Electromagnetically induced left handedness in a V-type four-level atomic system

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A closed four-level system in atomic vapor is proposed, which is made to possess left handedness by means of the technique of quantum coherence. The method of density matrix is utilized in view of the rotating-wave approximation and the effect of local-field in dense gas. The result of the numerical simulation shows that the negative permittivity and negative permeability of the medium can be achieved simultaneously (i.e., the left handedness) in a wider frequency band under appropriate parameter conditions. Furthermore, when analyzing the dispersion property of the left-handed material, we find that the probe beam can be controlled to change from superluminal to subluminal or *vice versa* via changing the detuning of the probe field.

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The propagation of electromagnetic waves in matter is characterized by the frequency-dependent relative dielectric permittivity ε_r and the magnetic permeability μ_r. Their product defines the index of refraction: $\varepsilon_\mathrm{r}\cdot\mu_\mathrm{r}=n^2$. Naturally, the wave vector **K**, the electric field **E**, and the magnetic field **H** forming a right-handed system in conventional materials have both positive permittivity and permeability simultaneously. However, in 1968, Veselago theoretically constructed an electromagnetic material, in which, the wave vector is opposite to the direction of energy propagation. Thus Veselago defined such a material as "left-handedness" material[1]. The left-handed material (LHM) has a negative refractive index when the permittivity and permeability are negative simultaneously. It is a new kind of material that offers a possibility of molding the flow of light inside media, and it has attracted considerable attention[2−12] because of its surprising, counterintuitive electromagnetic and optical properties. It can be verified that the left-handed media exhibit a number of peculiar electromagnetic and optical properties, such as the reversal of Doppler shift, anomalous refraction, negative Goos-Hanchen shift[5], reversed circular Bragg phenomenon[11], enhanced quantum interference, modified spontaneous emission rates, unusual photon tunneling effect[6], sub-wavelength focusing[7,12], and so on.

LHMs have been realized by several approaches, including artificial composite meta-materials[13], transmission line simulation[14], photonic crystal structures[10,15,16], chiral materials[17], and photonic resonant materials[18]. The former four methods require delicate manufacturing of spatially periodic structure. The last method is also a quantum optical way in which the physical mechanism is the quantum interference and coherence that arise from the transition process in a multilevel atomic system. It was proposed firstly in a three-level medium[18], which requires rigorous level condition. For the purposes of enhancing the freedom of choice of levels and making the scheme much more applicable to realistic system, Thommen *et al.* presented a proposal based on a coherent cross-coupling between electric and magnetic dipole transitions in a four-level scheme[16].

In this letter, we put forward a four-level dense atomic vapor scheme based on quantum coherence effects to realize left-handedness. In the system, we can dominate the relative permeability and relative permittivity by the electromagnetic interaction between multi-level atom and multi-mode optical field in our scheme, and the contribution of Lorentz-Lorenz local field of the four-level dense atomic vapor should be considered.

Now we discuss coherent effects in a four-level V scheme[19,20] as shown in Fig. 1. In this case, levels $|1\rangle$ and $|3\rangle$ have opposite parity and $\mathbf{d}_{31}=\langle 3|\,\widehat{\mathbf{d}}\,|1\rangle\neq 0$, where $\widehat{\mathbf{d}}$ is the electric dipole operator. The two levels $|3\rangle$ and $|4\rangle$ have the same parity with $\mu_{43}=\langle 4|\,\widehat{\mu}\,|3\rangle\neq 0$, where $\widehat{\mu}$ is the magnetic-dipole operator. Here transitions $|1\rangle\leftrightarrow|3\rangle$ are driven by a probe field with Rabi frequency Ω_2, while transitions $|1\rangle\leftrightarrow|2\rangle$ are driven by the coupling field with Rabi frequency Ω_1 and transitions $|2\rangle\leftrightarrow|4\rangle$ are driven by the pump field with Rabi frequency Ω_3. $\Delta_1=(\varepsilon_2-\varepsilon_1)/\hbar-\omega_1, \Delta_2=(\varepsilon_3-\varepsilon_1)/\hbar-\omega_2$, and $\Delta_3=(\varepsilon_4-\varepsilon_2)/\hbar-\omega_3$ denote the detunings between the frequencies of the transitions.

