Modified level set method with Canny operator for image noise removal

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The level set method is commonly used to address image noise removal. Existing studies concentrate mainly on determining the speed function of the evolution equation. Based on the idea of a Canny operator, this letter introduces a new method of controlling the level set evolution, in which the edge strength is taken into account in choosing curvature flows for the speed function and the normal to edge direction is used to orient the diffusion of the moving interface. The addition of an energy term to penalize the irregularity allows for better preservation of local edge information. In contrast with previous Canny-based level set methods that usually adopt a two-stage framework, the proposed algorithm can execute all the above operations in one process during noise removal.

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The use of the level set method[1,2] has become popular due to its flexibility and capability in modeling complex structures. The basic idea of this geometric approach is to evolve a higher dimensional implicit function, whose zero level set always represents the position of the propagating front according to a partial differential equation. It executes natural topological changes and allows efficient numerical solutions. Several previous studies[3−5] have referred to the application of the level set method to the issue of image noise removal. Malladi *et al.* have driven the image evolution under flows controlled by the min/max and mean curvature[3]. This algorithm is less sensitive to the nature of noise and is applicable to both salt-and-pepper grey-scale noise and full-image continuous noise. However, this algorithm penalizes high curvature value regardless of the curve regularity and could cause edge blurring. Gil *et al.* have presented a novel geometric flow that included a function measuring the degree of local irregularity in the curve[4]. It achieved nontrivial steady states representing a smooth model of level curves in a noisy image. Li *et al.* have defined a region-scalable fitting energy to be incorporated into a variational level set formulation with a regularization term[5], from which a curve evolution equation is derived for energy minimization. This region-based model can cope with intensity inhomogeneity, preserve regularity, and avoid expensive reinitialization.

All the above approaches concentrated on the selection, modification, and regularization of the speed function, a term of level set formulation determining the diffusion of the moving interface during the evolution. Actually, there also exist other methods to detect edge information that ensure contrast preservation in the process of noise removal[6,7] using the level set method. For example, combining the advantages of an edge detector and controlling the evolution direction are both promising choices. In this letter, a modified Canny operator-based level set method for image noise removal is presented.

Canny operator[8] is robust to noise and is probably the most widely used edge detector. Unless the preconditions are particularly suitable, it is difficult to find an edge detector, which performs significantly better than the Canny operator. Researchers have recently introduced the Canny operator into the level set method. Heydarian *et al.* have developed a semi-automated method to determine the desired object boundary in magnetic resonance (MR) and computed tomography (CT) images[9]. Rough object boundaries can be obtained manually by applying the Canny operator. Then, using the output of the genetic algorithm to fix the level set method parameters, accurate object boundaries can be detected automatically. Xia *et al.* have presented an optimal initialization scheme to improve the segmentation performance of the Chan-Vese model[10]. They first used the Canny operator to compute rough edges. By connecting edge points iteratively according to a local cost function, the final closed object contours have been generated using a morphological filter to remove noise and redundant edges. Qin *et al.* have used the maximum magnitude of edge gradient, which is the result of Canny processing, to replace the curve plane in the level set formulation and continuously evolve the moving interface for the precise segmentation of medical images[11]. This algorithm essentially produced rough initial contours with the Canny operator and then segmented the images using the level set method.

The two-stage framework, in which the Canny operator works for the initial conditions and the level set method works for the final results, has been applied in the previous studies mentioned above. This letter, however, adopts a different approach, in which the normal to edge direction is used to orient the level set evolution based on the idea of the Canny operator. This resulted in more preserved local edge information due to the penalized energy function. The key point is that the abovementioned operations can be executed in one process during the image noise removal.

The level set method is based on partial differential equation. Its main idea is that the moving interface is viewed as a zero level set in one higher dimensional space.

The evolution equation of the level set function φ can be written as[2]

$$\frac{\partial \varphi}{\partial t} + F\left|\nabla \varphi\right| = 0, \tag{1}$$

where F is a speed function determining the diffusion of the moving interface opposite to its normal direction.

