Beam steering approach for high-precision spatial light modulators

Lingjiang Kong (孔令讲)*, Ying Zhu (朱 颖), Yan Song (宋 艳), and Jianyu Yang (杨建宇)

School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China

**E-mail: uestc.kong@gmail.com*

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Steering accuracy is limited by the quantized phase modulation values and the number of phase pixels for spatial light modulators (SLMs). Conventional methods of beam steering lack optimum precision. In this letter, a beam steering approach based on horizontally moving phase steps is proposed. Compared with the conventional methods, this novel method is able to reduce the maximum normalized steering error in SLM significantly by a factor proportional to the number of pixels. In addition, steering errors of high steering angles can be drastically reduced by a factor proportional to the product of the number of pixels and the quantized phase levels; the number of high-precision steering angles increases with the number of pixels or the quantized phase levels increasing.

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Phase-only spatial light modulators (SLMs) can be used in beam correction, beam splitting, beam forming, and beam steering techniques that are utilized in laser beam weapons and optical communications, among others[1−7]. In free-space optical communication, sub-microradian pointing accuracy must be reached to maintain precise transportation between satellites with a distance of over ten thousand kilometers, and the bit error rate (BER) of the optical communication system decreases as the steering accuracy increases[8,9]. In deep space communication, pointing loss, which decreases as the steering accuracy increases, is an important measure of system performance[10,11]. Thus, it is necessary to improve the steering accuracy to obtain low pointing loss and low BER. However, most studies regarding SLMs have focused on steering efficiency, and little work has been published on steering accuracy[12−15]. In Ref. [16], a method was proposed based on the modification of the phase offset of all phase pixels to markedly improve the steering accuracy; however, the new wavefront achieved by this method followed the rules set by the conventional method. This means that the phase values of the phase pixels are always equal to the realized quantized phases closest to the ideal linear wavefront.

In this letter, we analyze the cause of steering error. After investigating the relationship between the mean slope and the wavefront phase step edges, a novel iterative optimization method is presented by modifying the wavefront. Simulation results indicate that the steering accuracy of the new steering method is significantly better than both the conventional method and the steering method in Ref. [16], especially for large steering angles.

In a one-dimensional (1D) SLM, each phase pixel can be regarded as a pixel with a quantized phase. In an electrically addressed SLM, each pixel takes on different phase delays through application of quantized voltages. SLMs can only perform the staircase phase pattern to approximate the ideal linear wavefront because of the limitation of the voltage quantization. The conventional beam steering method utilizes the quantized phase threshold, and divides the ideal linear wavefront into a staircase wavefront with several quantized phase levels; each phase pixel only performs the phase level closest to the ideal phase value[17].

Although the conventional method enables control of the beam deflection, it cannot achieve high-precision beam steering. The steering error is caused by the discaccord between the staircase wavefront and the ideal linear wavefront. Nevertheless, the fitting line from the linear fitting of the staircase wavefront can be regarded as a linear wavefront. The steering angle of the staircase wavefront is similar to that of the linear fitting wavefront. The steering angle can also be calculated by the slope k of the linear fitting wavefront. This indicates that the results of slope calculation is consistent with the results using the Fresnel diffraction integral equation to simulate the far field pattern of staircase wavefront and to obtain the peak intensity position corresponding to the steering angle[16].

As previously described, the analysis regarding the steering angles for the conventional steering method can be carried out using the method based on slope calculation. First, $\theta_{\max} = \arcsin(\lambda/2d)$, where d is the center-to-center spacing between adjacent phase pixels, θ is the aimed steering angle, and λ is the laser wavelength, is defined as the maximum angle of SLM. The distributions of realized and theoretical steering angles over the range $0 - \theta_{\max}$ with normalized coordinates are shown in Fig. 1, where M is the number of quantized steps between the phase 0 and 2π, and N is the number of phased array units. The maximum normalized steering error is $\varepsilon_{\text{norm,max}}$=0.1519, as shown at POS 1. Here, the normalized steering error $\varepsilon_{\text{norm}} = |\theta - \theta_{\text{stair}}|/\theta_{\text{spot}}$ is defined as the difference between the realized steering angle θ_{stair} and the theoretical steering angle θ divided by the beam spot size $\theta_{\text{spot}} = \lambda/(Nd)$. Evidently, the performance of the conventional beam steering method must be improved. The empirical formula for $\varepsilon_{\text{norm,max}} = 0.625M^{-1}$ was provided in Ref. [16].

