Entanglement dynamics of two atoms successively passing a cavity

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By means of concurrence, we investigate the dynamics of entanglement between two initially separate atoms in succession passing through a cavity and their interaction with a Fock state field. We then analyze the effects of the atomic coherence, photon number, and atomic motion on the time evolution of atom-atom entanglement. The results show that there can be entanglement between two separate atoms, and that the threshold time for the creation of the entanglement is controllable by the photon number, atomic motion, and field-mode structure.

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Quantum entanglement is not only one of the most remarkable features of quantum theory[1,2], but it has also provided a potentially fundamental resource for quantum information processes such as quantum key distribution[3,4], quantum teleportation[5,6], superdense coding[7,8], quantum computation[9,10], and entanglement swapping[11−13]. Therefore, it is important to study the production and preservation of entanglement between qubits that are well separated in implementing quantum protocols during information processing[14]. Bose *et al.* have recently discussed the interaction of a single qubit in a pure state with a thermal field, and have demonstrated that entanglement occurs mostly under such model[15]. Kim *et al.* have also shown that two atoms can be entangled through their interaction with a highly chaotic system[16]. The entanglement between two two-level atoms simultaneously interacting with a thermal field undergoing a two-photon process has been investigated[17]. Incidentally, entanglement can be fabricated in this proposal. The simultaneous interaction of two excited atoms with a Fock state field can never result in a two-atom entanglement[18]. Ghosh *et al.* have already examined the entanglement between two separate atoms interacting with a thermal field and a Fock state[19] . They have shown that the entanglement can be built up via atom-photon interactions inside the cavity. Moreover, Ficek *et al.* have predicted an interesting phenomenon of delayed birth of entanglement[20] . This feature is opposite to a recent and extensive discussion on the sudden death of entanglements[21−24]. Notably, threshold time is controllable by the distance between qubits. The participation of anti-symmetric states in the dynamics is also crucial for the creation of entanglement. The sudden birth of an entanglement deserves a more cautious study due to its fundamental importance in the controlled production of entanglement.

In this letter, we study the entanglement properties of two atoms passing through a cavity one after another, thereafter interacting with a Fock state radiation field inside the cavity. We find that the entanglement between the two separate atoms can be generated, and the threshold time for the creation of the entanglement can be controlled by the photon number, atomic motion, and field-mode structure.

The physical system under consideration is presented in Fig. 1. The system consists of two separate atoms passing through a cavity one after another. The interaction picture Hamiltonian of the joint atom-field system with the rotating wave approximation[25] can be written as

$$H_I = g\left(\sigma_+ a + a^\dagger \sigma_-\right) \quad (\hbar = 1), \tag{1}$$

where $a^\dagger$ and a are the creation and annihilation operators, respectively, of a single-mode cavity field; $\sigma_\pm$ and σ_z are the Pauli spin operators of the atom; ω is the field frequency; and g is the atom-field coupling constant.

We assume that at $t = 0$, the cavity field is previously prepared in the Fock state $|n\rangle$ and two spatially separated two-level atoms are initially in the coherent superposition state as

$$\left|\psi(0)_{a_1}\right\rangle = \cos(\theta_1/2)|e_1\rangle + \sin(\theta_1/2)\exp(\mathrm{i}\varphi_1)|g_1\rangle, \tag{2}$$

$$\left|\psi(0)_{a_2}\right\rangle = \cos(\theta_2/2)|e_2\rangle + \sin(\theta_2/2)\exp(\mathrm{i}\varphi_2)|g_2\rangle. \tag{3}$$

The physical procedure in this letter follows the procedure of Ref. [26] where a scheme of entangling two atoms successively passing through a cavity is proposed. We first consider the passage of the first atom, initially in the superposition state $\left|\psi(0)_{a_1}\right\rangle$ through the cavity and interacting for a time t. The atom-field wave function evolves with the interaction, as given in Eq. (1), to

$$\left|\psi(t)_{a_1-f}\right\rangle = M_1|e_1, n\rangle + M_2|g_1, n+1\rangle, \tag{4}$$

