Stochastic resonance in an optical bistable system subjected to cross-correlated additive white noise and multiplicative colored noise

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The stochastic resonance (SR) of an optical bistable system with cross-correlated additive white and multiplicative colored noises and periodic signal is studied using the unified colored noise approximation and the theory of signal-to-noise ratio (SNR). Results show that cross-correlation intensity λ enforces the SR of the system. The position of the peak on the SNR-τ curves moves to the right direction along with the increase of λ (τ is the self-correlation time of the multiplicative colored noise). We find the SR phenomenon in the SNR-D and SNR-Q curves (D and Q are the intensities of the additive and multiplicative noises, respectively), but not in the SNR-λ curves.

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Stochastic resonance (SR) has been proposed to explain the periodic recurrences of ice ages on the Earth[1,2]; as such, this phenomenon has been extensively studied from both the theoretical and experimental points of view. In the last two decades, research on signal processing in nonlinear systems with noise has revealed several interesting phenomena, the most important of which is SR[3]. SR was coined for the rather counterintuitive fact that the response of a nonlinear system to a periodic signal may be enhanced through the addition of an optimal amount of noise[4,5]. In order to describe SR, McNamara *et al.* suggested a master equation for populations in two stable states[6]. They considered the signal-to-noise ratio (SNR), which was the ratio of the δ peak height in the power spectrum to the noise background, in determining the SR effect. Zhou *et al.* suggested escape time distribution to describe SR[7]. The SR paradigm has drawn considerable attention in such fields as climatology, chemistry, laser physics, biophysics, physiology, solid-state physics, and even sociology[8−16]. In 2000, Jia *et al.* studied the SR phenomenon in a bistable system under the simultaneous action of multiplicative and additive noises using the adiabatic limit method[17]. Luo *et al.* studied SR in a bistable system driven by two different kinds of colored noises and found that there seemed to be a transition between one peak and two peaks in the curve of the SNR when either the noise correlation time or the coupling strength between the additive noise and the multiplicative noise was increased[18]. In 2007, Cao *et al.* studied the SR of periodically driven linear system with multiplicative white noise and periodically modulated additive white noise[19]. Du *et al.* investigated the SR phenomenon of a periodically driven time-delayed linear system with multiplicative white noise and periodically modulated additive white noise[20]. In 2008, Burada *et al.* presented a novel scheme for the appearance of SR when the dynamics of a Brownian particle took place in a confined medium[21]. Wu *et al.* studied coupled bistable oscillators with different sources of diversity, and found that the resonance was reduced, and even disappeared, as the correlation length between the diversity increased[22]. Applying the method of unified colored noise approximation, Zhao *et al.* investigated the phenomenon of entropic SR in a two-dimensional confined system driven by a transverse periodic force when colored fluctuation was included in the system[23].

Recently, optical bistability has attracted a great amount of interest and has given rise to numerous experimental and theoretical studies[24−26]. In optical systems, SR has also attracted wide interest. McNamara *et al.* first observed that the output SNR of a ring laser exhibited a maximum level versus the input noise intensity (i.e., SR). In this letter, the SR phenomenon in an optical bistable system with coupling between the additive white noise and multiplicative colored noise is investigated.

A model for purely absorptive optical bistability in an optical cavity has been introduced by Bonifacio *et al.*[27] for the input light amplitude y and the transmitted amplitude x, they derived the equation of motion for the dimensionless variables as

$$\frac{\mathrm{d}x}{\mathrm{d}t} = y - x - \frac{2cx}{1+x^2} = -\frac{\mathrm{d}U(x)}{\mathrm{d}t}, \quad (1)$$

with the potential $U(x) = -\int (y - x - \frac{2cx}{1+x^2})\mathrm{d}x$. In this equation, c is proportional to the inversion of the population of the atomic levels. For a large value of c, the input-output characteristics show the bistability. The potential $U(x)$ has two minima when the system exhibits optical bistability.

For a large value of c and a chosen input intensity $y = y_0$ within the regime of bistability, we take into account the fluctuations of input intensity y and inversion c and assume that the system is driven by a periodic signal, $A\cos\omega t$. Thus, the dimensionless form of the Langevin equation for this system can be shown as

$$\frac{\mathrm{d}x}{\mathrm{d}t} = y_0 - x - \frac{2cx}{1+x^2} + A\cos\omega t + \frac{2x}{1+x^2}\xi(t) + \eta(t), \quad (2)$$

where $\xi(t)$ and $\eta(t)$ are the Gaussian noises and are cor-

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related in the following forms:

$$\langle\eta(t)\rangle = \langle\xi(t)\rangle = 0, \quad (3)$$

$$\langle\xi(t)\xi(t')\rangle = \frac{Q}{2\tau}\exp(-\frac{|t-t'|}{\tau}), \quad (4)$$

$$\langle\eta(t)\eta(t')\rangle = D\delta(t-t'), \quad (5)$$

$$\langle\eta(t')\xi(t)\rangle = \langle\xi(t')\eta(t)\rangle = \lambda\sqrt{QD}\delta(t-t'), \quad (6)$$

where Q and D are the intensities of noises $\xi(t)$ and $\eta(t)$; τ is the self-correlation time of the multiplicative colored noise; $\delta(t-t')$ is the Kronecher delta function; λ is the correlation intensity between the additive and multiplicative noises with $|\lambda| \leq 1$.

