Comment on "Focusing of high polarization order axially-symmetric polarized beams" [Chin. Opt. Lett. 7, 938 (2009)]

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Received August 19, 2010

Some mathematical and conceptual errors contained in a recent letter [Chin. Opt. Lett. **7,** 938 (2009)] are pointed out.

OCIS codes: 050.1960, 050.1970, 260.5430, 110.2990, 140.3300.
doi: 10.3788/COL20100811.1110.

In a recent letter[1], Zhou *et al.* studied focusing of a special kind of cylindrical vector beams whose electric field amplitude in the incident plane before an aplanat is given by

$$\mathbf{E}(\rho,\varphi,z) = E(\rho)\,[\cos(Q\varphi+\varphi_0)\,\hat{\boldsymbol{\rho}} + \sin(Q\varphi+\varphi_0)\,\hat{\boldsymbol{\varphi}}], \quad (1)$$

where (ρ,φ,z) is a cylindrical coordinate system, the z axis is the beam axis, $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\varphi}}$ are unit vectors, Q is an integer, and φ_0 is a constant. They term this kind of beams "axially symmetric polarized beams (ASPB)". In my opinion, this terminology is inaccurate because axial symmetry is usually associated with an object that is invariant upon a rotation of any angle about the symmetry axis; therefore, ASPB describes only the case of $Q = 0$. However, this is not the point of this Comment. The purpose of this Comment is to point out some mathematical and conceptual errors of Ref. [1].

For the incident field given by Eq. (1), following the theory developed in Refs. [2] and [3], after some algebra but before carrying out any integration, one should obtain this expression for the electric field in the focal region:

$$\begin{aligned}\mathbf{E}(\rho_\mathrm{s},\varphi_\mathrm{s},z_\mathrm{s}) = C \int_{\theta_\mathrm{min}}^{\theta_\mathrm{max}}\int_0^{2\pi} & l(\theta)\cos^{1/2}\theta\exp[\mathrm{i}\,k\,z_\mathrm{s}\cos\theta - \mathrm{i}\,k\,\rho_\mathrm{s}\sin\theta\cos(\varphi-\varphi_\mathrm{s})] \\ & \{\,\hat{\boldsymbol{\rho}}_\mathrm{s}\,[\cos\theta\cos(Q\varphi+\varphi_0)\cos(\varphi-\varphi_\mathrm{s}) - \sin(Q\varphi+\varphi_0)\sin(\varphi-\varphi_\mathrm{s})] \\ & +\ \hat{\boldsymbol{\varphi}}_\mathrm{s}\,[\cos\theta\cos(Q\varphi+\varphi_0)\sin(\varphi-\varphi_\mathrm{s}) + \sin(Q\varphi+\varphi_0)\cos(\varphi-\varphi_\mathrm{s})] \\ & +\ \hat{\mathbf{z}}_\mathrm{s}\,[\sin\theta\cos(Q\varphi+\varphi_0)]\,\}\,\sin\theta\mathrm{d}\theta\mathrm{d}\varphi\,, \end{aligned} \quad (2)$$

where C is a constant and the meanings of all other undefined symbols can be found in Refs. [1−3]. Comparing Eq. (1) of Ref. [1] [henceforth Eq. (1−1), and so on] with Eq. (2), we see two differences. First, there is a sign difference in the arguments of the exponential functions. Apparently Ref. [1] inherits the sign error from Refs. [2] and [3]. Second, one term in the radial component and another in the azimuth component of Eq. (2) are missing in Eq. (1−1). While the first error is unimportant for most purposes, the second may be significant when the field distribution is to be computed accurately. Carrying out the integration in φ, one further obtains

$$\begin{aligned}\mathbf{E}(\rho_\mathrm{s},\varphi_\mathrm{s},z_\mathrm{s}) = \pi\,\mathrm{i}^{Q+1}C \int_{\theta_\mathrm{min}}^{\theta_\mathrm{max}} & l(\theta)\cos^{1/2}\theta\exp(\mathrm{i}\,k\,z_\mathrm{s}\cos\theta) \\ & (\hat{\boldsymbol{\rho}}_\mathrm{s}\ \cos(Q\varphi_\mathrm{s}+\varphi_0)\{\,\cos\theta\,[J_{Q+1}(-k^*) - J_{Q-1}(-k^*)] + J_{Q+1}(-k^*) + J_{Q-1}(-k^*)\,\} \\ & +\hat{\boldsymbol{\varphi}}_\mathrm{s}\ \sin(Q\varphi_\mathrm{s}+\varphi_0)\{\,\cos\theta\,[J_{Q+1}(-k^*) + J_{Q-1}(-k^*)] + J_{Q+1}(-k^*) - J_{Q-1}(-k^*)\,\} \\ & -2\mathrm{i}\,\hat{\mathbf{z}}_\mathrm{s}\ \cos(Q\varphi_\mathrm{s}+\varphi_0)\sin\theta\,J_Q(-k^*)\,)\ \sin\theta\,\mathrm{d}\theta\,, \end{aligned} \quad (3)$$

where $k^* = k\rho_\mathrm{s}\sin\,\theta$. When $\varphi_0 = 0$ and $Q = 0$, Eq. (3) agrees with Eq. (12) of Ref. [4], which is free of the sign error mentioned above. If all $-k^*$ in Eq. (3) is changed to k^*, the resulting formula in the special case of $\varphi_0 = 0$ but arbitrary Q agrees with Eq. (1) of Ref. [5]. In any case, without the two lost terms Eq. (1−1) leads to an expression substantially different from Eq. (3).

For the numerical results shown in Figs. 3−7, the authors use the uniform apodization function given by Eq. (1−2), which is constant in the entire spherical aperture. This amounts to using an incident beam of nonzero on-axis transverse field. However, the symmetry of a rotationally symmetric vector beam requires that its transverse electric field component be zero on the beam axis. Furthermore, from Maxwell's equations it can be shown that, for the incident beam given by Eq. (1), $E(\rho)$ must tend to 0 at least as fast as $\rho \rightarrow 0$ if $Q = 0$ or -2 and at least as fast as $\rho^2 \rightarrow 0$ if $|Q + 1| > 1$. So, the incident beam used by the authors in Ref. [1] is nonphysical, and even if Eq. (1−1) were correct the numerical results in Figs. 3−6 would still be incorrect. Finally, the absolute value of the transmission function given by Eq. (1−3) for the designed diffractive optical element is 1.0 in the central zone. As a result of using the nonphysical apodization function, the simulation result shown in Fig. 7 corresponds to a nonphysical situation.

References

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