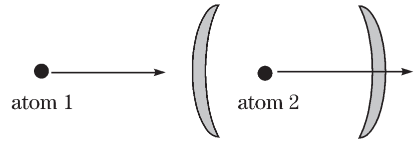

Fig. 1. System with two separate atoms passing through a cavity one after another.

where

$$M_1 = \cos(\theta_1/2)\cos\left(\sqrt{n+1}gt\right) - \mathrm{i}\sin(\theta_1/2)\exp(\mathrm{i}\varphi_1)\sin\left(\sqrt{n+1}gt\right), \quad (5)$$

$$M_2 = -\mathrm{i}\cos(\theta_1/2)\sin\left(\sqrt{n+1}gt\right) + \sin(\theta_1/2)\exp(\mathrm{i}\varphi_1)\cos\left(\sqrt{n+1}gt\right). \quad (6)$$

At a later time, the second atom that is initially in the superposition state $\left|\psi(0)_{a_2}\right\rangle$ passing through the cavity, also interacts at time t. A correlation develops between the two atoms via the cavity field. The joint state of the two atoms and the field is given by

$$\left|\psi(t)_{a_1-a_2-f}\right\rangle = x_1\left|e_1,e_2,n\right\rangle + x_2\left|e_1,g_2,n+1\right\rangle + x_3\left|g_1,e_2,n+1\right\rangle + x_4\left|g_1,g_2,n+2\right\rangle, \quad (7)$$

where

$$x_1 = M_1 N_1, \quad (8)$$

$$x_2 = M_1 N_2, \quad (9)$$

$$x_3 = M_2 N_3, \quad (10)$$

$$x_4 = M_2 N_4, \quad (11)$$

$$N_1 = \cos(\theta_2/2)\cos\left(\sqrt{n+1}gt\right) - \mathrm{i}\sin(\theta_2/2)\exp(\mathrm{i}\varphi_2)\sin\left(\sqrt{n+1}gt\right), \quad (12)$$

$$N_2 = -\mathrm{i}\cos(\theta_2/2)\sin\left(\sqrt{n+1}gt\right) + \sin(\theta_2/2)\exp(\mathrm{i}\varphi_2)\cos\left(\sqrt{n+1}gt\right), \quad (13)$$

$$N_3 = \cos(\theta_2/2)\cos\left(\sqrt{n+2}gt\right) - \mathrm{i}\sin(\theta_2/2)\exp(\mathrm{i}\varphi_2)\sin\left(\sqrt{n+2}gt\right), \quad (14)$$

$$N_4 = -\mathrm{i}\cos(\theta_2/2)\sin\left(\sqrt{n+2}gt\right) + \sin(\theta_2/2)\exp(\mathrm{i}\varphi_2)\cos\left(\sqrt{n+2}gt\right). \quad (15)$$

For a tripartite system of the two atoms and the cavity, we are particularly interested in calculating the entanglement of the two-atom state after the atoms pass through the cavity. The information on such entanglement can be obtained based on the reduced density matrix $\rho_{a_1a_2}(t)$ by tracing the field variables from Eq. (7). The reduced density matrix $\rho_{a_1a_2}(t)$ then takes the form based on $|e_1,e_2\rangle$, $|e_1,g_2\rangle$, $|g_1,e_2\rangle$, and $|g_1,g_2\rangle$ states as

$$\rho_{a_1a_2}(t) = \begin{pmatrix} |x_1|^2 & 0 & 0 & 0 \\ 0 & |x_2|^2 & x_2x_3^* & 0 \\ 0 & x_2^*x_3 & |x_3|^2 & 0 \\ 0 & 0 & 0 & |x_4|^2 \end{pmatrix}. \quad (16)$$

