## All-optical realization of $\times$ NOR logic gate using chaotic semiconductor lasers under phase modulator control

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The theoretical construction of the fundamental  $\times$  NOR gate using two injection semiconductor laser diodes due to mutual coupling is presented. Two laser diodes that are commonly driven by a monochromatic light beam result in chaos; however, chaotic synchronization between the two lasers may be achieved by coupling them. The all-optical logic gate is finally implemented by synchronizing or un-synchronizing appropriately the two chaotic states under a phase modulator (PM) control. Numerical results validate the feasibility of the method.

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Chaotic semiconductor laser has generated tremendous interest due to its potential applications in secure optical communications<sup>[1]</sup>, such as optical chaotic cryptography. In addition to chaotic communications, a new research area involving chaotic laser dynamics based on computation has received much attention. Chaotic systems can be applied to perform simple mathematical calculations<sup>[2]</sup> and simulate logic operations<sup>[3]</sup>. The systems can perform universal computing if they have the capability of emulating the NOR gate. Using the appropriate combinations of the NOR gate, all possible functions can thus be accomplished<sup>[2]</sup>. Recently, Chlouverakis *et al.* have presented an optoelectronic NOR logic gate system<sup>[4]</sup> that has made it possible to achieve chaotic laser application in NOR logic gate.

In this letter, the optoelectronic setup presented by Chlouverakis *et al.* is improved as an all-optical system by using external optical phase modulation of the chaotic optical field. This method does not affect system performance and the dynamic behaviors of the two lasers. The implementation of the fundamental  $\times$  NOR gate is presented by using an all-optical scheme based on chaotic laser synchronization and phase modulators (PMs). Due to all the optical system and external optical modulation, the system is appropriately integrated as a device for optical computation; it has higher modulation rate superior to the electronic systems.

Optical injection phenomena in lasers are governed by the beating and injection quantity of the injected signal. For this, specific phenomena, such as the gain-varying effect in the semiconductor lasers, must be taken into account. The resulting dynamic behaviors have been investigated theoretically and experimentally, after which a route leading from stability to chaos has been found by increasing injection levels. Different regimes have been described, including self-pulse, quasi-period, and chaos<sup>[5-9]</sup>. Recently, chaotic behaviors of such systems have also been proposed for chaotic lidar and secure applications<sup>[10-17]</sup>. In this letter, we transform a master laser (ML) light which drives two slave lasers (SLs) into chaotic states<sup>[4]</sup>. Synchronization is achieved between the two chaotic lasers by coupling the two SLs mutually. The equations for this synchronization are given as  $^{[1,4-10,14-17]}$ 

$$\begin{aligned} \frac{\mathrm{d}E_{1}}{\mathrm{d}t} &= \frac{1}{2}(G_{1} - \gamma_{\mathrm{p}})E_{1} + \frac{K}{\tau_{L}}E_{\mathrm{m}}\cos\phi_{1} \\ &+ \frac{k_{1}}{\tau_{L}}E_{2}\cos(\phi_{2} + \varphi_{2} - \phi_{1} - \varphi_{1}), \\ \frac{\mathrm{d}\phi_{1}}{\mathrm{d}t} &= \frac{1}{2}\beta_{\mathrm{c}}(G_{1} - \gamma_{\mathrm{p}}) + \frac{K}{\tau_{L}}\frac{E_{\mathrm{m}}}{E_{1}}\sin(-\phi_{1}) \\ &+ \frac{k_{1}}{\tau_{L}}\frac{E_{2}}{E_{1}}\sin(\phi_{2} + \varphi_{2} - \phi_{1} - \varphi_{1}) - \Delta\omega, \quad (1) \\ \frac{\mathrm{d}N_{1}}{\mathrm{d}t} &= \frac{I}{q} - \gamma_{\mathrm{e1}}N_{1} - G_{1}V_{\mathrm{p}}E_{1}^{2}, \\ \frac{\mathrm{d}E_{2}}{\mathrm{d}t} &= \frac{1}{2}(G_{2} - \gamma_{\mathrm{p}})E_{2} + \frac{K}{\tau_{L}}E_{\mathrm{m}}\cos\phi_{2} \\ &+ \frac{k_{2}}{\tau_{L}}E_{1}\cos(\varphi_{1} + \phi_{1} - \varphi_{2} - \phi_{2}), \\ \frac{\mathrm{d}\phi_{2}}{\mathrm{d}t} &= \frac{1}{2}\beta_{\mathrm{c}}(G_{2} - \gamma_{\mathrm{p}}) + \frac{K}{\tau_{L}}E_{\mathrm{m}}\sin(-\phi_{2}) \\ &+ \frac{k_{2}}{\tau_{L}}\frac{E_{1}}{E_{2}}\sin(\varphi_{1} + \phi_{1} - \varphi_{2} - \phi_{2}) - \Delta\omega, \quad (2) \\ \frac{\mathrm{d}N_{2}}{\mathrm{d}t} &= \frac{I}{q} - \gamma_{\mathrm{e2}}N_{2} - GV_{\mathrm{p}}E_{2}^{2}, \end{aligned}$$

where footnotes "1" and "2" represent the lasers 1 and 2, respectively; E,  $\phi$ , and N indicate the slowly varying field amplitude, the phase, and the carrier number, respectively; G represents the mode gain,  $V_{\rm p}$  is the mode volume of laser;  $E_{\rm m}$  is the optical field amplitude of ML light;  $\gamma_{\rm p}$  represents the photon loss rate;  $\Delta \omega$  represents the frequency detuning between ML and SLs;  $\tau_L$  represents the round-trip time in the cavity length L; I is the drive current; q is the unit charge;  $\beta_{\rm c}$  is the optical linewidth enhancement factor;  $\gamma_{\rm e}$  represents the total carrier loss rate; K is the optical injection factor, where the injection level of ML is determined by K; k is the optical coupling factor, where the coupling-injection level of the two SLs is determined by k; and  $\varphi$  is the phase shift produced by the PM.