Under the rotating-wave approximation and the dipole approximation, the density matrix elements are given as follows:

$$\begin{aligned}\frac{\partial\rho_{11}}{\partial t}=&-\mathrm{i}\Omega_1\rho_{12}-\mathrm{i}\Omega_2\rho_{13}+\mathrm{i}\Omega_1\rho_{21}\\&+\mathrm{i}\Omega_2\rho_{31}+2\gamma_{31}\rho_{33}+2\gamma_{21}\rho_{22},\end{aligned}$$

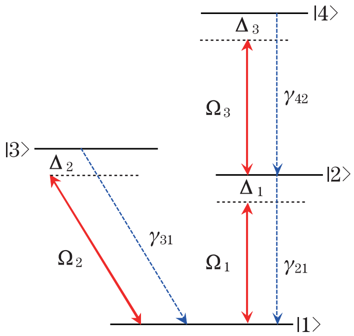

Fig. 1. Model of four-level V atom system interacting with three light fields.

$$\begin{aligned}
\frac{\partial\rho_{22}}{\partial t} &= \mathrm{i}\Omega_1\rho_{12} - \mathrm{i}\Omega_3\rho_{24} - \mathrm{i}\Omega_1\rho_{21} \\
&+ \mathrm{i}\Omega_3\rho_{42} - 2\gamma_{21}\rho_{22} + 2\gamma_{42}\rho_{44}, \\
\frac{\partial\rho_{44}}{\partial t} &= \mathrm{i}\Omega_3\rho_{24} - \mathrm{i}\Omega_3\rho_{42} - 2\gamma_{42}\rho_{44}, \\
\frac{\partial\rho_{12}}{\partial t} &= -\gamma_{21}\rho_{12} + \mathrm{i}\Delta_1\rho_{12} + \mathrm{i}\Omega_1\rho_{22} \\
&- \mathrm{i}\Omega_1\rho_{11} + \mathrm{i}\Omega_2\rho_{32} - \mathrm{i}\Omega_3\rho_{14}, \\
\frac{\partial\rho_{13}}{\partial t} &= \mathrm{i}\Omega_1\rho_{23} + \mathrm{i}\Omega_2\rho_{33} - \mathrm{i}\Omega_2\rho_{11} \\
&+ (\mathrm{i}\Delta_2 - \gamma_{31})\rho_{13}, \\
\frac{\partial\rho_{14}}{\partial t} &= -\mathrm{i}\Omega_3\rho_{12} + \mathrm{i}\Omega_2\rho_{34} + \mathrm{i}\Omega_1\rho_{24} \\
&+ (\mathrm{i}\Delta_1 + \mathrm{i}\Delta_3 - \gamma_{42})\rho_{14}, \\
\frac{\partial\rho_{23}}{\partial t} &= \mathrm{i}\Omega_1\rho_{13} - \mathrm{i}\Omega_2\rho_{21} + \mathrm{i}\Omega_3\rho_{43} \\
&- (\mathrm{i}\Delta_1 - \mathrm{i}\Delta_2 + \gamma_{31} + \gamma_{21})\rho_{23}, \\
\frac{\partial\rho_{24}}{\partial t} &= \mathrm{i}\Omega_3\rho_{44} - \mathrm{i}\Omega_3\rho_{22} + \mathrm{i}\Omega_1\rho_{14} \\
&+ (-\gamma_{42} - \gamma_{21} + \mathrm{i}\Delta_3)\rho_{24}, \\
\frac{\partial\rho_{34}}{\partial t} &= -\mathrm{i}\Omega_3\rho_{32} + \mathrm{i}\Omega_2\rho_{14} + (\mathrm{i}\Delta_1 - \mathrm{i}\Delta_2 \\
&+ \mathrm{i}\Delta_3 - \gamma_{31} - \gamma_{41})\rho_{34},
\end{aligned} \tag{1}$$

where γ_{31}, γ_{21}, and γ_{42} are the spontaneous decay rates for transitions $|3\rangle \leftrightarrow |1\rangle, |2\rangle \leftrightarrow |1\rangle$, and $|4\rangle \leftrightarrow |2\rangle$, respectively. We have assumed a closed atomic system, i.e., $\sum\limits_{i=1}^{4} \rho_{ii} = 1$.