We refer to concurrence, which is widely accepted for any two qubits system, to measure the degree of entanglement between the atoms. Concurrence $C(t)$ introduced by Wooters[27] is defined as

$$C(t) = \max\left[0, \lambda_1(t) - \lambda_2(t) - \lambda_3(t) - \lambda_4(t)\right], \quad (17)$$

where $\lambda_i(t)$ are the eigenvalues, in a decreasing order, of the Hermitian matrix $\left[\sqrt{\rho}\boldsymbol{\rho}\sqrt{\rho}\right]^{1/2}$ with $\boldsymbol{\rho} = \sigma_y \otimes \sigma_y\rho^*\sigma_y \otimes \sigma_y$, and σ_y is the Pauli matrix. The range of concurrence is from 0 to 1. The larger the concurrence is, the stronger the entanglement becomes. Concurrence $C(t) = 0$ corresponds to an unentangled state whereas $C(t) = 1$ is for the maximally entangled state. For a system described by the density matrix (16), the concurrence has a simple explicit expression:

$$C(t) = 2\max\left(0, |x_2x_3^*| - |x_1x_4^*|\right). \quad (18)$$

When the atomic motion is taken into account, the effective Hamiltonian of the model with rotating-wave approximation[28,29] can be written as

$$H = gf(z)\left(\sigma_+ a + a^\dagger\sigma_-\right)(\hbar = 1), \quad (19)$$

where $f(z)$ is the shape function of the cavity field mode. We restrict the investigations to the atomic motion along z-axis; that is, the z-dependence of the field-mode function is considered. Atomic motion can be incorporated into the system through the following relationship[29,30]:

$$f(z) \to f(\nu t), \quad (20)$$

where ν is the atomic motion velocity. In this regard, the transformation TEM_{mnp} is defined as[29]

$$f(z) = \sin(p\pi\nu t/L), \quad (21)$$

where p represents the number of half-wave lengths of the field mode inside a cavity of length L. We first consider the passage of the first atom, initially in the superposition state $\left|\psi(0)_{a_1}\right\rangle$, through the cavity. The atom-field wave function evolves with the interaction, as in Eq. (19), to

$$\left|\psi(t)_{a_1-f}\right\rangle = M_1\left|e_1,n\right\rangle + M_2\left|g_1,n+1\right\rangle, \quad (22)$$

where M_1 and M_2 have the same expressions shown in Eqs. (5) and (6), and the only difference is $t \to \phi(t)$. For a particular choice of the atomic motion velocity $\nu = gL/\pi$ the time-dependent function $\phi(t)$ can be written as

$$\phi(t) = \int_0^t f(\nu t')\mathrm{d}t' = \frac{1}{pg}\left[1 - \cos(pgt)\right]. \quad (23)$$

As the calculation procedure is the same for the atomic motion case, the same expression of the concurrence is therefore presented. The only difference is that $t \to \phi(t)$ when the atomic motion is taken into account.

From Eqs. (18) and (23), we discuss the effects of the atomic coherence, photon number, and atomic motion on the time evolution of atom-atom entanglement with an initial Fock state field in the cavity. To this end, we plot the concurrence as a function of the scaled time gt in Figs. 2–4, for which the atomic motion is neglected. Figure 5 shows the concurrence as a function of the scaled time gt when the atomic motion is taken into account. In Fig. 6, we compare the results when the atomic motion is neglected and thereafter considered.

Figure 2 shows the concurrence as a function of the scaled time gt for the first atom that is initially in a different state and the second atom at an excited state

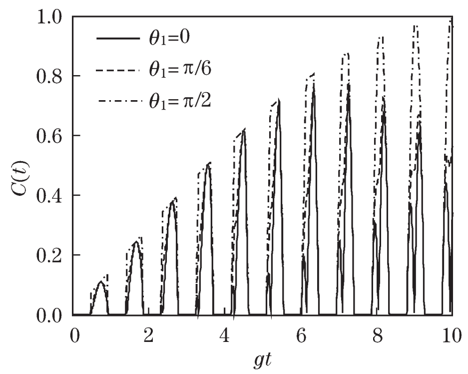

Fig. 2. Evolution of the concurrence as a function of scaled time gt. The field is initially in a Fock state with photon number $n = 10$ and the first atom is initially in different states, $\theta_1 = 0, \varphi_1 = 0$ (solid line), $\theta_1 = \pi/6, \varphi_1 = 0$ (dot line) $\theta_1 = \pi/2, \varphi_1 = 0$ (dash-dot line). The second atom is in the excited state ($\theta_2 = 0, \varphi_2 = 0$).