According to Eq. (2) and using the unified colored noise approximation[28], the corresponding Fokker-Planck equation is written as

$$\frac{\partial P(x,t)}{\partial t} = L_{\mathrm{FP}}P(x,t), \quad (7)$$

$$L_{\mathrm{FP}} = -\frac{\partial}{\partial x}F(x,\tau) + \frac{\partial^2}{\partial x^2}G(x,\tau). \quad (8)$$

The drift coefficient $F(x,\tau)$ and the diffusion coefficient $G(x,\tau)$ are given by

$$F(x,\tau) = \frac{f(x)}{C(x,\tau)} + \frac{K'(x)}{C^2(x,\tau)} - \frac{C'(x,\tau)K(x)}{C^3(x,\tau)}, \quad (9)$$

$$G(x,\tau) = \frac{K(x)}{C^2(x,\tau)}, \quad (10)$$

and

$$f(x) = y_0 - x - \frac{2cx}{1+x^2} + A\cos\omega t, \quad (11)$$

$$K(x) = Q\frac{4x^2}{(1+x^2)^2} + 2\lambda\sqrt{QD}\frac{2x}{1+x^2} + D, \quad (12)$$

$$C(x,\tau) = 1 - \tau[f'(x) - \frac{1-x^2}{(1+x^2)x}f(x)]. \quad (13)$$

We only consider the stationary state, and the steady-state probability density of Eq. (7) can be obtained as

$$P_{\mathrm{st}}(x) = N\frac{C(x,\tau)}{\sqrt{K(x)}}\exp\Big(\int\frac{f(x)C(x,\tau)}{K(x)}\mathrm{d}x\Big) = N\frac{C(x,\tau)}{\sqrt{K(x)}}\exp\Big(-\frac{\Phi(x,\tau)}{D}\Big), \quad (14)$$

where N is the normalization constant. Here, $\Phi(x,\tau)$ is the generalized potential, and

$$\Phi(x,\tau) = -D\int\frac{f(x)C(x,\tau)}{K(x)}\mathrm{d}x. \quad (15)$$

The mean first passage time $T_\pm$ of the process $x(t)$ to reach the state $x_\mp$ ($x_\mp$ represents the two stable states) with initial condition of $x(t=0) = x_\pm$ is

$$T_\pm = \frac{2\pi}{\sqrt{|V''(x_u)V''(x_{\pm1})|}}\exp[\Phi(x_u,\tau) - \Phi(x_\pm,\tau)]. \quad (16)$$

Based on $W_\pm = T_\pm^{-1}$, the transition rates $W_\pm$ can be obtained out of $x_\pm$[29].

The system is subjected to a time-dependent signal, $A\cos\omega t$, up to the first-order of its amplitude (assumed to be small), and the transition rates can be expanded as follows by the two-state model theory:

$$W_+ = W_{+0} - W_{+1}A\cos\omega t, \quad (17)$$

$$W_- = W_{-0} + W_{-1}A\cos\omega t. \quad (18)$$

According to the theory of McNamara *et al.*[29], the master equation governing the evolution of $n_\pm(t)$ is written as

$$\dot{n}_+ = -\dot{n}_- = W_-(t)n_- - W_+(t)n_+ = W_-(t) - [W_+(t) + W_-(t)]n_+. \quad (19)$$

Once Eq. (19) is integrated, the correlation function and the power spectrum can be calculated[30], and the expression of SNR is given by

$$\mathrm{SNR} = \frac{A^2\pi(W_{+0}W_{-1} + W_{-0}W_{+1})^2}{4W_{+0}W_{-0}(W_{+0} + W_{-0})}, \quad (20)$$

where

$$W_{+0} = W_+|_{A\cos(\omega t)=0}, W_{-0} = W_-|_{A\cos(\omega t)=0}, \quad (21)$$

$$W_{+1} = -\frac{\mathrm{d}W_+}{\mathrm{d}(A\cos(\omega t))}\Big|_{A\cos(\omega t)=0},$$

$$W_{-1} = \frac{dW_-}{\mathrm{d}(A\cos(\omega t))}\Big|_{A\cos(\omega t)=0}. \quad (22)$$

By using the expression in Eq. (20) of SNR, the effects of the self-correlation time τ and the cross-correlation intensity λ on the SNR of the system can be analyzed by numerical calculation.