The steering angle is determined by the mean slope of the wavefront; thus, the steering accuracy is closely related to the mean slope of the wavefront. Analyzing the

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cause of steering errors for conventional method is necessary in order to propose an improved steering method. The enlarged drawing of wavefront that corresponds to the maximum steering error at POS 1 of Fig. 1 is shown in Fig. 2.

In Fig. 2, the error between the steering angles of the staircase wavefront and the ideal linear wavefront is $\theta_{\text{error}} = |\theta - \theta_{\text{stair}}|$, where $\theta_{\text{stair}} = \arcsin(\frac{\lambda k}{2\pi d})$ and $\theta = \arcsin(\frac{\lambda k_{\text{I}}}{2\pi d})$. Hence, the steering error can also be written as

$$\theta_{\text{error}} = \left|\arcsin\left(\frac{\lambda k_{\text{I}}}{2\pi d}\right) - \arcsin\left(\frac{\lambda k}{2\pi d}\right)\right|, \quad (1)$$

where k is the slope of the linear fitting wavefront for the staircase wavefront, and k_{I} is the slope of the ideal wavefront. As shown in Fig. 2, the number of the left phase pixels with the phase value of 0 is small, and the number of the right phase pixels with the phase value of 1.57 rad is excessively large. Thus, the staircase wavefront increases the mean slope of the whole wavefront, causing θ_{error}. This indicates that the difference between k and k_{I} yields θ_{error}. The mean slope k becomes close to the slope k_{I} of the ideal linear wavefront by modifying the location of the wavefront steps; the steering error further decreases and the steering accuracy further improves.

Based on the analysis above, the modification of the mean slope k can reduce the mean slope error $k_{\text{error}} = |k_{\text{I}} - k|$ to the minimum, leading to the further minimization of the angle error θ_{error}. The mean slope can be obtained by the slope of the linear fitting wavefront of the staircase wavefront $U(n)$. If the fitting line is $U_{\text{fit}}(n) = a + kn$, where $n = 1, 2, \cdots, N$ and $a = \overline{U} - k\overline{n}$, then the slope can be written as

$$k = \frac{\overline{nU} - \overline{n} \cdot \overline{U}}{\overline{n^2} - (\overline{n})^2}, \quad (2)$$

where $\overline{U} = \frac{1}{N}\sum_{n=1}^{N} U(n)$, $\overline{n} = \frac{1}{N}\sum_{n=1}^{N} n = \frac{N+1}{2}$, $\overline{n^2} = \frac{1}{N}\sum_{n=1}^{N} n^2 = \frac{(N+1)(2N+1)}{6}$, $\overline{nU} = \frac{1}{N}\sum_{n=1}^{N} nU(n)$. Thus, the mean slope of the staircase wavefront can be obtained from Eq. (2).

The relationship between the quantized staircase wavefront of SLM and phase delays is shown in Fig. 3. If p represents the number of phase levels and a_i represents the serial number of the phase pixels at the extreme right

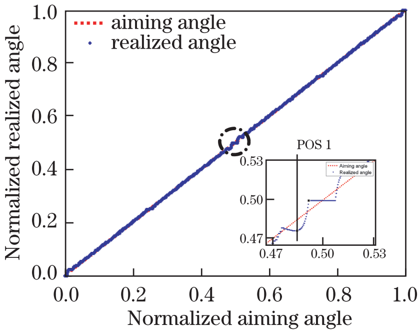

Fig. 1. Distributions of the theoretical and realized steering angles for the conventional method over ($M = 4$, $N = 64$).