Fig. 1. Schematics of the chaotic synchronization laser setup for all-optical  $\times$  NOR gate implementation. C<sub>M</sub>, C<sub>1</sub>, and C<sub>2</sub>: couplers.



Fig. 2. Diagram of the achieved chaotic synchronization.

× NOR realization based on the setup is schematically shown in Fig. 1. SLs 1 and 2 were used to inject a fraction of their chaotic output into the opposite laser<sup>[4]</sup> after a fraction of the chaotic outputs  $E_1$  and  $E_2$  were modulated by the PMs, respectively, to control synchronization or un-synchronization between the SLs. The laser parameters are listed in Ref. [5].

Chaotic synchronization can be performed between the lasers if the SLs are identical and the two modulators perform the same modulation depth. In our simulations, we took  $k_1 = k_2 = 0.0091$  to accomplish the synchronization between the two SLs. The two roads of the injection light are taken as symmetry, the lengths  $L_1$  and  $L_2$  of the two paths are equal, and these two SLs are made as a back-to-back system to avoid delaying time<sup>[4]</sup>. Synchronization can be achieved between the two chaotic lasers, in which the parameters are taken as  $E_{\rm m} = 0.125 E_{\rm s}$  ( $E_{\rm s}$  is the optical field amplitude at saturation),  $\Delta \omega = 0.9$  GHz, K = 0.03,  $k_1 = k_2 = 0.0091$ , and  $\varphi_1 = \varphi_2 = 0$  (Fig. 2). Thus, the system can be performed for the  $\times$  NOR gate.

In our simulations, the phase shift values of  $\varphi_1$  or  $\varphi_2$  were set between 0 and  $\pi$  by controlling the PM. This technique is based on the external optical phase modulation used in optical communication and the chaos shift key (CSK), which is widely used in chaotic cryptography<sup>[1,14-17]</sup>. This external optical phase modulation does not affect the laser performance<sup>[14-17]</sup>. However, CSK is a kind of internal modulation of the chaotic laser resulting in an unstable and degenerated laser performance and capability. External optical phase mod-

ulation has more advantages compared with CSK. The technique can be performed on the system for NOR gate. Synchronization occurs if  $\varphi_1 = \pi$  and  $\varphi_2 = \pi$  or  $\varphi_1 = 0$ and  $\varphi_2 = 0$ , that is,  $\varphi_1 = \varphi_2$ . On the other hand, synchronization error or obvious un-synchronization occurs if  $\varphi_1 = 0$  and  $\varphi_2 = \pi$  or  $\varphi_1 = \pi$  and  $\varphi_2 = 0$ , that is,  $\varphi_1 \neq \varphi_2$ . From a simple subtraction of the two chaotic outputs  $(E_1 \text{ and } E_2)$ , we can easily observe the logic operations of the  $\times$  NOR gate. The results are illustrated in Fig. 3, where  $S_1$  and  $S_2$  denote the modulation signals of the PMs, either  $S_1 = 1$  or  $S_2 = 1$  stands for the  $\pi$ phase shift of the modulated light, and either  $S_1 = 0$ or  $S_2 = 0$  stands for the zero phase shift of the modulated light. Figures 3(a) and (b) show the modulation signals. Figure 3(c) shows the subtraction of  $E_2$  from  $E_1$ . When the system is synchronized, namely,  $E_1 = E_2$ and  $S_1 = S_2$ , then the logical operation is performed  $(S_1 = S_2)$  and the logical output is 1. When  $S_1 \neq S_2$ , a synchronization error is enhanced, the detection of the unsynchronization regions is thus simplified, in which the logical output is 0 (Fig. 3(d)). Based on the results, the synchronization can be accomplished in less than 10 ns. We focused on the synchronization time of the system used in the  $\times$  NOR gate.

It is essential to study the effects of parameter mismatch and noise on synchronization. In our simulations, mismatches of  $\pm 1\% E_{\rm m}$ ,  $\pm 0.5\% I$ , and  $0.1\% (\varphi_1 - \varphi_2)$ , resulted in synchronization errors below 1%, 10%, and 10%, respectively. Meanwhile, white noise with a square of 0.1%  $E_{\rm m}$  resulted in a synchronization error below 0.5%, making it possible to detect the synchronized region. The mismatch can be permitted within a certain range. Moreover, the delay time for the chaotic reinjection from one laser to the other is negligible if the distance is less than 20 cm<sup>[1,4]</sup>, which is experimentally achievable. Given that  $L_1 = L_2$ , the two delay times are the same, and even with the consideration of the two delay times, synchronization can still occur between the two SLs.

In conclusion, we have demonstrated that chaotic synchronization scheme using coupling-injection lasers can be used in implementing the fundamental  $\times$  NOR gate. When a couple of modulators perform the phase modulation of chaotic light from the two SLs to the opposite laser, the logic gate is finally implemented by detecting synchronization or un-synchronization between the chaotic states. However, the system has been substan-



Fig. 3. Simulation realization of the  $\times$  NOR logic gate. (a) Modulation signal of PM<sub>1</sub>; (b) modulation signal of PM<sub>2</sub>; (c) demodulation; (d) logical output.

tially limited by the resynchronization time, indicating the advantage of using such a setup to establish a  $\times$  NOR gate performing at a few nanoseconds. This system presents the importance of optical chaotic system as a fundamental all-optical  $\times$  NOR logic gate in the field of dynamical universal computing.

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