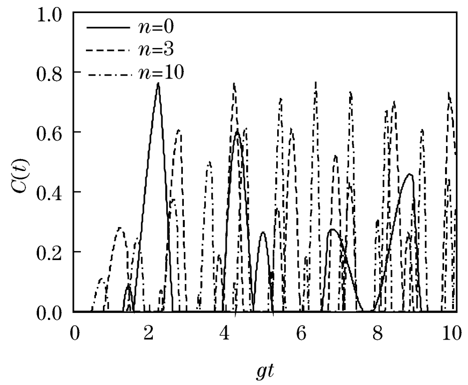

Fig. 3. Evolution of the concurrence as a function of scaled time gt. The field is initially in a Fock state with different photon numbers, $n = 0$ (solid line), $n = 3$ (dot line), and $n = 10$ (dash-dot line), for which both atoms are initially in the excited state ($\theta_1 = 0, \varphi_1 = 0, \theta_2 = 0, \varphi_2 = 0$).

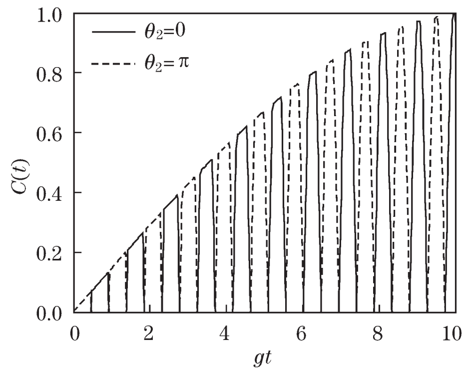

Fig. 4. Evolution of the concurrence as a function of scaled time gt. The field is initially in a Fock state with photon number $n = 10$, the first atom is initially in the superposition state $|\psi(0)_{a_1}\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle + |g_1\rangle)$, and the second atom is initially in the excited state $\theta_2 = 0, \varphi_2 = 0$ (solid line) and ground state $\theta_2 = \pi, \varphi_2 = 0$ (dot line).

having a photon number of $n = 10$. Clearly, no entanglement could be observed at earlier time, until suddenly, at some finite time, an entanglement would start to build up. The threshold time for the creation of the entanglement is independent of atomic coherence for the first atom. Inasmuch as maximum concurrence exists, maximum entanglement degree occurs when the first atom is initially in a superposition state ($\theta_1 = \pi/2, \varphi_1 = 0$) (see the dash-dot line in Fig. 2). The comparison of the solid curve, dot curve, and dash-dot line in Fig. 2 shows that the atomic coherence of the first atom increases the degree of the atom-atom entanglement.

To show the influence of the photon number of the field on the atom-atom entanglement, we plot in Fig. 3 the time evolution of the concurrence for both atoms that are initially in the excited state. We observe no entanglement at earlier times but rather an entanglement suddenly building up at some finite time. Furthermore, the threshold time for the creation of the entanglement becomes shorter as the photon number n increases. However, the maximum of the atom-atom entanglement decreases as photon number n increases.

Figure 4 illustrates the concurrence as a function of the scaled time gt for the first atom that is initially in the superposition state $|\psi(0)_{a_1}\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle + |g_1\rangle)$ for which the photon number n= 10. The solid line and the dot line show the time evolution of the atom-atom entanglement when the second atom is initially prepared in the excited and ground states, respectively. It is obvious from Fig. 4 that the concurrence builds up as time develops, implying that entanglement of two atoms can be immediately created when the second atom is initially in the ground state. The comparison of the solid line and the dot line shows that the concurrence displays the complementary behaviors when the second atom is initially prepared in the ground and excited states, respectively.