For the problem of image noise removal, φ is replaced by the image data I_{m}, and F is usually a sort of curvature flow k which depends on I_{m}. Thus, Eq. (1) is rewritten as

$$\frac{\partial I_{\mathrm{m}}}{\partial t} + F\left(k\right) \cdot \left|\nabla I_{\mathrm{m}}\right| = 0. \tag{2}$$

According to the definition of Canny operator[8], for step edges, the shape of optimized edge detector deduced by Canny is similar to the first order derivative of the Gaussian function. Based on the symmetric and decomposable properties of the two-dimensional (2D) Gaussian function, it is easy to compute the convolution of the directional derivative on any direction and image.

Suppose the 2D Gaussian function is

$$G\left(x, y, \sigma\right) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right). \tag{3}$$

Its first order directional derivative on some direction $\mathbf{n}$ can be written as

$$G_n = \frac{\partial G}{\partial n} = \mathbf{n} \cdot \nabla G, \tag{4}$$

where $\nabla G = \begin{bmatrix} \partial G/\partial x \\ \partial G/\partial y \end{bmatrix}$. Let the image I_{m} be convoluted with operator G_n. When $G_n * I_{\mathrm{m}}$ achieves the maximum value, $\mathbf{n}$ is oriented normal to the direction of a detected edge. Although this direction is not known a priori, it can be estimated well from the smoothed gradient direction as expressed by

$$\mathbf{n} = \frac{\nabla\left(G * I_{\mathrm{m}}\right)}{\left|\nabla\left(G * I_{\mathrm{m}}\right)\right|}. \tag{5}$$

At such an edge point, the edge strength is the magnitude of

$$\left|G_n * I_{\mathrm{m}}\right| = \left|\nabla\left(G * I_{\mathrm{m}}\right)\right|. \tag{6}$$

With the first order differential coefficients of the smoothed image, the edge strength M can be calculated by

$$M = \sqrt{E_x^2 + E_y^2}, \tag{7}$$

where $E_x = \frac{\partial G}{\partial x} * I_{\mathrm{m}}$ and $E_y = \frac{\partial G}{\partial y} * I_{\mathrm{m}}$.

To combine the advantages of the Canny operator in preserving edge information during image noise removal, $\mathbf{n}$ is used to replace ∇I_{m} in Eq. (2). The speed function $F(k)$ in Eq. (2) should also be selected by considering the edge strength M. Based on Eqs. (2), (5), and (7), the level set evolution equation with the Canny operator can be obtained as

$$\frac{\partial I_{\mathrm{m}}}{\partial t} + F_{\mathrm{c}} \cdot \left|\frac{\nabla\left(G * I_{\mathrm{m}}\right)}{\left|\nabla\left(G * I_{\mathrm{m}}\right)\right|}\right| = 0, \tag{8}$$

$$F_{\mathrm{c}} = \begin{cases} \text{mean} \quad \text{flow} & \max\left(M\right) < T \\ \text{min/max} \quad \text{flow} & \text{otherwise} \end{cases}, \tag{9}$$

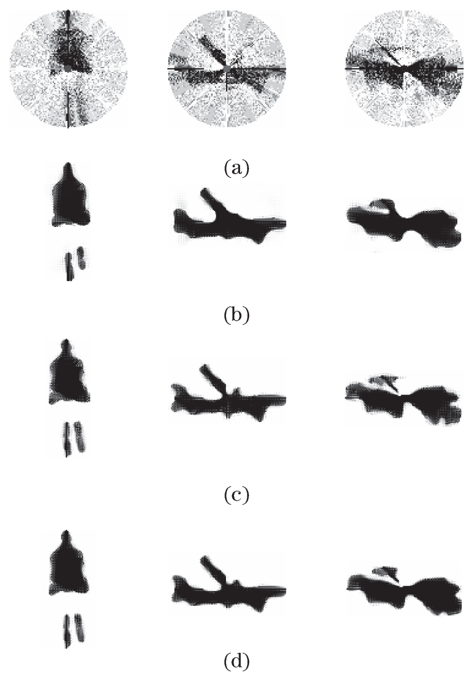

Fig. 1. Performances of different level set algorithms for noise removal. (a) Noisy images; (b) results by the algorithm of Eq. (2); (c) results by the algorithm of Eq. (8); (d) results by the proposed algorithm of Eq. (13).

where $\max\left(M\right)$ is the maximum of edge strength in 3×3 neighborhoods of the central point and T is the low threshold for Canny optimized algorithm.

