## Modeling of silicon-nanocrystal formation in amorphous silicon/silicon dioxide multilayer structure

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The formation process of silicon-nanocrystals (Si-NCs) in the amorphous silicon/silicon dioxide (a-Si/SiO<sub>2</sub>) multilayer structure during thermal annealing is theoretically studied with a modified model based on the Gibbs free energy variation. In this model, the concept of average effective interfacial free energy variation is introduced and the whole formation process consisting of nucleation and subsequent growth is considered. The calculating results indicate that there is a lower limit of the silicon layer thickness for forming Si-NCs in a-Si/SiO<sub>2</sub> multilayer, and the oxide interfaces cannot constrain their lateral growth. Furthermore, by comparing the results for a-Si/SiO<sub>2</sub> and a-Si/SiN<sub>x</sub> multilayers, it is found that the constraint on the crystal growth from the dielectric interfaces depends on the difference between interfacial free energies.

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Efficient silicon-based light source is one of the most popular topics in the studies on silicon (Si) photonics that has attracted many attentions due to its compatibility with existing semiconductor fabrication techniques. For this topic, various approaches have been explored, such as porous Si<sup>[1,2]</sup> and Si nanocrystals (Si-NCs) in bulk Sirich compounds<sup>[3-5]</sup> and multilavers<sup>[6,7]</sup>. Among them,</sup></sup> Si-NCs are holding the attention for their strong quantum confinement effects with stable optical properties, especially when multilayer structure is considered as a method to independently regulate the size and density of Si-NCs, and improve light emission with substantially lower driving voltage<sup>[6]</sup>. Nevertheless, compared with that in bulk Si-rich compounds<sup>[8]</sup>, the formation of Si-NCs in ultrathin amorphous Si (a-Si) layers covered with thin dielectric layers, such as SiO<sub>2</sub> and Si<sub>3</sub>N<sub>4</sub>, i.e., a-Si multilayers, has not been widely studied. The existing theories  $^{[9,10]}$  cannot well clarify the whole formation process of Si-NCs in multilayers by modeling nucleation and subsequent growth separately. In this letter, for better understanding and control of the Si-NC formation, a modified model is developed to unify the formation process based on the Gibbs free energy variation. According to the calculations using the proposed model, a lower limit for the a-Si layer thickness and incapability of constraint on the lateral growth of Si-NCs from the oxide interfaces are discovered in  $a-Si/SiO_2$  multilayers. Furthermore, the model could be readily applied to estimate the differences between interfacial free energies, which correspond to different dielectric materials. By comparing the results of  $a-Si/SiO_2$  and  $a-Si/SiN_x$  multilayers, we find that the constraint on the Si-NC growth from the dielectric interfaces depends on the difference between interfacial free energies.

In general, the formation of Si-NCs includes nucleation and the following growth, induced either by furnace annealing or pulsed laser<sup>[11]</sup>. Nucleation occurs homogeneously in the interior of the uniform amorphous phase or heterogeneously at preferential sites such as impurities, interfaces, and defects. After nucleation, Si-NCs would grow up with sufficient external thermal energy.

Here, we divide the whole formation process into two successive stages, i.e., pre-touch and post-touch of the dielectric interfaces. The multilayer is simplified as an a-Si/dielectric sandwich structure as shown in Figs. 1 and 2. In the pre-touch stage, a Si-NC is assumed to nucleate symmetrically in the middle of the a-Si layer with spherical shape with radius r. As in Ref. [9], spacing l is introduced to separate the crystal nucleus from the dielectric interfaces. When the Si-NC keeps growing up and intersects the top and bottom interfaces, the post-touch stage will begin. However, it will be shaped like a drum with lateral diameter beyond the layer thickness d. Moreover, we define  $\gamma_{\rm ac}$  as the interfacial free energy per unit area between the a-Si phase and the crystalline Si (c-Si) phase,  $\gamma_{dc}$  as that between the dielectric material and the c-Si phase, and  $\gamma_{da}$  as that between the dielectric material and the a-Si phase. According to the deductions in Ref. [9], we can then achieve the effective interfacial free energies  $\gamma_{dc}^{eff}$  and  $\gamma_{da}^{eff}$ :



Fig. 1. Schematic diagram of the Si-NC in the pre-touch stage.



