# Elimination of vertical parallax in multi－view parallax images 

Aihong Wang（王爱红）${ }^{1}$ ，Qionghua Wang（王琼华）$)^{1,2 *}$ ，Yuhong Tao（陶宇虹）${ }^{1}$ ，and Dahai Li（李大海）${ }^{1}$<br>${ }^{1}$ School of Electronics and Information Engineering，Sichuan University，Chengdu 610065，China<br>${ }^{2}$ State Key Laboratory of Fundamental Science on Synthetic Vision，Sichuan University，Chengdu 610065，China<br>＊E－mail：qhwang＠scu．edu．cn

Received May 11， 2010


#### Abstract

In three－dimensional displays，large vertical parallax in parallax images is disadvantageous to stereo vision due to the presence of visual fatigue．Based on the principle that homologous points in different parallax images correspond to the same object point，a method is proposed to eliminate the vertical parallax in multi－view parallax images．The coordinate mapping relationship between a standard parallax image and an awaiting rectification parallax image is established according to the coordinates of the image points of the rectangular calibration board vertices．Experiments are conducted，and results prove that the proposed method is reliable．


OCIS codes：100．6890，150．6910， 330.1400.
doi：10．3788／COL20100811．1047．

The realization of three－dimensional displays ${ }^{[1-4]}$ has been a long fostered dream of mankind simply because the world is three－dimensional．The basic principle of three－dimensional displays is that they can show parallax images with slightly different contents for each eye．The sensation of stereoscopic depth arises from this differ－ ence；this is called horizontal parallax．Parallax images are captured by a stereo camera consisting of multiple single cameras in either parallel configuration or toed－in configuration ${ }^{[5]}$ ．Toed－in configurations introduce key－ stone distortions that produce vertical parallax ${ }^{[6,7]}$ ．In theory，parallel configurations do not produce vertical parallaxes．In real life，however，this configuration can－ not be precisely employed．Large vertical parallaxes af－ fect stereo vision，exacerbate stereoscopic display effects， and cause visual fatigues ${ }^{[8]}$ ．Some techniques have been proposed to eliminate vertical parallaxes．These methods were mostly based on epipolar constraints ${ }^{[9]}$ or mathe－ matical transformations ${ }^{[10,11]}$ ．The method based on epipolar constraints requires caculating the fundamental matrix accurately，and the process is complicated．The method based on mathematical transformation needs some intrinsic and extrinsic parameters of the stereo camera．
This letter proposes a method to eliminate vertical parallaxes based on the principle that homologous points in different parallax images correspond to the same ob－ ject point．The method does not use any parameter of the stereo camera，resulting in a fast implementation process．
A rectangular board，regarded as a calibration object， was fixed in the object space and imaged on the sensors of the multiple single cameras．
Figure 1 shows the schematic of the rectangular board with points $A, B, C$ ，and $D$ being its four vertices．A normalized coordinate system $x-o-y$ is established．Ori－ gin $o$ is located on the upper left corner of the board． The coordinates of vertices $A, B, C$ ，and $D$ are $(0,0)$ ， $(0,1),(1,1)$ ，and $(1,0)$ ，respectively．
Figure 2 shows the schematic of a parallax image cap－
tured by a single camera of the stereo camera．A co－ ordinate system $u-o^{\prime}-v$ can be established．Origin $o^{\prime}$ is located on the upper left corner of the parallax image． The coordinate unit of the coordinate system $u-o^{\prime}-v$ is pixel．Points $A^{\prime}, B^{\prime}, C^{\prime}$ ，and $D^{\prime}$ are the image points of the four vertices of the rectangular board，and their coordinates are $\left(u_{A^{\prime}}, v_{A^{\prime}}\right),\left(u_{B^{\prime}}, v_{B^{\prime}}\right),\left(u_{C^{\prime}}, v_{C^{\prime}}\right)$ ，and （ $u_{D^{\prime}}, v_{D^{\prime}}$ ），respectively．
Lines $C^{\prime} B^{\prime}$ and $D^{\prime} A^{\prime}$ intersect at point $O\left(u_{O}, v_{O}\right)$ ． As line $C B$ is parallel to line $D A$ in the $x$ coordinate， point $O$ is considered the image point of the negative infinite object point in the $x$ coordinate．Similarly，lines $B^{\prime} A^{\prime}$ and $C^{\prime} D^{\prime}$ intersect at point $Q\left(u_{Q}, v_{Q}\right)$ ．As line $B A$ is parallel to line $C D$ in the $y$ coordinate，point $Q$ is considered the image point of the negative infinite object


