A model for LED spectra at different drive currents

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A mathematical model for the spectra of monocolor light-emitting diodes (LEDs) and phosphor-coated white LEDs at different drive currents is established. The simulation program of the color rendering of a white light LED cluster is developed based on this model. The program can predict not only the spectral power distribution and color rendering index (CRI), but also the number of LEDs, drive currents, input power, and luminous efficacy of a white light LED cluster at a given color temperature according to the requirement of the luminous flux. The experimental results show that the relative spectral power distributions (SPDs) and chromaticity coordinates of the model LED are very close to that of the real LED at different drive currents. Moreover, the correlated color temperature (CCT), CRI, special color rendering index (R9) luminous flux, input power, and luminous efficacy of the white light LED cluster predicted by simulation are also very close to the measured values. Furthermore, a white/red cluster with high rendering (CCT = 2903 K, CRI = 91.3, R9 = 85) and a color temperature tunable warm-white/red/green/blule LED cluster with high rendering (CCT = 2700−6500 K, CRI > 90, R9 > 96) are created.

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One of the most important characteristics of light sources for general lighting is color rendering. Color rendering is a property of a light source, which shows how natural the colors of objects look under the given illumination. If color rendering is poor, the light source will not be useful for general lighting. This is an important aspect to be considered in developing white light-emitting diodes (LEDs) for general lighting. To create white light from LEDs, two distinct approaches have been adopted: the use of individual primary-color LEDs (e.g., red, green, and blue, also known as RGB), and the combination of wavelength down-converting phosphor with a blue or ultraviolet LED. Color rendering index (CRI) can reach above 90 in any color temperature region from 2700 to 13000 K by matching the ratio of red and green phosphors and silica gel[1]. The light-extraction efficiency of LEDs has been improved by double-layer or multi-layer phosphor package structures[2]. The multichip RGB approach allows for high color rendering, chromaticity-point stabilization, and color adjustment[3−5]. However, the strong temperature dependence of performance and high cost are among its main disadvantages. It is possible to maintain precise and stable color temperature control using optical and thermal feedbacks[5]. Meanwhile, phosphor-coated white LEDs have the advantages of being relatively low cost and having great color stability over a wide range of temperatures[6]. These can also be tuned to achieve higher CRIs and luminous efficacies of radiation through dual-blue emitting active regions as opposed to single-blue white sources[7]. In order to analyze the numerous designs of white light LED clusters, mathematical models for relative spectral power distribution (SPD) of LEDs at a given drive current have been developed[3−5]. However, the drive current affects the peak wavelength, half spectral width of the monocolor LED, the relative SPD of the phosphor-coated white LED, and the color rendering of white light LED clusters. Thus, simulated programs developed by these models cannot predict the drive currents, luminous flux, input power, and luminous efficacy of white light LED clusters. In this letter, the mathematical model for the spectra of monocolor LEDs and phosphor-coated white LEDs at different drive currents is established, and the simulation program of the color rendering of the white light LED cluster is developed based on this model. The simulations and verifications of the two white light LED clusters are also presented.

Based on the established mathematical model for the spectra of monocolor LED, its relative SPD, $S_{\mathrm{LED}}(\lambda, \lambda_0, \Delta\lambda)$, is given by

$$S_{\mathrm{LED}}(\lambda, \lambda_0, \Delta\lambda) = [g(\lambda, \lambda_0, \Delta\lambda) + k_1 g(\lambda, \lambda_0, \Delta\lambda)^{k_2}] /(1 + k_1), \quad (1)$$

where

$$g(\lambda, \lambda_0, \Delta\lambda) = \exp[-(\lambda - \lambda_0)^2/(\Delta\lambda)^2],$$

$$\Delta\lambda = \begin{cases} \Delta\lambda_1, (\lambda < \lambda_0) \\ \Delta\lambda_2, (\lambda \geq \lambda_0) \end{cases},$$

$$k_i = \begin{cases} k_i^1, (\lambda < \lambda_0) \\ k_i^2, (\lambda \geq \lambda_0) \end{cases} \quad (i = 1,2),$$