Fig. 2. Schematic diagram of the Si-NC in the post-touch stage.

$$\gamma_{\rm dc}^{\rm eff} = \gamma_{\rm ac} + (\gamma_{\rm dc} - \gamma_{\rm ac}) e^{-l/l_0}, \qquad (1)$$

$$\gamma_{\rm da}^{\rm eff} = \gamma_{\rm da} e^{-l/l_0},\tag{2}$$

where the spacing distance between the crystal nucleus and the dielectric interface l=d/2-r, and  $l_0$  is the screening length that is strongly associated with the interatomic force between the dielectric material and c-Si or a-Si. In the longitudinal direction, the effective interfacial free energy variation  $\Delta \gamma_{\perp}^{\text{eff}}$  is given by  $\gamma_{\text{dc}}^{\text{eff}} - \gamma_{\text{da}}^{\text{eff}}$ , while the interfacial free energy variation remains constant as  $\gamma_{\text{ac}}$  in the latitudinal direction. Then we introduce the average effective interfacial free energy variation  $\Delta \gamma_{\text{eff}}^{\text{eff}}$  which equals  $(\Delta \gamma_{\perp}^{\text{eff}} + \gamma_{\text{ac}})/2$ , and assume that the variation is isotropic. Therefore, the crystal surface at the same distance from the closer dielectric interface has the same interfacial free energy variation of  $\overline{\Delta \gamma_{\text{eff}}}$  in any direction.

With the above assumptions, the Gibbs free energy variations of the Si-NC in the pre-touch and post-touch stages can be derived:

$$\Delta G_{\rm pre-touch} = -\Delta G_{\rm ac} V + \sum \overline{\Delta \gamma^{\rm eff}} \Delta S \, (r \le d/2), \quad (3)$$

$$\Delta G_{\text{post-touch}} = -\Delta G_{\text{ac}} V + \sum \overline{\Delta \gamma^{\text{eff}}} \Delta S + (\gamma_{\text{dc}} - \gamma_{\text{da}}) S (r \ge d/2), \quad (4)$$

where

$$\overline{\Delta\gamma^{\text{eff}}} = (\Delta\gamma_{\perp}^{\text{eff}} + \gamma_{\text{ac}})/2 = (\gamma_{\text{dc}}^{\text{eff}} - \gamma_{\text{da}}^{\text{eff}} + \gamma_{\text{ac}})/2$$
$$= \gamma_{\text{ac}} + \frac{\gamma_{\text{dc}} - \gamma_{\text{da}} - \gamma_{\text{ac}}}{2} e^{-(d/2 - r)/l_0}, \qquad (5)$$

and  $\Delta G_{\rm ac}$  is the difference between the Gibbs free energies per unit volume of amorphous and crystalline bulk phase, V is the volume of the crystal cluster, S is the sum of contact areas between the cluster and the upper/lower dielectric interfaces, and  $\Delta S$  is the ribbon-like area element of its spherical surface, which stands for a region with approximately the same interfacial free energy variation of  $\overline{\Delta \gamma^{\rm eff}}$ . This can be regarded as a pure mathematical technique. Using integral calculus, we can further deduce the following results from Eqs. (3)–(5):

$$\Delta G_{\rm pre-touch} = -\Delta G_{\rm ac} V + \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \left[ \gamma_{\rm ac} + \frac{\gamma_{\rm dc} - \gamma_{\rm da} - \gamma_{\rm ac}}{2} \mathrm{e}^{-(d/2 - r\sin\theta)/l_0} \right] (2\pi r\cos\theta) r \mathrm{d}\theta, \tag{6}$$

$$\Delta G_{\text{post-touch}} = -\Delta G_{\text{ac}}V + \int_{-\arccos(d/2r)}^{+\arccos(d/2r)} \left[ \gamma_{\text{ac}} + \frac{\gamma_{\text{dc}} - \gamma_{\text{da}} - \gamma_{\text{ac}}}{2} \mathrm{e}^{-(d/2 - r\sin\theta)/l_0} \right] (2\pi r\cos\theta) r \mathrm{d}\theta + (\gamma_{\text{dc}} - \gamma_{\text{da}}) S, \quad (7)$$

and then

$$\Delta G_{\rm pre-touch} = -\Delta G_{\rm ac} \times \frac{4}{3} \pi r^3 + \{\gamma_{\rm ac} \times 4\pi r^2 + (\gamma_{\rm dc} - \gamma_{\rm da} - \gamma_{\rm ac}) \times 4\pi r l_0 \times [e^{-(d/2 - r)/l_0} - e^{-d/2l_0}]\},\tag{8}$$

$$\Delta G_{\text{post-touch}} = -\Delta G_{\text{ac}} \times \left( \pi r^2 d - \frac{\pi d^3}{12} \right) + \left[ \gamma_{\text{ac}} \times 2\pi r d + \left( \gamma_{\text{dc}} - \gamma_{\text{da}} - \gamma_{\text{ac}} \right) \times 4\pi r l_0 \times (1 - e^{-d/2l_0}) \right] + \left( \gamma_{\text{dc}} - \gamma_{\text{da}} \right) \times 2\pi \left( r^2 - \frac{d^2}{4} \right).$$
(9)