To penalize the local irregularity in the curves, an energy function E is introduced as

$$E = \mu \cdot g \cdot \delta_{\varepsilon}\left(I_{\mathrm{m}}\right), \tag{10}$$

where g is an edge indicator function[12], $\delta_{\varepsilon}\left(I_{\mathrm{m}}\right)$ is the univariate Dirac function, and μ is an adjusting factor. In practice, g and $\delta_{\varepsilon}\left(I_{\mathrm{m}}\right)$ are defined as

$$g = \frac{1}{1 + \left|\nabla G * I_{\mathrm{m}}\right|}^{2}, \tag{11}$$

$$\delta_{\varepsilon}\left(I_{\mathrm{m}}\right) = \frac{1}{2\varepsilon} \cdot \left[1 + \cos\left(\frac{\pi \cdot I_{\mathrm{m}}}{\varepsilon}\right)\right]. \tag{12}$$

Both μ and ε take the value of 1.5 for all the experiments in this letter. Adding Eqs. (10) to (8), the modified Canny operator-based level set equation is

$$\frac{\partial I_{\mathrm{m}}}{\partial t} + F \cdot \left|\frac{\nabla\left(G * I_{\mathrm{m}}\right)}{\left|\nabla\left(G * I_{\mathrm{m}}\right)\right|}\right| + \mu \cdot g \cdot \delta_{\varepsilon}\left(I_{\mathrm{m}}\right) = 0. \tag{13}$$

This achieves good performance both in the whole image for noise removal and in the local area for edge preservation.

The first experiment started with three noisy images, in which the objects were "man", "airplane", and "car" (Fig. 1(a)). Different level set algorithms for noise removal were then applied for comparison. Figure 1(b) gives the results of the basic level set evolution as described in Eq. (2). Although most of the noise in the whole image was cleared out, the information in some detailed parts, e.g., the legs in "man", the tail fin in "airplane", and the roof in "car", were not preserved well.

Figure 1(c) illustrates the results of the initial Canny operator-based level set method as explained in Eq. (8). When the global noise disappeared, the local information was preserved. However, this method could not remove the noise around the detailed parts completely. Figure 1(d) shows the results of the proposed algorithm in this letter as interpreted in Eq. (13). With the addition of the penalized energy function, the modified Canny operator-based level set algorithm achieved the best performance; it not only removed the noise well but also preserved the edge information more.

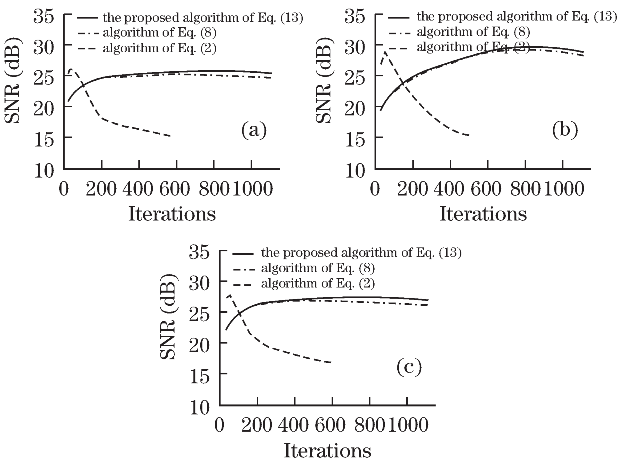

Fig. 2. Quantitative comparison of different level set algorithms for noise removal for the noisy images of (a) "man" (b) "airplane", and (c) "car".