Figure 5 shows the concurrence as a function of the scaled time gt for both atoms initially in the excited state and at a photon number of $n = 10$ for which the atomic motion is taken into account. Clearly, the atomic motion leads to the periodic evolution of the atom-atom entanglement, and an increase in parameter p resulting in not only a shortening of the evolution period of the time behavior of concurrence, but also a decrease in amplitude of atom-atom entanglement. Physically, these features can be attributed to the change in the atom-field interaction time due to atomic motion. When the atomic motion is considered, from Eq. (23), we have $g\phi(t) = (1/p)[1 - \cos(pgt)]$. Parameter $g\phi(t)$ is a periodical function on the scaled time gt with a period of $2\pi/p$. This periodicity of $g\phi(t)$ on the scaled time gt leads to the periodicities of evolution of the concurrence. When $p = 1$, the time behavior of the concurrence evolves in period 2π (see the solid line in Fig. 5) whereas when $p = 2$, the atom-atom entanglement evolves in period π (see the dot line in Fig. 5).

In Fig. 6, we plot the time evolution of the concurrence for both atoms that are initially in the excited state and of the photon number $n = 10$. The solid line and the dot line in Fig. 6 show the time evolution of the atom-atom entanglement when the atomic motion is neglected and then considered thereafter, respectively. The threshold time for the creation of the entanglement is shorter when atomic motion is neglected. Furthermore, when atomic motion is taken into account, the maximum of the concurrence decreases and the atomic motion leads to the periodic evolution of the atom-atom entanglement.

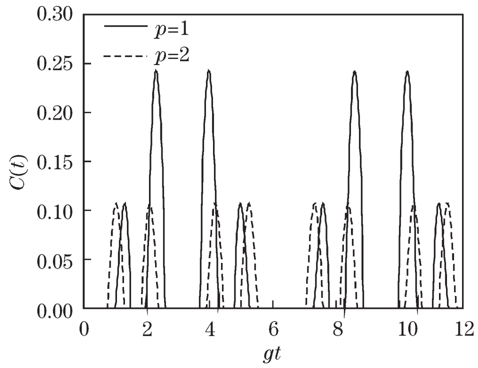

Fig. 5. Effects of the atomic motion and field-mode structure on the evolution of the concurrence for the field-mode structure parameter $p = 1$ (solid line), $p = 2$ (dot line). The field is initially in a Fock state with photon number $n= 10$ and both atoms are initially in the excited state ($\theta_1 = 0, \varphi_1 = 0, \theta_2 = 0, \varphi_2 = 0$).

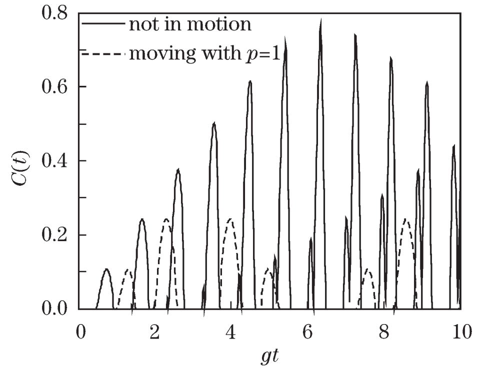

Fig. 6. Evolution of the concurrence as a function of the scaled time gt. The atomic motion is neglected (solid line) and considered for $p = 1$ (dot line). The field is initially in a Fock state with photon number $n = 10$ and both atoms are initially in the excited state ($\theta_1 = 0, \varphi_1 = 0, \theta_2 = 0, \varphi_2 = 0$).