In a dilute vapor, there are few differences between the macroscopic fields and the local fields, and both fields act on any atom (molecule or group of molecules)[21]. But in dense media with closely packed atoms (molecules), the polarization of neighboring atoms (molecules) gives rise to an internal field at any given atom in addition to the average macroscopic fields so that the total fields at the atom are different from the macroscopic fields[22]. The chosen vapor with atomic concentration $N = 1 \times 10^{21}\,\mathrm{m}^{-3}$ should be dense so that one can consider the local field effect, which results from the dipole-dipole interaction between neighboring atoms. With the formula of the atomic electric polarizations $\gamma_\mathrm{e} = 2\mathbf{d}_{31}\rho_{13}/\varepsilon_0 E_P$, where $E_P = \hbar\Omega_2/\mathbf{d}_{31}$, one can arrive at

$$\gamma_\mathrm{e} = \frac{2\mathbf{d}_{31}^2\rho_{13}}{\varepsilon_0\hbar\Omega_2}. \tag{2}$$

In the same way, we can obtain the explicit expression for the atomic magnetic polarizability by using the formulae of the atomic magnetic polarizations $\gamma_\mathrm{m} = 2\mu_0\mu_{34}\rho_{43}/B_P$[21], and the relation between the microscopic local electric and magnetic fields $E_P/B_P = c$:

$$\gamma_\mathrm{m} = \frac{2c\mu_0\mu_{34}\mathbf{d}_{31}\rho_{43}}{\hbar\Omega_2}, \tag{3}$$

where μ_0 is the permeability of vacuum, and c is the speed of light in vacuum. In order to achieve a significant magnetic response, the transition frequency corresponding to $|4\rangle - |3\rangle$ should be approximately equal to the frequency of the probe light.

However, what we are interested in is the macroscopic physical quantities such as the electric and magnetic susceptibilities, which are the electric permittivity and magnetic permeability. The Clausius-Mossotti relation between the electric permittivity and the magnetic permeability can reveal the connection between the macroscopic and microscopic quantities. According to the Clausius-Mossotti relation[22], one can obtain the electric susceptibility of the atomic vapor medium $\chi_\mathrm{e} = N\gamma_\mathrm{e} \cdot (1 - \frac{N\gamma_\mathrm{e}}{3})^{-1}$.

The relative electric permittivity of the atomic medium is

$$\varepsilon_\mathrm{r} = 1 + \chi_\mathrm{e} = \frac{1 + 2/3N\gamma_\mathrm{e}}{1 - 1/3N\gamma_\mathrm{e}}. \tag{4}$$

Meanwhile, the magnetic Clausius-Mossotti relation[23] $\gamma_\mathrm{m} = \frac{1}{N}\left(\frac{\mu_\mathrm{r}-1}{\frac{2}{3}+\frac{\mu_\mathrm{r}}{3}}\right)$ shows the connection between the macroscopic magnetic permeability μ_r and the microscopic magnetic polarizations γ_m. The relative magnetic permeability of the atomic vapor medium is

$$\mu_\mathrm{r} = \frac{1 + 2/3N\gamma_\mathrm{m}}{1 - 1/3N\gamma_\mathrm{m}}. \tag{5}$$

In LHMs, the expression of the medium refractive index is defined as[1]

$$n = -\sqrt{\varepsilon_\mathrm{r}\mu_\mathrm{r}}. \tag{6}$$

The expression of the absorption coefficient A in LHMs is[24]

$$A = 2\pi \mathrm{Im}(-\sqrt{\varepsilon_\mathrm{r}\mu_\mathrm{r}}). \tag{7}$$

From the above analysis, we can obtain the expressions for the electric permittivity and the magnetic permeability of the coherent atomic vapor medium.

In numerical analysis, for simplicity, all the parameters are scaled by γ and we assume $\gamma_{31} = \gamma_{21} = 0.5\gamma, \gamma_{42} = 0, \Delta_1 = \Delta_3 = 0; \Omega_1 = 10\gamma, \Omega_2 = 0.01\gamma, \Omega_3 = \gamma; \gamma = 100\,\mathrm{MHz}$.

It can be seen from Fig. 2(a) that the relative dielectric permittivity has a negative real part in the probe frequency detuning range [-1.2γ, -0.2γ]. It is also shown that the real part of relative magnetic permeability is negative in the range [-2.72γ, 5.0γ] in Fig. 2(b). Therefore, the four-level coherent atomic vapor we have considered can exhibit simultaneously negative permittivity and permeability in the range [-1.2γ, -0.2γ]. The medium shows left-handed effect at the moment.