So far, the relationships between the Gibbs free energy variation and the crystal size as well as the layer thickness have been obtained. In Eqs. (8) and (9), the first negative terms in the right hand show the energy that would be released by forming the crystal volume while the remaining positive terms show the energy that would be consumed to form the crystal surface interfacing with the ambient. As a result, the Gibbs free energy variation reveals the overall energy gain or cost during the Si-NC formation. Additionally, the impact of the dielectric layers on the formation is just embodied in  $\gamma_{dc} - \gamma_{da}$ .

With Eqs. (8) and (9), the layer-thickness dependence of the Gibbs free energy variations in the two stages can be solved numerically. Here, we adopt the parameter values from Ref. [9] for a common multilayer structure of a-Si/SiO<sub>2</sub> as seen in Table 1.

In the pre-touch stage, the maximum of  $\Delta G_{\text{pre-touch}}$ is defined as the nucleation barrier, denoted as  $\Delta G_{\text{c}}$ . The corresponding nucleus size is the critical radius  $r_{\text{c}}$ . When  $r \geq r_{\text{c}}$ , Si-NC can be formed; otherwise the crystal

Table 1.	Paramete	r Values	for	$a-Si/SiO_2$	and	
$a-Si/SiN_x$ Multilayers						

Multilayer	$\Delta G_{\rm ac}$	$\gamma_{ m ac}$	$\gamma_{ m dc} - \gamma_{ m da}$	$l_0$
Structure	$(eV/nm^3)$	$(eV/nm^2)$	$(eV/nm^2)$	(nm)
$a-Si/SiO_2$	5.081	1.440	2.491	0.64
$a-Si/SiN_x$	8.85	1.46	151.1	0.64

The parameter values for a-Si/SiO<sub>z</sub> and a-Si/SiN<sub>x</sub> multilayers are obtained from Refs. [10] and [11], respectively. But for a-Si/SiN<sub>x</sub> multilayers,  $l_0$  is set to 0.64 nm in our model.



Fig. 3. Nucleation barrier  $\Delta G_c$  and corresponding critical radius  $r_c$  versus a-Si layer thickness d in the pre-touch stage for a-Si/SiO<sub>2</sub> multilayers.



Fig. 4. Gibbs free energy variation in the post-touch stage  $\Delta G_{\text{post-touch}}$  versus lateral radius r of the crystal cluster with various a-Si layer thicknesses d from 1.5 to 24 nm for a-Si/SiO<sub>2</sub> multilayers.

nucleus would be unstable with the tendency to shrink. We acquire  $\Delta G_c$  by searching the maximum of  $\Delta G_{\text{pre-touch}}$  with varying radii of Si-NC at a certain thickness of a-Si layer. The calculated  $\Delta G_c$  and the corresponding  $r_c$  versus the layer thickness d are plotted in Fig. 3. The figure shows a cross point of the line  $r_c = d/2$  (dash) and the curve of  $r_c$ , which significantly indicates a lower limit (~1.5 nm) of the layer thickness for the Si-NC formation under the condition of  $r_c \leq d/2$  or equivalently  $l \geq 0$  that must be implemented in our model. In addition, the thinner the Si layer, the higher the nucleation barrier or thermal energy that would be required for the nucleation.

Consequently, only when the a-Si layer thickness is no smaller than the limit can the post-touch stage occur. In the post-touch stage, the values of  $\Delta G_{\text{post-touch}}$  are calculated with various r, subjected to  $d \geq 1.5$  nm. Figure 4 displays that  $\Delta G_{\text{post-touch}}$  is negative and decreases with increasing crystal radius, which indicates that the Si-NC tends to grow up with no lateral constraint from



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Fig. 5. Difference between the interfacial free energies  $\gamma_{dc}$  and  $\gamma_{da}$  versus lower limit of the a-Si layer thickness  $d_{\min}$ .



Fig. 6. Nucleation barrier  $\Delta G_c$  and corresponding critical radius  $r_c$  versus difference between the interfacial free energies  $\gamma_{\rm dc}$  and  $\gamma_{\rm da}$  with the a-Si layer thickness d of 3 nm for a-Si/SiO<sub>2</sub> multilayers.



Fig. 7. Gibbs free energy variation in the post-touch stage  $\Delta G_{\text{post-touch}}$  versus lateral radius r of the crystal cluster with various a-Si layer thicknesses d from 15 to 50 nm for a-Si/SiN<sub>x</sub> multilayers.

the oxide interfaces as the formation is favored.