λ_0 is the peak wavelength, $\Delta\lambda_1$ is the left half spectral width which is $2\int_{380\ \mathrm{nm}}^{\lambda_0} S_{\mathrm{LED}}(\lambda)\mathrm{d}\lambda$, $\Delta\lambda_2$ is the right half spectral width which is $2\int_{\lambda_0}^{780\ \mathrm{nm}} S_{\mathrm{LED}}(\lambda)\mathrm{d}\lambda$, $\Delta\lambda_0$ is the half spectral width which is $[(\Delta\lambda_1 + \Delta\lambda_2)/2]$, and k_i $(i = 1,2)$ represents the characteristic parameters of the spectral shape. Figure 1 shows an example of this LED model compared with the SPD of a typical blue LED. The average residual sum of squares, Chi^2/DoF, for the model and real SPDs of blue LED is 1×10^{-6}. The difference of the chromaticity coordinates (u, v) of the model and those of real blue LED, $\mathrm{d}C = \sqrt{(\Delta u)^2 + (\Delta v)^2}$, is 6×10^{-7}.

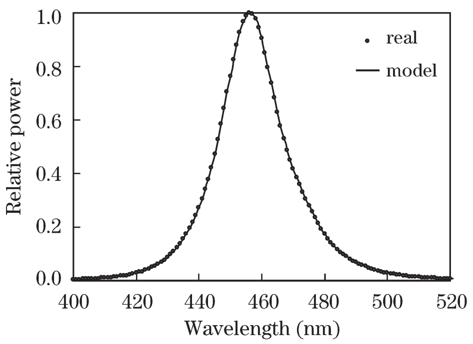

Fig. 1. SPDs of a real and model blue LED.

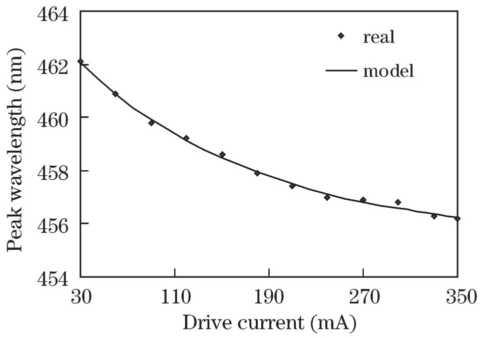

Fig. 2. Curve of λ_0 versus I_F for a typical blue LED.

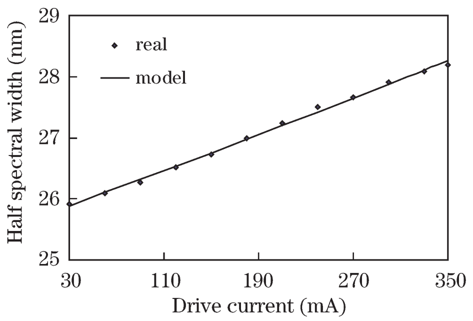

Fig. 3. Curve of $\Delta\lambda_0$ versus I_F for a typical blue LED.

LED drive current affects the peak wavelength and half spectral width of monocolor LED. The functional relationship of the peak wavelength λ_0 and the drive current I_F, λ_0 (I_F), is given by

$$\lambda_0(I_F) = A_{\lambda_0} \exp(B_{\lambda_0} I_F) + C_{\lambda_0}, \quad (2)$$

where A_{λ_0}, B_{λ_0}, and C_{λ_0} are function parameters of $\lambda_0(I_F)$. The curve of λ_0 versus I_F for a typical blue LED is shown in Fig. 2. Chi^2/DoF for the curve of λ_0 versus I_F of blue LED is 1×10^{-2}.