Fig．1．Schematic of the rectangular board regarded as a cal－ ibration object．


Fig．2．Schematic of a parallax image captured by a single camera of the stereo camera．
point in the $y$ coordinate. The coordinates, $\left(u_{O}, v_{O}\right)$ and $\left(u_{Q}, v_{Q}\right)$, can be calculated based on the condition of three collinear points. The formulas for the equations are given as

$$
\begin{align*}
\frac{v_{D^{\prime}}-v_{A^{\prime}}}{u_{D^{\prime}}-u_{A^{\prime}}} & =\frac{v_{D^{\prime}}-v_{O}}{u_{D^{\prime}}-u_{O}}  \tag{1a}\\
\frac{v_{C^{\prime}}-v_{B^{\prime}}}{u_{C^{\prime}}-u_{B^{\prime}}} & =\frac{v_{C^{\prime}}-v_{O}}{u_{C^{\prime}}-u_{O}}  \tag{1b}\\
\frac{v_{B^{\prime}}-v_{A^{\prime}}}{u_{B^{\prime}}-u_{A^{\prime}}} & =\frac{v_{B^{\prime}}-v_{Q}}{u_{B^{\prime}}-u_{Q}}  \tag{2a}\\
\frac{v_{C^{\prime}}-v_{D^{\prime}}}{u_{C^{\prime}}-u_{D^{\prime}}} & =\frac{v_{C^{\prime}}-v_{Q}}{u_{C^{\prime}}-u_{Q}} \tag{2b}
\end{align*}
$$

Assume that the coordinates of an arbitrary object point $P$ on the rectangular board are $\left(x_{P}, y_{P}\right)$. Line $P E$ is parallel to the $y$ coordinate and it intersects line $B C$ at point $E$. The coordinates are $\left(x_{P}, 1\right)$. Line $P F$ is parallel to the $x$ coordinate and it intersects line $D C$ at point $F$. The coordinates are $\left(1, y_{P}\right)$. Points $P$ and $E$ have identical $x$ coordinates, and points $P$ and $F$ have identical $y$ coordinates.

Point $P^{\prime}$ in the parallax image is the image point of object point $P$, and its coordinates are ( $u_{P^{\prime}}, v_{P^{\prime}}$ ). Lines $O C^{\prime}$ and $Q P^{\prime}$ intersect at point $E^{\prime}\left(u_{E^{\prime}}, v_{E^{\prime}}\right)$. According to the principle of perspective projection, point $E^{\prime}$ is the image point of object point $E$. If point $E^{\prime}$ coincides with point $O$, the $x$ coordinate of object point $P$ is a negative infinite. If point $E^{\prime}$ coincides with point $B^{\prime}$, the $x$ coordinate of the object point $P$ is 0 . If point $E^{\prime}$ coincides with point $C^{\prime}$, the $x$ coordinate of object point $P$ is 1 . Based on the aforementioned three conditions, the following equation describing the relationship between the $x$ coordinate of object point $P$ and the $u$ coordinate of image point $E^{\prime}$ is established:

$$
\begin{align*}
x_{P}= & \frac{1}{u_{E^{\prime}}-u_{O}}\left[\left(u_{E^{\prime}}-u_{O}\right)-\left(u_{B^{\prime}}-u_{O}\right)\right] \\
& \frac{u_{C^{\prime}}-u_{O}}{\left(u_{C^{\prime}}-u_{O}\right)-\left(u_{B^{\prime}}-u_{O}\right)} . \tag{3}
\end{align*}
$$