However, the difference between the interfacial free energies  $\gamma_{dc}$  and  $\gamma_{da}$ ,  $\gamma_{dc} - \gamma_{da}$ , related to a certain dielectric layer, is not a well-known parameter and is usually obtained by fitting experimental data. With our model, its value could be simply derived from the lower limit  $d_{\min}$  of the a-Si layer thickness, as shown in Fig. 5. It can be found that  $\gamma_{dc} - \gamma_{da}$  increases with  $d_{\min}$  and on the other hand, nucleation barrier and critical radius increase with  $\gamma_{dc} - \gamma_{da}$ , as displayed in Fig. 6. Therefore, for those dielectric layers that cause higher energy consumption to expand the crystal surface, the layer thickness would affect the Si-NC formation more dramatically.

The proposed model could also be used for other a-Si

multilayer structures. With the use of the parameter values from Ref. [10] as also seen in Table 1, we analyze the Si-NC formation in a-Si/SiN $_x$  multilayers. According to the trends of  $\Delta G_{\text{post-touch}}$  in Fig. 7, the lateral growth of the Si-NC would be constrained by the nitride interfaces when the a-Si layer thickness is below  $\sim 35$  nm, since  $\Delta G_{\text{post-touch}}$  moves from negative to positive with the crystal growing and the formation is not favorable in the end. Such results are consistent with those in Ref. [10]. Meanwhile, the lower limit for the layer thickness could be estimated as > 4 nm from Fig. 5. However, Si-NCs have been experimentally observed while the layer thickness is below that limit<sup>[10]</sup>. It may be attributed to the effects for nonstoichiometry of the dielectric layer  $(SiN_x,$ x < 4/3), including the silicon diffusion, bond relaxation, and rearrangement during high-temperature annealing, and the reduced inhomogeneous interfacial strain. These effects are not included in our model and may weaken the influence of the nitride interfaces and the limitation to the a-Si layer thickness. The value of  $\gamma_{\rm dc} - \gamma_{\rm da}$  for a-Si/SiN<sub>x</sub> (151.1 eV/nm<sup>2</sup>) is consequently much higher than that for  $a-Si/SiO_2$  (2.491 eV/nm<sup>2</sup>). In short, according to the calculating results for both multilayers, there remains a lower bound for the a-Si layer thickness but the constraint on the Si-NC growth from the dielectric interfaces would depend on the difference of  $\gamma_{\rm dc} - \gamma_{\rm da}$ .

In conclusion, we investigate the whole formation process of Si-NCs in a-Si/SiO<sub>2</sub> multilayers from nucleation to following growth with a modified model based on the Gibbs free energy variation. The calculating results indicate that there is a lower limit for the a-Si layer thickness below which the Si-NC formation cannot occur for a-Si/SiO<sub>2</sub> multilayers and the lateral growth of Si-NCs cannot be constrained by the oxide interfaces, which should be accounted in order to better regulate the size and density of Si-NCs in the multilayer structure and further exploit as much as possible the luminescent potential of Si quantum dots.

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## References

- 1. L. T. Canham, Appl. Phys. Lett. 57, 1046 (1990).
- X. Lu, J. Hou, M. Xiang, Z. Jia, and F. Zhong, Chin. Opt. Lett. 8, 618 (2010).
- M. Molinari, H. Rinnert, and M. Vergnat, Appl. Phys. Lett. 82, 3877 (2003).
- H. Zhang, L. Lin, and S. Jiang, Chin. Opt. Lett. 7, 332 (2009).
- M. Xie, Z. Yuan, B. Qian, and L. Pavesi, Chin. Opt. Lett. 7, 319 (2009).
- M. Wang, A. Anopchenko, A. Marconi, E. Moser, S. Prezioso, L. Pavesi, G. Pucker, P. Bellutti, and L. Vanzetti, Physica E 41, 912 (2009).
- F. Delachat, M. Carrada, G. Ferblantier, A. Slaoui, C. Bonafos, S. Schamm, and H. Rinnert, Physica E 41, 994 (2009).
- C. Spinella, S. Lombardo, and F. Priolo, J. Appl. Phys. 84, 5383 (1998).
- M. Zacharias and P. Streitenberger, Phys. Rev. B 62, 8391 (2000).
- L. Zhang, K. Chen, L. Wang, W. Li, J. Xu, X. Huang, and K. Chen, J. Phys.: Condens. Matter 14, 10083 (2002).
- K. Chen, X. Huang, J. Xu, and D. Feng, Appl. Phys. Lett. 61, 2069 (1992).