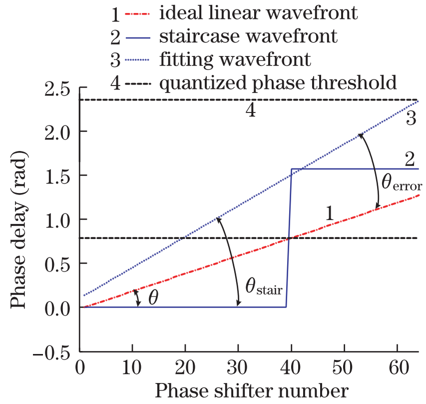

Fig. 2. Output wavefront at POS 1 with the maximal steering error.

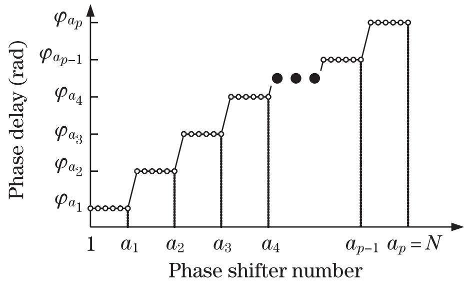

Fig. 3. θ uantized staircase wavefront wavefront of SLM.

position of phase step with the phase level $\varphi_{a_i}(i = 1, 2, \cdots, p)$, then the phase distribution of the staircase wavefront in a single period can be denoted as $U(n) = \phi(n)$,

$$U(n) = \begin{cases} \varphi_{a_1} & 1 \leqslant n \leqslant a_1 \\ \varphi_{a_2} & a_1 + 1 \leqslant n \leqslant a_2 \\ \vdots & \\ \varphi_{a_p} & a_{p-1} + 1 \leqslant n \leqslant a_p (a_p = N) \end{cases}. \quad (3)$$

From Eqs. (2) and (3), the simplified mean slope is

$$k = \frac{6}{N^2 - 1}\sum_{n=1}^{N-1} n\left(1 - \frac{n}{N}\right)(\phi_{n+1} - \phi_n). \quad (4)$$

Using Eq. (4), the mean slope is obtained by the superposition of different weights of the phase pixels. With the weight $A_n = n(1 - n/N)$ of the nth phase pixel, the mean slope can be written as

$$k = C\sum_{n=1}^{N-1} A_n(\phi_{n+1} - \phi_n), \quad (5)$$

where $C = 6/(N^2 - 1)$ and C is constant when the total number of phase pixels is fixed. The distribution of weight $A_n = n(1 - n/N)$ is shown in Fig. 4 (supposing $N = 64$).

Afterward we can obtain: 1) The weight is distributed in the form of a parabola. Thus, the maximal weight is located at the center of the SLM at $N/2 = 32$, and the weights at the two ends of the SLM are the smallest. 2) The horizontal movement of the step edges at the center of the liquid crystal phased array (LCPA) has the least influence on the modification of the mean slope, while the movements of the step edges at the two ends have the greatest influence on the modification of the mean slope. This is because the first derivative at the peak of a parabola is the smallest and the first derivatives at the two ends are the largest. 3) The mean slope of step edges with serial numbers smaller than $N/2$ increases when moving to the right, and decreases when moving to the left. In contrast, the mean slope of step edges with serial numbers larger than $N/2$ decreases when moving to the right, and increases when moving to the left.

The effect of the phase step edges with horizontal movement toward the mean slope can be analyzed as described above. As shown in Fig. 5, step edges are produced between phase pixels with different phase values; hence, the total number of step edges for the staircase output wavefront is $p-1$. Here, the serial number of the first phase pixel on the left side of the step edge is used to show the position of the step edge. The serial number of the phase pixel at the qth step edge is a_q. In Fig. 5(a), the right movement of the qth step edge denotes the phase of the (a_q+1)th phase pixel that decreases by one phase step. This indicates that the phase has been modified from φ_{a_q+1} to $\varphi_{a_q+1}-\Delta\varphi$, where $\Delta\varphi = 2\pi/M$. In Fig. 5(b), the left movement of the qth step edge denotes the phase of (a_q+1)th phase pixel that increases by one phase step. This indicates that the phase has been modified from ϕ_q to $\phi_q+\Delta\phi$. Under normal conditions, $\phi_{a_q+1}-\phi_{a_q}$ is equal to the integral multiple of $\Delta\phi$.