In conclusion, we have studied the dynamical properties of entanglement between two two-level atoms that are spatially separated from each other passing through a cavity one after another. We have investigated the influences of the atomic coherence, photon number, and atomic motion on the time evolution of atom-atom entanglement when the cavity is initially in a Fock state. We have found that the entanglement between the two separate atoms is created via atom-photon interactions inside the cavity even though no single atom interacts directly with another. Furthermore, the threshold time for the creation of the entanglement is independent of the atomic coherence of the first atom and becomes shorter as the photon number increases. The atomic motion leads to the periodic evolution of the atom-atom entanglement, and an increase in parameter p results in not only a shortening of the evolution period of the time behavior of concurrence but also a decrease in the amplitude of the atom-atom entanglement. In addition, our results obtained may have potential application in the field of quantum information processes due to its significance in the creation of entanglement.

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References

1. A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer Academic, Dordrecht, 1993).
2. G. Alber, T. Beth, M. Horodecki, P. Horodecki, R. Horodecki, M. Rötteler, W. Weinfurter, R. Werner, and A. Zeilinger, *Quantum Information: An Introduction to Basic Theoretical Concepts and Experiments* (Springer, Berlin, 2001).
3. A. E. Ekert, Phys. Rev. Lett. **67,** 661 (1991).
4. Y. Yu and Z. Zhang, Acta Opt. Sin. (in Chinese) **28,** 556 (2008).
5. C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70,** 1895 (1993).
6. X. Xiao and L. Yang, Acta Opt. Sin. (in Chinese) **28,** 1812 (2008).
7. C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69,** 2881 (1992).
8. S. Mozes, J. Oppenheim, and B. Reznik, Phys. Rev. A **71,** 012311 (2005).
9. J. J. Garcia-Ripoll, P. Zoller, and J. I. Cirac, Phys. Rev. A **71,** 062309 (2005).
10. A. Biswas and G. S. Agarwal, Phys. Rev. A **68,** 054303 (2003).
11. O. Glöckl, S. Lorenz, C. Marquardt, J. Heersink, M. Brownnutt, C. Silberhorn, Q. Pan, P. van Loock, N. Korolkova, and G. Leuchs, Phys. Rev. A **68,** 012319 (2003).
12. M. Yang, W. Song, and Z.-L. Cao, Phys. Rev. A **71,** 034312 (2005).
13. H. Li, F. Li, Y. Yang, and Q. Zhang, Phys. Rev. A **71,** 022314 (2005).
14. M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
15. S. Bose, I. Fuentes-Guridi, P. L. Knight, and V. Vedral, Phys. Rev. Lett. **87,** 050401 (2001).
16. M. S. Kim, J. Lee, D. Ahn, and P. L. Knight, Phys. Rev. A **65,** 040101(R) (2002).
17. L. Zhou, H. S. Song, and C. Li, J. Opt. B: Quantum Semiclass. Opt. **4,** 425 (2002).
18. T. E. Tessier, I. H. Deutsch, A. Delgado, and I. Fuentes-Guridi, Phys. Rev. A **68,** 062316 (2003).
19. B. Ghosh, A. S. Majumdar, and N. Nayak, Int. J. Quantum Inf. **5,** 169 (2007).
20. Z. Ficek and R. Tanaś, Phys. Rev. A **77,** 054301 (2008).
21. T. Yu and J. H. Eberly, Phys. Rev. Lett. **93,** 140404 (2004).
22. T. Yu and J. H. Eberly, Phys. Rev. Lett. **97,** 140403 (2006).
23. J. H. Eberly and T. Yu, Science **316,** 555 (2007).
24. T. Yu and J. H. Eberly, Science **323,** 598 (2009).
25. E. T. Jaynes and F. W. Cummings, Proc. IEEE **51,** 89 (1963).
26. S. J. D. Phoenix and S. M. Barnett, J. Mod. Opt. **40,** 979 (1993).
27. W. K. Wootters, Phys. Rev. Lett. **80,** 2245 (1998).
28. A. Joshi and S. V. Lawande, Phys. Rev. A **42,** 1752 (1990).
29. R. R. Schlicher, Opt. Commun. **70,** 97 (1989).
30. M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, 1974).