The functional relationship of the half spectral width $\Delta\lambda_0$ and the drive current I_F, $\Delta\lambda_0(I_F)$, is given by

$$\Delta\lambda_0(I_F) = A_{\Delta\lambda} + B_{\Delta\lambda} I_F, \quad (3)$$

where $A_{\Delta\lambda}$ and $B_{\Delta\lambda}$ are the function parameters of $\Delta\lambda_0(I_F)$. $\Delta\lambda_1$ and $\Delta\lambda_2$ can also be expressed by Eq. (3). The curve of $\Delta\lambda_0$ versus I_F for a typical blue LED is shown in Fig. 3. Chi^2/DoF for the curve of $\Delta\lambda_0$ versus I_F of blue LED is 1×10^{-3}.

In this letter, the LED drive current affects the characteristic parameters k_i (i=1,2) of the spectral shape

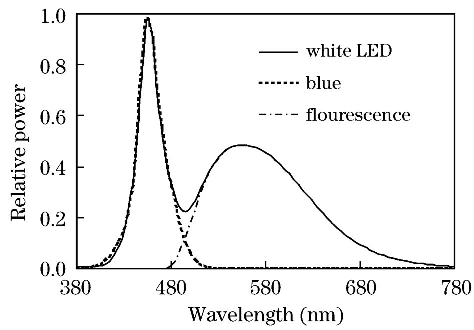

Fig. 4. White, blue, and fluorescence spectra of the phosphor-coated white LED.

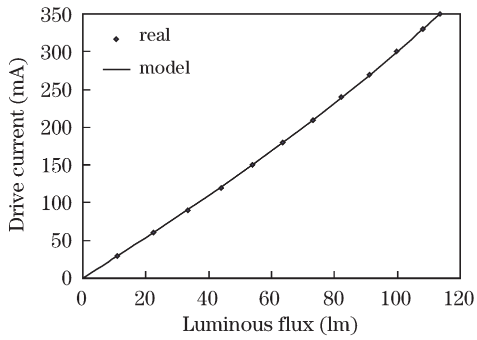

Fig. 5. Curve of I_F versus Φ for a typical white LED.

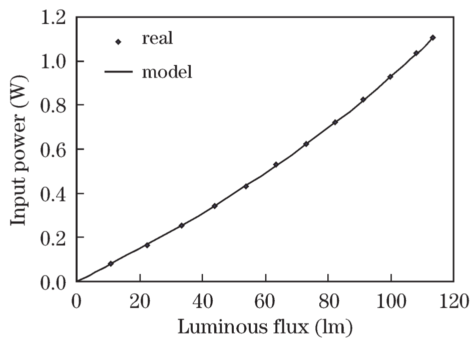

Fig. 6. Curve of P_{in} versus Φ for a typical white LED.

for monocolor LED. The functional relationship between the characteristic parameter k_i and the drive current I_F, $k_i(I_F)$, is given by

$$k_i(I_F) = A_i \exp(B_i I_F) + C_i, \quad (4)$$

where A_i, B_i, and C_i are the function parameters of $k_i(I_F)$.

Four monocolor LEDs (red, yellow, green, and blue) were tested. Their parameters are shown in Table 1. Chi^2/DoF for the model and real SPDs of these LEDs at different drive currents are shown in Table 2. dCs of the model and real LEDs at different drive currents are shown in Table 3. The results show that the SPDs and chromaticity coordinates of the model LED are very close to those of real LED.

A mathematical model for the spectra of phosphor-coated white LED was established in this work. The relative SPD of phosphor-coated white LED, $S_W(\lambda)$, is given by