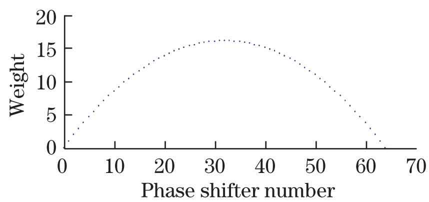

Fig. 4. Weight values distribution of phase pixels ($N = 64$).

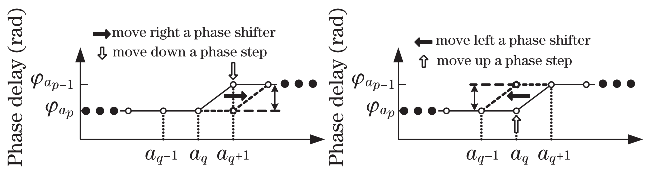

Fig. 5. Relocation of the step edges to the (a) right and (b) left.

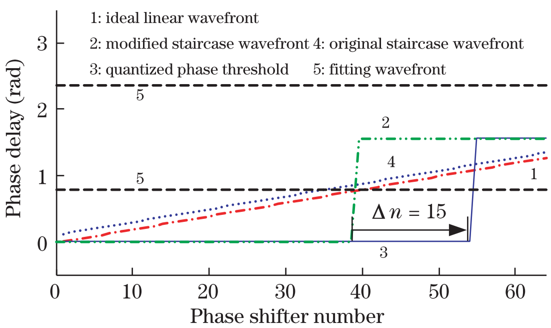

Fig. 6. Method to reduce the steering error through the modification of the wavefront.

According to Eq. (5), the mean slope without movement can be expressed as

$$k_0 = C\Big[\sum_{n=1}^{a_q-1} A_n(\phi_{n+1}-\phi_n) + A_{a_q}(\phi_{a_q+1}-\phi_{a_q}) + \sum_{n=a_q+1}^{N-1} A_n(\phi_{n+1}-\phi_n)\Big]. \quad (6)$$

After moving the step edges to the right, the mean slope becomes

$$k_{\mathrm{R}} = C\Big\{\sum_{n=1}^{a_q-1} A_n(\varphi_{n+1}-\varphi_n) + A_{a_q}[(\varphi_{a_q+1}-\Delta\varphi) - \varphi_{a_q}] + A_{a_q+1}[\varphi_{a_q+2}-(\varphi_{a_q+1}-\Delta\varphi)] + \sum_{n=a_q+2}^{N-1} A_n(\varphi_{n+1}-\varphi_n)\Big\}, \quad (7)$$

where, $\Delta k_{a_q\mathrm{R}}$ is defined as the modification of the mean slope after the qth step edge moves a phase pixel toward the right. Thus,

$$\Delta k_{a_q\mathrm{R}} = k_{\mathrm{R}} - k_0 = C\Delta\phi(A_{a_q+1} - A_{a_q}). \quad (8)$$

Similarly, $\Delta k_{a_q\mathrm{L}}$ is defined as the modification of the mean slope after the qth step edge moves a phase pixel towards the left. Thus,

$$\Delta k_{a_q\mathrm{L}} = C\Delta\phi(A_{a_q-1} - A_{a_q}). \quad (9)$$

According to Eqs. (8) and (9), the modification of the mean slope caused by the horizontal movement of the step edges is related to the differential value of weight A_n.

In this letter, we develop a method based on the modification of the wavefront. The mean slope can be modified by shifting the position of the steps of the wavefront, which in turn can be accomplished by moving the phase step edges. The steering accuracy can thereby be improved. This method enables the independent movement of the phase steps, which is performed by horizontally moving the step edges. The method proposed in Ref. [16] enables the movement of the phase steps in whole, which is performed by selecting the approximate optimal initial phase offset to produce the ideal linear wavefront. The new wavefront achieved by this method deviates from the rules set by the conventional method; the phase values of phase pixels are not always equal to the realized quantized phases closest to the ideal linear wavefront. When the phase steps are few, the exhaustive method can be used to calculate the distances and directions of movement to carry out the process of modifying wavefront. As shown in Fig. 6, if the step edges of the original staircase wavefront is moved 15 phase pixels to the right, the optimal steering accuracy can be achieved. However, when the number of phase steps is large, this exhaustive method is no longer applicable. Therefore, the development of a new iterative algorithm is necessary in modifying the wavefront to improve steering accuracy.