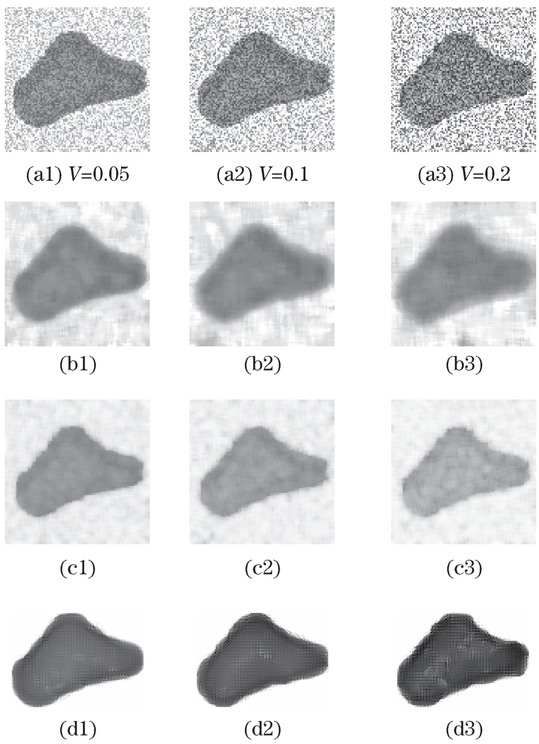

Fig. 3. Superior performance of the proposed noise removal algorithm. (a1)−(a3) the noisy images; (b1)−(b3) noise removal by median filtering; (c1)−(c3) noise removal by contra-harmonic mean filtering; (d1)−(d3) noise removal by the proposed algorithm of Eq. (13).

To assess the performance of the proposed algorithm quantitatively, the parameter of signal-to-noise ratio (SNR) is defined as

$$\mathrm{SNR} = 10 \lg \left(\frac{\|I_{\mathrm{ev}}\|}{\|I_{\mathrm{o}} - I_{\mathrm{ev}}\|} \right), \tag{14}$$

where I_{o} denotes the original image, and I_{ev} denotes the evolution of the noisy image. The higher the SNR, the greater the effect of the noise removal algorithm. Figure 2 shows the quantitative comparison results, and the proposed algorithm has the highest SNR. The SNR almost does not change after a proper number of iterations, indicating the good noise removal stability of the proposed algorithm, which can automatically stop smoothing at some optimal point. Continued application produced no further changes.

The second experiment was conducted in order to validate the superiority of the proposed noise removal algorithm further. It began with three levels of noisy images, in which the uniformly distributed random noise had different variances V(i.e., V=0.05, 0.1, 0.2), as shown in Figs. 3(a1)−(a3). The proposed algorithm and two frequently used filtering techniques, i.e., contra-harmonic mean filtering and median filtering, were then applied to smooth the noisy images. Their corresponding results are given in Figs. 3(b1)−(b3), (c1)−(c3), and (d1)−(d3). The results of the modified Canny operator-based level set method appeared to be far superior compared with those of the two other methods.

The third experiment tested the applicability of the proposed algorithm to different kinds of noises. Figures 4(a1)−(a3) show the added Gaussian white noise, salt and pepper noise, and multiplicative noise to "rice" image, respectively. The corresponding results of the modified Canny operator-based level set method for noise removal are shown in Figs. 4(b1)−(b3). The proposed algorithm gave satisfactory performances for all the noises, indicating that it can deal with various noises robustly.

In conclusion, this letter presents a modified level set method for image noise removal. Based on the idea of a

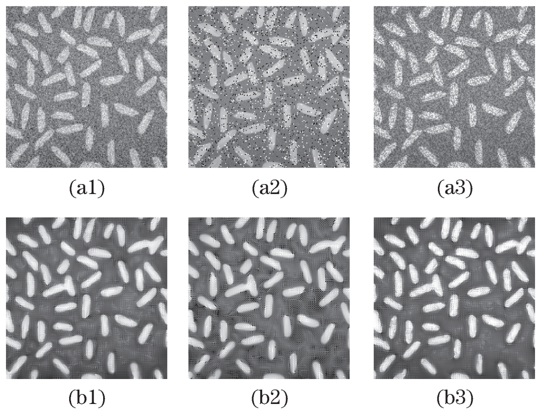

Fig. 4. Proposed noise removal algorithm that can deal with various noises. (a1) Gaussian white noise, (a2) salt and pepper noise, (a3) multiplicative noise; (b1) result after 270 iterations, (b2) result after 220 iterations, (b3) result after 290 iterations.

Canny operator, the edge strength has been taken into account in choosing curvature flows for the speed function. In addition, the normal to the edge direction has been used to orient the diffusion of the moving interface. With the addition of an energy term to penalize the irregularity, this algorithm not only removes noise in the whole image but also preserves edge information in the local area. All the above operations can be executed in one process during the evolution. The experimental results have shown that the proposed algorithm performs effectively in noise removal. It is also stable for continuous smoothing and can deal with various noises.

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