$$S_W(\lambda) = S_B(\lambda) + S_F(\lambda), \quad (5)$$

Table 1. Parameters of Monocolor LEDs

LED	λ_0			$\Delta\lambda_1$		k_1^1			k_1^2		
	A_{λ_0}	B_{λ_0}	C_{λ_0}	$A_{\Delta\lambda}$	$C_{\Delta\lambda}$	A_1^1	B_1^1	C_1^1	A_1^2	B_1^2	C_1^2
Red	627.12	9.46×10^{-6}	0	30.5	19.90	0.1582	−0.0085	1.7220	0.9855	−0.0085	6.9403
Yellow	595.57	7.34×10^{-6}	0	30.2	18.91	1.4151	0.0001	0.0000	0.7000	−0.0095	6.4750
Green	10.96	-7.45×10^{-3}	514	30.8	36.81	9.3850	0.0002	0.0000	1.3725	−0.0461	5.3000
Blue	8.38	-5.61×10^{-3}	455	30.1	20.25	0.7617	−0.0288	1.6600	0.6189	−0.0288	4.7700
LED	λ_0			$\Delta\lambda_2$		k_2^1			k_2^2		
	A_{λ_0}	B_{λ_0}	C_{λ_0}	$A_{\Delta\lambda}$	$C_{\Delta\lambda}$	A_2^1	B_2^1	C_2^1	A_2^2	B_2^2	C_2^2
Red	627.12	9.46×10^{-6}	0	30.5	14.25	2.5089	0.0004	0.0000	5.9287	0.0001	0.0000
Yellow	595.57	7.34×10^{-6}	0	30.2	14.25	3.1765	0.0002	0.0000	0.6720	−0.0095	4.7960
Green	10.96	-7.45×10^{-3}	514	30.4	36.81	1.2082	0.0006	0.0000	2.3553	−0.0461	6.3000
Blue	8.38	-5.61×10^{-3}	455	30.1	22.91	1.6600	0.9576	0.0003	4.7700	4.5226	−0.0288

Table 2. Chi^2/DoF for the Model and Real SPDs of Monocolor LEDs ($\times10^{-5}$)

I_F (mA)	30	60	90	120	150	180	210	240	270	300	330	350
Red LED	1.8	2.5	2.0	1.7	1.6	1.7	1.7	1.8	1.4	1.4	1.4	1.4
Yellow LED	1.6	1.4	1.2	1.2	1.1	1.2	1.1	1.5	1.1	1.6	1.1	1.2
Green LED	3.8	6.3	5.5	4.5	4.4	4.6	4.4	4.1	4.0	3.9	4.0	4.0
Blue LED	0.1	1.8	3.8	3.5	2.5	1.6	0.9	0.6	0.4	0.2	0.1	0.1

Table 3. dC of the Model and Real Monocolor LEDs ($\times10^{-4}$)

I_F (mA)	30	60	90	120	150	180	210	240	270	300	330	350
Red LED	1	3	3	2	2	3	2	3	2	2	2	2
Yellow LED	1	5	4	3	2	2	3	1	2	3	1	1
Green LED	0	4	2	0	2	3	4	4	3	1	0	0
Blue LED	0	1	4	3	1	0	1	4	4	3	1	0

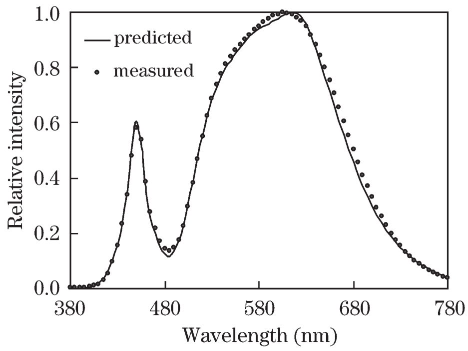

Fig. 7. SPDs of the phosphor-coated white LED.

where $S_W(\lambda)$, $S_B(\lambda)$, and $S_F(\lambda)$ are the white, blue, and fluorescence spectra of phosphor-coated white LED, respectively. $S_B(\lambda)$ can be expressed by Eq. (1). $S_F(\lambda)$ can be determined by $\sum_{380}^{475}[S_W(\lambda)-S_B(\lambda)]^2\rightarrow\min$. $S_W(\lambda)$, $S_B(\lambda)$, and $S_F(\lambda)$ of a typical phosphor-coated white LED are shown in Fig. 4. Chi^2/DoF for the model and real SPDs of the LED is 3×10^{-5} and dC of the model and real white LEDs is 1×10^{-4}.