In Eq. (5), the position of step edges is coordinated and the mean slopes are related. This relationship can be used to control the modification of wavefront, and improve the steering accuracy. Combining Eqs. (8) and (9), we define

$$\Delta \boldsymbol{k}_{\mathrm{R}} = \{\Delta k_{1\_\mathrm{R}}, \Delta k_{2\_\mathrm{R}}, \cdots, \Delta k_{N-1\_\mathrm{R}}\},$$
$$\Delta \boldsymbol{k}_{\mathrm{L}} = \{\Delta k_{1\_\mathrm{L}}, \Delta k_{2\_\mathrm{L}}, \cdots, \Delta k_{N-1\_\mathrm{L}}\},$$

where $\Delta \boldsymbol{k}_{\mathrm{R}}$ and $\Delta \boldsymbol{k}_{\mathrm{L}}$ contain the entire modifications of the mean slope with all possible horizontal movements of step edges. The concrete process of the iterative modification of the wavefront is presented in the following steps:

1) Based on the aimed steering angle θ, the output staircase wavefront $U(n) = \phi_n$ can be calculated using the conventional method, where $\phi_n = \mathrm{round}(\phi_n^{\mathrm{ideal}} \frac{M}{2\pi}) \frac{2\pi}{M}$, $n = 1, 2, \cdots, N$, and $\phi_n^{\mathrm{ideal}} = \frac{2\pi d}{\lambda} n \sin\theta$.

2) The mean slope error can be calculated as $k_{\mathrm{error}} = k_{\mathrm{I}} - k$, where k is the mean slope of staircase wavefront, and k_{I} is the mean slope of ideal wavefront,

$$k_{\mathrm{I}} = \Delta\varphi^{\mathrm{ideal}} = \frac{2\pi d}{\lambda} \sin\theta.$$

3) The statistics of all the positions of the step edges of the staircase wavefront are obtained and recorded by array $\boldsymbol{Q}$, and the step edge position is defined as the serial number of the first phase pixel at the left of step edge.

4) In array $\boldsymbol{Q}$, a step edge at the position of q_{R} always exists. Thus,

$$\sigma_{\mathrm{R}} = \min\left[\left|k_{\mathrm{error}} - \Delta \boldsymbol{k}_{\mathrm{R}}(\boldsymbol{Q})\right|\right] = \left|k_{\mathrm{error}} - \Delta \boldsymbol{k}_{\mathrm{R}}(q_{\mathrm{R}})\right|. \quad (10)$$

At the same time, a step edge at the position of q_{L} always exists. Thus,

$$\sigma_{\mathrm{L}} = \min\left[\left|k_{\mathrm{error}} - \Delta \boldsymbol{k}_{\mathrm{L}}(\boldsymbol{Q})\right|\right] = \left|k_{\mathrm{error}} - \Delta \boldsymbol{k}_{\mathrm{L}}(q_{\mathrm{L}})\right|. \quad (11)$$

5) When $\sigma_{\mathrm{R}} > |k_{\mathrm{error}}|$ and $\sigma_{\mathrm{L}} > |k_{\mathrm{error}}|$, the mean slope error $|k_{\mathrm{error}}|$ cannot be further reduced. Hence, the iteration should be stopped, and the angle errors are obtained in step 7.

6) If $\sigma_{\mathrm{R}} < \sigma_{\mathrm{L}}$, $|k_{\mathrm{error}}|$ can be reduced by the step edge to the right. Therefore, the step edge at q_{R} is horizontally moved to the right. If $\sigma_{\mathrm{R}} \geqslant \sigma_{\mathrm{L}}$, $|k_{\mathrm{error}}|$ can be reduced by moving the step edge to the left. Thus, the step edge at q_{L} is horizontally moved to the left. After the horizontal movement, step 2 is repeated to continue the iteration.

