpha,\beta,\tau)\right]^{\mathrm{T}}\mathrm{d}u\mathrm{d}v. \quad (5)$$

i.e.,

$$J(\alpha,\beta,\tau)=\iint \frac{1}{h(u,v,\alpha,\beta,\tau)}\left[\frac{\partial}{\partial\tau}h(u,v,\alpha,\beta,\tau)\right]^2\mathrm{d}u\mathrm{d}v, \quad (6)$$

where

$$\frac{\partial}{\partial\tau}h(u,v,\alpha,\beta,\tau)=F[\mathrm{i}(x^2+y^2)P(x,y,\alpha,\beta,\tau)]F^*[P(x,y,\alpha,\beta,\tau)]+F^*[\mathrm{i}(x^2+y^2)P(x,y,\alpha,\beta,\tau)]F[P(x,y,\alpha,\beta,\tau)]. \quad (7)$$

As the field depth in the object space corresponds to the focus depth in the image space, the object distance l is used as variable instead of the defocus parameter τ. Then $P(x,y,\alpha,\beta,\tau)$ can be substituted by $P(x,y,\alpha,\beta,l)$, which can be obtained directly from the optical design software, such as Zemax and Code V. The proper parameters α,β of pupil phase mask will be obtained if they make

$$JF(\alpha,\beta)=\int[J(\alpha,\beta,l)]^2\mathrm{d}l \quad (8)$$

reach its minimum. However, the restorability must be taken into consideration. In this letter, we use a function of MTF as the metric of restorability. It is well known that the OTF is the Fourier transform of the PSF and the MTF is the magnitude of the OTF:

$$M(m,n,\alpha,\beta,l)=|F[h(u,v,\alpha,\beta,l)](m,n)|, \quad (9)$$

where (m,n) is a point in frequency plane. Considering about penalty factor μ, Eq. (8) is modified as

$$JF(\alpha,\beta)=\int[J(\alpha,\beta,l)]^2\mathrm{d}l-\mu\iiint M(m,n,\alpha,\beta,l)\mathrm{d}m\mathrm{d}n\mathrm{d}l. \quad (10)$$

The parameters α,β of the pupil phase mask can be acquired by minimizing Eq. (10).

The simulation of the optical system is shown in Fig. 1. Four object positions: 5000, 4000, 3000, and 2000 mm are considered. Here, we assume that the system is focusing when the object distance is 5000 mm.

Figure 2(a) describes the MTF of standard system; Fig. 2(c) describes the MTF of the WFC system with $\alpha=0.0037342$ and $\beta=0.00076923$, which is the optimization result based on FI metric ($JF=9.2005\times10^{-10}$); the MTF of the WFC system with $\alpha=0.0005$ and $\beta=0$ ($JF=3.4782\times10^{-8}$) is described as a reference in Fig. 2(b).

Images formed by these systems are described in Table 1. The first row of Table 1 corresponds to the images whose object distance is 5000 mm (focusing) and the second one is the results of 2000 mm (defocusing). It is obvious that the defocusing image of the standard system is very poor, not only because the cut-off frequency of MTF is very low, but also because MTF is sensitive to the object distance shown in Fig. 2(a).

WFC systems improve defocusing image quality to a great extent without obvious deterioration to the focusing image as Table 1 shows. Comparing the reference WFC system and the optimized WFC systems, the latter shown in Fig. 2(c) has a better defocusing performance because it is more insensitive than the former as shown in Fig. 2(b).

In conclusion, WFC system can extend the field depth of optical system while maintaining the quality of the focusing image. In order to design the optimization pupil mask we take the MTF of the practical optical system into consideration and get the system's FI. The employment of optimization method based on this FI metric in WFC will further improve the defocusing image quality. Therefore, WFC is an appealing technology and will be

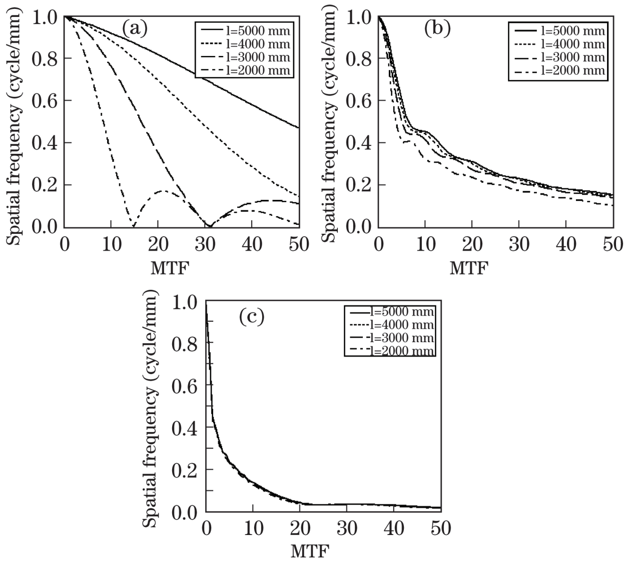
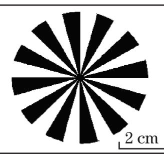
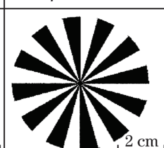
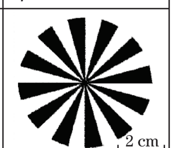
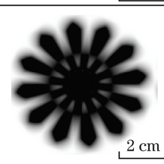
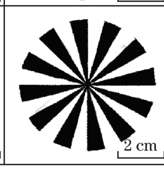
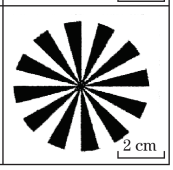


Fig. 2. MTF of (a) the standard system, (b) the WFC system with $\alpha=0.0005$ and $\beta=0$, and (c) the WFC system with $\alpha=0.0037342$ and $\beta=0.00076923$.

Table 1. Images by Standard Systems and WFC Systems

Object Distance (mm)	Standard System	WFC System with α=0.0005, β=0	WFC System with α=0.0037342, β=0.00076923
(a) 5000	 2 cm	 2 cm	 2 cm
(b) 2000	 2 cm	 2 cm	 2 cm

widely used in various practical optical systems in the future.

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