The drive current also affects the relative SPD of phosphor-coated white LED. Equations (2)–(4) can be applied to the blue spectrum of phosphor-coated white LED. The functional relationship of the fluorescence spectrum $S_F(\lambda)$ and the drive current I_F, $S_F(\lambda, I_F)$, is given by

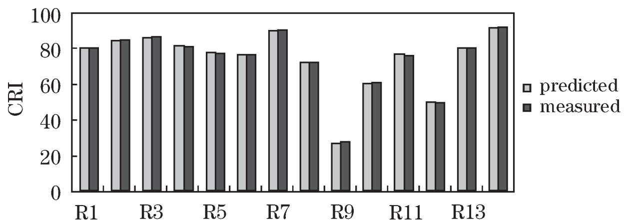

Fig. 8. CRIs of the warm-white LED.

$$S_F(\lambda, I_F) = S_F(\lambda, I_{F\max}) + A_F\exp(B_F I_F), \qquad (6)$$

where A_F and B_F are the function parameters of $S_F(\lambda, I_F)$. Chi^2/DoF for the model and real SPDs of phosphor-coated white LEDs at different drive currents are shown in Table 4. dCs of the model and real phosphor-coated white LEDs at different drive currents are shown in Table 5. The results show that the SPDs and chromaticity coordinates of model white light LED are very close to those of the real white light LED.

Table 4. Chi^2/DoF for the Model and Real SPDs of Phosphor-Coated White LEDs ($\times 10^{-5}$)

I_F (mA)	30	60	90	120	150	180	210	240	270	300	330	350
WW LED	0.5	1.3	1.5	2.0	3.0	1.7	1.0	0.9	0.7	0.6	0.5	0.5
NW LED	0.4	0.4	0.5	0.7	0.7	0.7	1.3	0.7	0.6	0.5	0.5	0.4
CW LED	0.6	1.6	2.3	3.2	3.1	2.8	2.5	2.1	2.0	1.6	1.3	1.2

*WW, NW, and CW LEDs refer to warm-white, neutral-white, and cool-white LEDs, respectively.

Table 5. dC of the Model and Real Phosphor-Coated White LEDs ($\times 10^{-4}$)

I_F (mA)	30	60	90	120	150	180	210	240	270	300	330	350
WW LED	1	1	1	0	0	1	2	2	1	1	0	0
NW LED	0	0	0	1	0	1	2	1	1	1	1	1
CW LED	1	2	3	2	3	2	2	2	2	1	1	1

Table 6. CCT, CRI, R9, Φ, P_{in}, and η of the White/Red Cluster

	CCT	CRI	R9	Φ (lm)	P_{in} (W)	η (lm/W)
Predicted	2909	90.3	84	457.4	5.94	77.0
Measured	2903	91.3	85	464.9	5.91	78.6

Table 7. Predicted and Measured Results of WW/Red/Green/Blue Cluster (P_{in}=8 W)

	N	CCT (K)	2699	2998	3497	3999	4497	4995	5699	6505
WW LED	4	I_F (mA)	330	321	300	279	261	245	217	200
Red LED	3	I_F (mA)	263	226	184	156	136	121	106	94
Green LED	4	I_F (mA)	157	182	216	241	259	271	297	303
Blue LED	1	I_F (mA)	0	26	82	136	187	235	277	339
Predicted Results	CRI	94.3	94.4	94.3	94.1	93.8	93.6	92.6	92.3	
	R9	97.6	98.3	98.4	98.5	98.3	98.2	98.2	98.7	
	Φ (lm)	477.1	476.9	469.4	458.9	448.0	437.2	421.0	406.7	
	P_{in} (W)	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	
	η (lm/W)	59.6	59.6	58.7	57.4	56.0	54.6	52.6	50.8	
Measured Results	CCT (K)	2639	2977	3523	4010	4492	4979	5676	6422	
	CRI	94.8	94.2	93.6	93.4	93.1	92.9	92.0	91.6	
	R9	96.0	97.5	98.5	98.6	98.8	98.3	97.8	99.2	
	Φ (lm)	485.2	471.1	452.8	441.4	429.3	417.0	398.0	383.0	
	P_{in} (W)	7.5	7.5	7.5	7.5	7.5	7.6	7.6	7.6	
	η (lm/W)	64.4	62.7	60.2	58.7	56.9	55.1	52.5	50.4	