7) After iteration, k is calculated. The steering angle errors can be obtained according to k_{I} and θ_{error} in Eq. (1).

The flowchart of the iterative modification is shown in Fig. 7. The main purpose of the iteration is to minimize the mean slope error $k_{\mathrm{error}} = |k_{\mathrm{I}} - k|$. In Eq. (1) when k_{error} has been reduced to the minimum, θ_{error} can also reach its minimum.

In order to verify the proposed iterative method, we iteratively modified the wavefront for each steering angle in Fig. 1; the results are shown in Fig. 8. The modified realized steering angles are significantly close to the theoretical angles. At POS 1, the maximal normalized steering error is $\varepsilon_{\mathrm{norm,max}}$=0.004, and is reduced by 38 times compared with the conventional method. The intensity distributions and the corresponding enlarged drawing are shown in Fig. 9. The performance of wavefront modification with various combinations of M and N is also presented. The steering error after the iterative modification is shown in Fig. 10.

In Fig. 10, the modified steering error $\varepsilon_{\mathrm{norm,max}}^{\mathrm{opt}} \approx 2.6N^{-1}M^{-1}$ is greatly reduced compared with $\varepsilon_{\mathrm{norm,max}} = 0.625M^{-1}$ caused by the conventional method. However, when the steering angles are small, the stair steps in the single period are few. Therefore, the stair steps used to modify the wavefront iteratively are insufficient, and the modification accuracy is not significantly obvious. When the number of the stair steps is sufficiently large, the modified steering error can be stable. Compared with the results of Ref. [16], the following conclusions can be obtained:

1) Compared with the conventional method, the method based on the iterative modification of the wavefront can reduce the maximum normalized steering error by a factor of $\varepsilon_{\mathrm{norm,max}}/\varepsilon_{\mathrm{norm,max}}^{\mathrm{opt}} \approx 0.625M^{-1}/2.6N^{-1}M^{-1} \approx 0.25\ N$. Thus, the reduction is dependent on the number of the phase pixels.

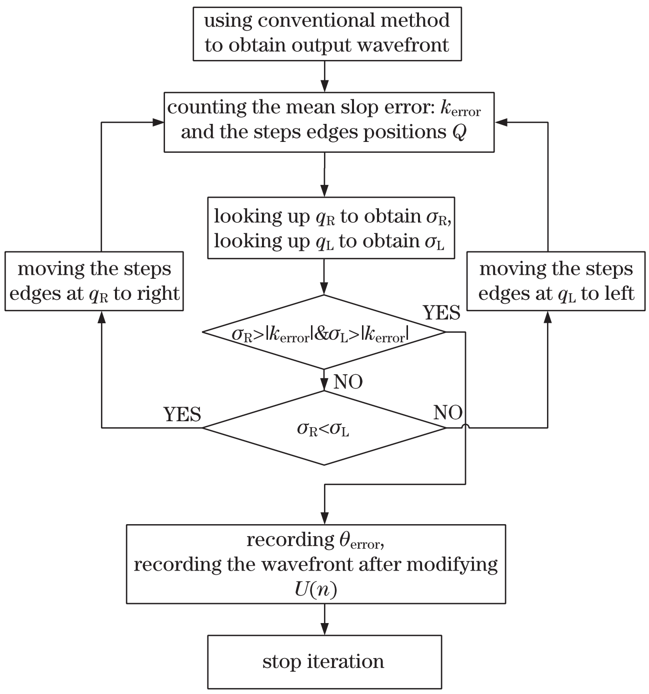

Fig. 7. Flowchart of the iterative modification.

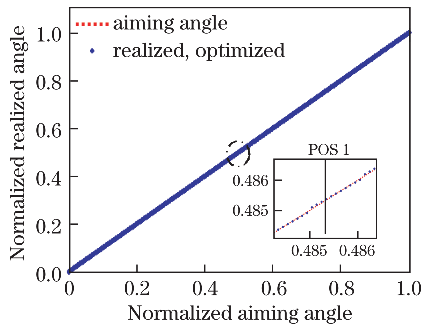

Fig. 8. Modified steering angles distribution with respect to the normalized aiming angle $\theta/\theta_{\max}$ ($M = 4$, $N = 64$).