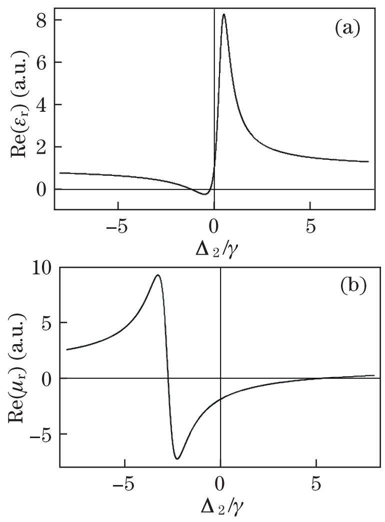

Fig. 2. Real parts of (a) the relative permittivity ε_r and (b) the relative permeability μ_r versus the detuning of the probe field Δ_2.

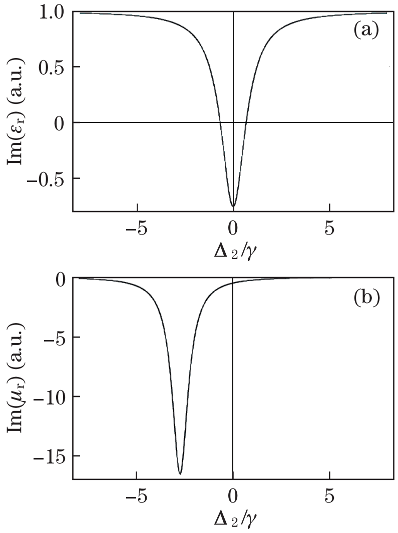

Fig. 3. Imaginary parts of (a) the relative permittivity ε_r and (b) the relative permeability μ_r versus the detuning of the probe field Δ_2.

A weak absorption peak appears in the resonance region in Fig. 3(a). In Fig. 3(b), the probe light exhibits electromagnetically induced transparency, and the imaginary part of the relative permeability shows a negative peak at the probe field detuning $\Delta_2 = -2.8\gamma$.

Figure 4(a) shows that medium absorption coefficient is negative in the probe frequency detuning range [-1.2γ, -0.3γ], and that the absorption is small in a certain range near the resonance region, especially in the case of $\Delta_2 = -0.73\gamma$, where the absorption is zero. In our scheme, the absorption is reduced, even to zero. The main application limitation of the LHMs is the large amount of dissipation and absorption[23]. Particularly, the resolution of perfect lens[7] is obviously decreased because of the absorption. Maybe our scheme is a potential application in high resolution imaging and beam refocusing.

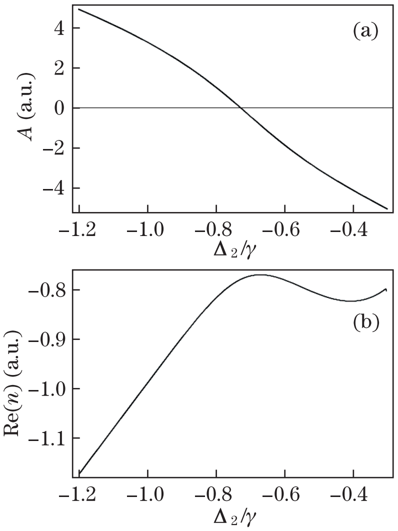

Fig. 4. (a) Absorption coefficient and (b) refractive index versus the detuning of the probe field Δ_2 (all parameters are the same as those in Fig. 2).

Figure 4(b) shows the dispersion properties of the vapor media. It exhibits the different dispersive properties at two sides of it, namely, normal dispersion in the ranges [-1.2γ, -0.675γ], [-0.4γ, -0.3γ] and anomalous dispersion in the range [-0.675γ, -0.4γ] of the probe frequency detuning. According to the group velocity definition $v_g = c/\mathrm{Re}[(n + \omega\mathrm{d}n/\mathrm{d}\omega)]$[21], the superluminal propagation will occur in [-0.675γ, -0.4γ] and the subluminal propagation in [-1.2γ, -0.675γ], [-0.4γ, -0.3γ]. Therefore, maybe we can manipulate the probe beam to change from superluminal to subluminal or *vice versa* in this LHM.

In conclusion, the quantum system with an interaction between a closed V-type four-level dense atomic vapor and multi-mode light fields is adopted to possess left handedness by means of the technique of quantum coherence. The negative permittivity and negative permeability of medium can be achieved simultaneously in a wider frequency band under the appropriate parameter conditions. We also discuss how the medium affects light absorption and gain, and the gain properties of the LHM may be a scheme to solve the main application limitation of LHMs because of the dissipation and absorption. It may have potential applications in improvements of the perfect lens resolution[7,25], beam focusing[26], and so on. Furthermore, by analyzing the dispersion property of the LHM, we can also manipulate the probe beam to change from superluminal to subluminal or *vice versa* in this material.

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