To predict the drive current of LED at a given luminous flux, the functional relationship between the drive current I_F and luminous flux Φ, $I_F(\Phi)$, is given by

$$I_F(\Phi) = k_I \Phi^{\gamma}(1 + c_I \Phi^2), \quad (7)$$

where k_I, c_I, and γ are the function parameters of $I_F(\Phi)$. The curve of I_F versus Φ for a typical white LED is shown in Fig. 5. Chi^2/DoF for the curve of I_F versus Φ for a typical white LED is 6×10^{-2}.

To predict the input power of LED at a given luminous flux, the functional relationship between the input power P_{in} and luminous flux Φ, $P_{in}(\Phi)$, is given by

$$P_{in}(\Phi) = k_P \Phi^{\gamma'}(1 + c_P \Phi^2), \quad (8)$$

where k_P, c_P, and γ' are the function parameters of $P_{in}(\Phi)$. The curve of P_{in} versus Φ for a typical white LED is shown in Fig. 6. Chi^2/DoF for the curve of P_{in} versus Φ for a typical white LED is 2×10^{-4}.

To analyze the possible performance of the white light LED cluster, a simulation program was developed according to the principles of additive color mixture and the Commission International del' Eclairage (CIE) method of measuring and specifying color rendering properties of light sources. The simulation program can predict the SPD of phosphor-coated white LEDs by spectra mixture of blue chip, yellow, green, and red phosphor according to the requirements of the CCT and CRI. Thus,

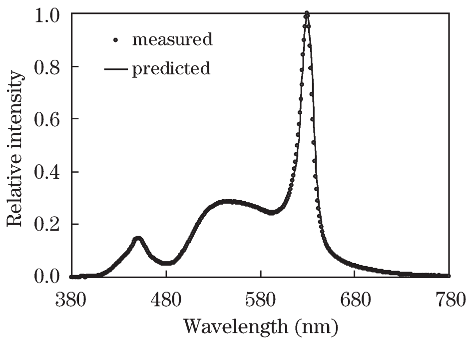

Fig. 9. SPDs of the W/R cluster.

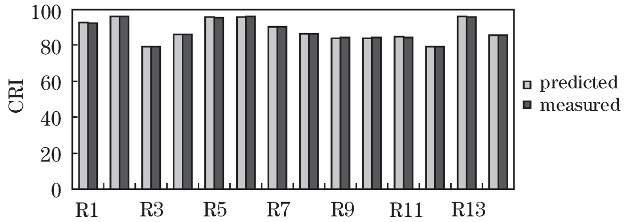

Fig. 10. CRIs of the W/R cluster.

the program can predict not only the SPDs, chromaticity coordinates, and CRIs, but also the number of LEDs (N), drive currents ($I_{\rm F}$), input power ($P_{\rm in}$), and the luminous efficacy (η) according to the requirements of the CCT and luminous flux (Φ).

To validate the simulation for phosphor-coated white LED, a real warm-white LED (CCT = 3183 K, u = 0.2429, v = 0.3487, CRI = 80.4, excitation wavelength λ_0 = 451.0 nm) was simulated by our program. The simulated and measured SPD and CRIs of phosphor-coated white LED are shown in Figs. 7 and 8, respectively. The simulation results show that the predicted spectra are very close to the real spectra of the warm-white LED, and that the predicted CRIs are almost equal to the measured values.