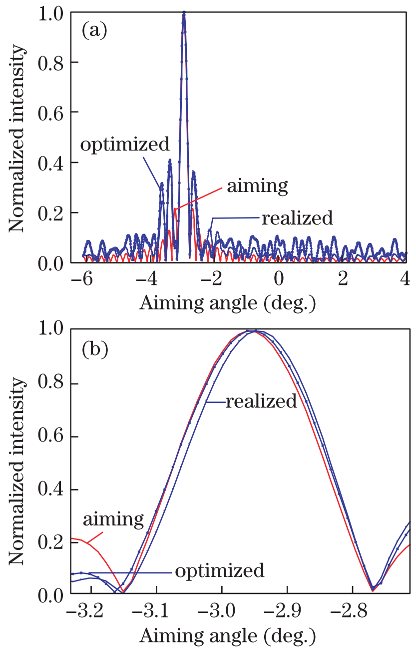

Fig. 9. (a) Intensity distributions with respect to aiming angle and (b) enlarged draw.

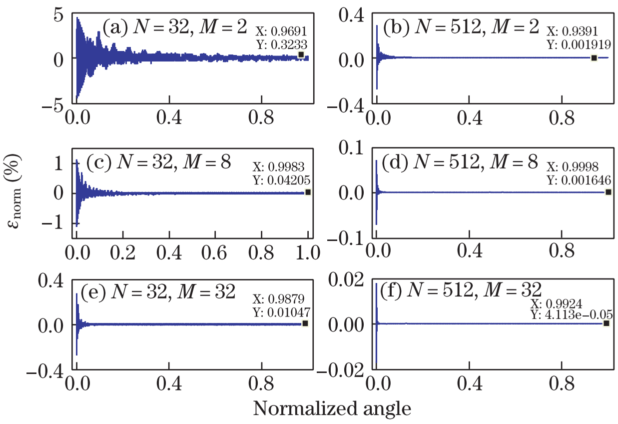

Fig. 10. Modified steering error with different combinations of N and M.

2) As an example, when $N = 32$ and $M = 8$, the mean normalized steering error caused by the conventional method over the angle region $0 - \theta_{\max}$ is $\bar{\varepsilon}_{\text{norm}} \approx 1.395\%$. However, the method based on the iterative modification of the wavefront can reduce the mean normalized steering error by multiplying

$$\bar{\varepsilon}_{\text{norm}}/\bar{\varepsilon}^{\text{opt}}_{\text{norm,max}} \approx 1.395\%/0.029\% \approx 257 \approx M \times N.$$

3) As an example, when $N = 32$ and $M = 8$, the method based on the iterative modification of the wavefront only yields the same results as Ref. [16] in small angles. However, with the angle range of $0.2\theta_{\max} - \theta_{\max}$, the maximum normalized steering error caused by the method proposed in Ref. [16] is $\varepsilon_{\text{norm,max}} \approx 0.0117\%$. The method based on the iterative modification of the wavefront can reduce the maximum normalized steering error by multiplying $\varepsilon_{\text{norm,max}}/\varepsilon^{\text{opt}}_{\text{norm,max}} \approx 0.0117\%/0.00109\% \approx 11$. Compared with the method proposed in Ref. [16], the method based on the iterative modification of the wavefront can significantly improve the steering accuracy.

4) When M or N increases, the number of steering angles with steering errors that has been reduced by the product of $N \times M$ will increase.

In conclusion, we have mainly addressed the beam steering problem, and proposed a novel iterative modification wavefront, which can reduce the error of mean slope by moving the step edges horizontally, thus improving the steering accuracy. Several numerical trails show that normalized accuracy error with the proposed algorithm is much less than that with conventional methods. Meanwhile, the steering error can be further reduced by increasing the number of phase pixels. Moreover, the steering accuracy, especially of high steering angles, can be significantly improved using the proposed method.

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