In Fig. 1, we present the SNR as a function of the self-correlation time τ of the multiplicative noise for different values of cross-correlation intensity λ between the two types of noises. The existence of a maximum level in the curves of SNR-τ is the identifying characteristic of the SR phenomenon. The height of the peak increases accordingly with the increase in the value of λ. This means that the increase in the correlation intensity between noises can cause the SR phenomenon to be prominent. The position of the peak on the SNR-τ curves also moves to the right direction along with the increase of λ.

The curves of the SNR with respect to the correlation

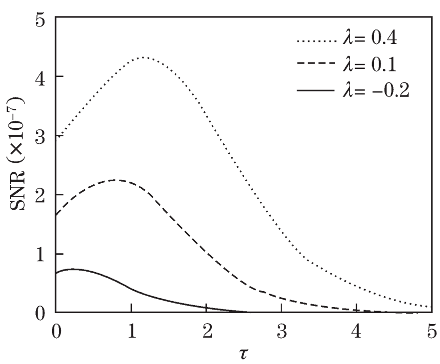

Fig. 1. SNR of the system as a function of the self-correlation time τ with different values of λ. The parameters are chosen as $D = Q = 0.2$, $\omega = 0.004$, and $A = 0.003$.

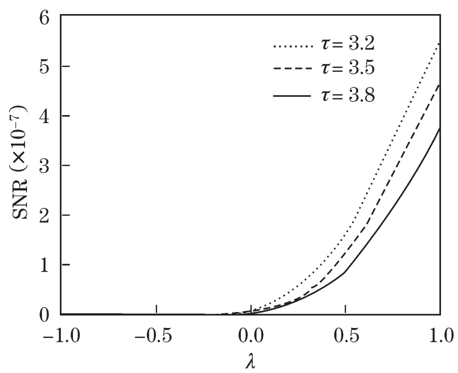

Fig. 2. SNR of the system as a function of the cross-correlation intensity λ with different values of τ. The parameters are chosen as $Q = 0.2$, $D = 0.4$, $\omega = 0.004$, and $A = 0.003$.

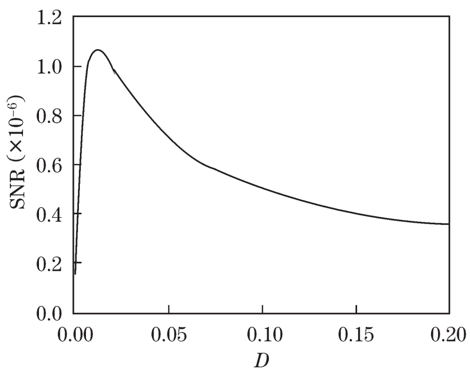

Fig. 3. SNR of the system as a function of the additive noise intensity D. The parameters are chosen as $Q = 0.2$, $\lambda = 0.4$, $\tau = 0.4$, $\omega = 0.004$, and $A = 0.003$.

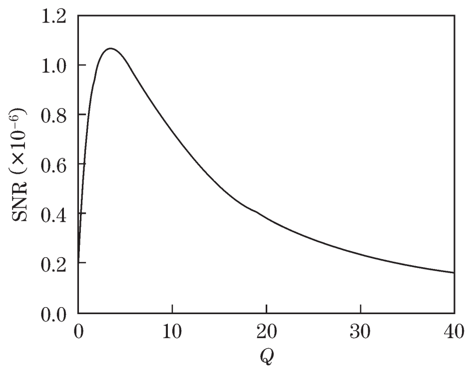

Fig. 4. SNR of the system as a function of the multiplicative noise intensity Q. The parameters are chosen as $D = 0.2$, $\lambda = 0.4$, $\tau = 0.4$, $\omega = 0.004$, and $A = 0.003$.

intensity λ between the two types of noises for different values of self-correlation time τ of the multiplicative noise are plotted in Fig. 2. As the figure shows, no SR phenomenon occurs in the SNR-λ curves. SNR increases along with increasing λ, whereas it decreases along with increasing self-correlation time τ.

As shown in Figs. 3 and 4, there is a peak in the SNR-D and SNR-Q curves, indicating that SR also occurs during the variation of SNR with both the additive noise intensity D and the multiplicative noise intensity Q, respectively.

In conclusion, the effects of the self-correlation time τ, the cross-correlation intensity λ, and the additive and multiplicative noise intensities D and Q on the SR of an optical bistable system are investigated. Using the expression of SNR, we find that λ enforces the SR of the system. The position of the peak on the SNR-τ curves moves to the right direction along with the increase of λ. In addition, the SR phenomenon can be found in the SNR-D and SNR-Q curves, but not in the SNR-λ curves.

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