To validate simulation for the white light cluster, white/red LED (W/R cluster) and warm-white/red/green/blue LED (WW/R/G/B cluster) were tested in our laboratory.

The W/R clusters with white light LEDs (excited wavelength λ_0 = 450.0 nm, CCT = 4587 K, Φ = 95.1 lm, $P_{\rm in}$ = 1.15 W, and η = 82.7 lm/W at $I_{\rm F}$ = 350 mA) and red LEDs (λ_0 = 631.9 nm, Φ = 37.6 lm, $P_{\rm in}$ = 0.75 W, and η = 50.1 lm/W at $I_{\rm F}$ = 350 mA) were simulated according to CCT = 2900 K and Φ = 450 lm. The predicted numbers of white light and red LEDs were four and three, respectively, while the predicted drive currents of white light and red LEDs were 350 and 220 mA, respectively. The predicted and measured SPDs and CRIs of the W/R cluster are shown in Figs. 9 and 10, respectively. The CCT, CRI, Φ, $P_{\rm in}$, and η of the W/R cluster are shown in Table 6.

The WW/R/G/B clusters ($P_{\rm in}$ = 8 W) with the WW LEDs (excited wavelength λ_0 = 454.0 nm, CCT = 3162 K, Φ = 79.2 lm, $P_{\rm in}$ = 1.21 W, and η = 65.5 lm/W at $I_{\rm F}$ = 350 mA), the red LEDs (λ_0 = 633.6 nm, Φ = 32.7 lm, $P_{\rm in}$ = 0.75 W, and η = 43.6 lm/W at $I_{\rm F}$ =350 mA), the green LEDs (λ_0 = 522.9 nm, Φ = 45.2 lm, $P_{\rm in}$ = 1.08 W, and η = 41.9 lm/W at $I_{\rm F}$ = 350 mA), and the blue LEDs (λ_0 = 454.3 nm, Φ = 12.8 lm, $P_{\rm in}$ = 1.21 W, and η = 10.6 lm/W at $I_{\rm F}$ = 350 mA) were simulated. The predicted and measured results are shown in Table 7.

The experimental results show that the correlated color temperature, CRIs, luminous flux, input power, and luminous efficacy of white light LED cluster predicted using simulation are very close to the measured values. Furthermore, CRI and R9 of the W/R and WW/R/G/B clusters are over 90. This result is the first to be created successfully in a laboratory as far as we know.

In conclusion, a mathematical model for spectra of monocolor and phosphor-coated white LEDs at different drive currents is established. The simulation program of the color rendering of the white light LED cluster is developed based on this model. The program can predict both the SPDs and CRIs, as well as the numbers of LEDs, drive currents, input power, and luminous efficacy of a white light LED cluster at a given color temperature according to the requirement of luminous flux. As far as we know, our laboratory is the first to create successfully the W/R cluster with high rendering (CCT = 2903 K, CRI = 91.3, R9 = 85) and a color temperature tunable WW/R/G/B cluster with high rendering (CCT = 2700–6500 K, CRI $>$ 90, R9 $>$ 96).

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References

1. H. Wu, C. Wang, and S. He, Acta Opt. Sin. (in Chinese) **28,** 1777 (2008).
2. B. Li, D. Zhang, Y. Huang, Z. Ni, and S. Zhuang, Chin. Opt. Lett. **8,** 221 (2010).
3. Y. Ohno, Proc. SPIE **5530,** 88 (2004).
4. S. Chhajed, Y. Xi, Y. L. Li, T. Gessmann, and E. F. Schubert, J. Appl. Phys. **97,** 054506 (2005).
5. K. Man and I. Ashdown, Proc. SPIE **6337,** 633702 (2006).
6. E. F. Schubert, *Light Emitting Diodes* (Cambridge University Press, Cambridge, 2003).
7. R. Mirhosseini, M. F. Schubert, S. Chhajed, J. Cho, J. K. Kim, and E. F. Schubert, Opt. Express **17,** 10806 (2009).