Lines $Q C^{\prime}$ and $O P^{\prime}$ intersect at point $F^{\prime}\left(u_{F^{\prime}}, v_{F^{\prime}}\right)$. Similarly, according to the principle of perspective projection, point $F^{\prime}$ is the image point of object point $F$. If point $F^{\prime}$ coincides with point $Q$, the $y$ coordinate of the object point $P$ is a negative infinite. If point $F^{\prime}$ coincides with point $D^{\prime}$, the $y$ coordinate of the object point $P$ is 0 . If point $F^{\prime}$ coincides with point $C^{\prime}$, the $y$ coordinate of the object point $P$ is 1 . Based on the abovementioned three conditions, the following equation describing the relationship between the $y$ coordinate of object point $P$ and the $v$ coordinate of image point $F^{\prime}$ is established:

$$
\begin{align*}
y_{P}= & \frac{1}{v_{F^{\prime}}-v_{Q}}\left[\left(v_{F^{\prime}}-v_{Q}\right)-\left(v_{D^{\prime}}-v_{Q}\right)\right] \\
& \frac{v_{C^{\prime}}-v_{Q}}{\left(v_{C^{\prime}}-v_{Q}\right)-\left(v_{D^{\prime}}-v_{Q}\right)} . \tag{4}
\end{align*}
$$

In Eqs. (3) and (4), the coordinates $u_{E^{\prime}}$ and $v_{F^{\prime}}$ are unknown. However, they are related to the coordinates of image point $P^{\prime}$. Based on the condition of the three collinear points, the coordinates ( $u_{E^{\prime}}, v_{E^{\prime}}$ ) and ( $u_{F^{\prime}}, v_{F^{\prime}}$ )
can be respectively calculated using

$$
\begin{align*}
& \frac{v_{E^{\prime}}-v_{Q}}{u_{E^{\prime}}-u_{Q}}=\frac{v_{P^{\prime}}-v_{Q}}{u_{P^{\prime}}-u_{Q}},  \tag{5a}\\
& \frac{v_{E^{\prime}}-v_{O}}{u_{E^{\prime}}-u_{O}}=\frac{v_{B^{\prime}}-v_{O}}{u_{B^{\prime}}-u_{O}},  \tag{5b}\\
& \frac{v_{F^{\prime}}-v_{O}}{u_{F^{\prime}}-u_{O}}=\frac{v_{P^{\prime}}-v_{O}}{u_{P^{\prime}}-u_{O}},  \tag{6a}\\
& \frac{v_{F^{\prime}}-v_{Q}}{u_{F^{\prime}}-u_{Q}}=\frac{v_{D^{\prime}}-v_{Q}}{u_{D^{\prime}}-u_{Q}} . \tag{6b}
\end{align*}
$$

The relationship between the coordinates of object point $P$ and the coordinates of its corresponding image point $P^{\prime}$ is thereby established.

The image points of the same object point $P$ in different parallax images are called homologous points. Thus, the calculated coordinates $\left(x_{P}, y_{P}\right)$ corresponding to the homologous points are identical. As such, the parallax image captured by a specific single camera of the stereo camera is regarded as a standard parallax image. This could rectify the parallax images captured by the other single cameras of the stereo camera.
The rectification process involves three steps. Firstly, for each image point in the standard parallax image, calculate the coordinates of its corresponding object point using Eqs. (3) and (4). Secondly, for each object point calculated in the first step, calculate the coordinates of its image point in the rectification parallax image using Eqs. (3) and (4). Hence, the coordinate mapping relationship between the standard parallax image captured by a specific single camera and the awaiting rectification parallax image captured by another single camera can be established. Finally, remove the rectangular board and shoot the scene using the same stereo camera. By using the coordinate mapping relationship obtained in the second step, sample the awaiting rectification parallax image in order to produce a new parallax image, which has no vertical parallax compared with the standard parallax image.

A stereo camera, including two single cameras, was used to shoot the parallax images. The setup of the stereo camera is shown in Fig. 3. The captured parallax images are shown in Figs. 4(a) and (b). The left parallax image was used as the standard parallax image, as well as to rectify the right parallax image by using the proposed method. The rectified right parallax image is shown in Fig. 4(c). The resolution of the parallax images was $512 \times 384$ (pixel). Five groups of homologous points are marked in Fig. 4, and their coordinates in the vertical direction are listed in Table 1.

The original vertical parallax corresponds to the difference in the vertical coordinates of the homologous points between the captured left and right parallax images. The rectified vertical parallax corresponds to the difference in the vertical coordinates of the homologous points between the left parallax image and the rectified right parallax image. As shown in Table 1, by using the propose method, the vertical parallax was reduced. We synthesized an unrectified stereoscopic image using the left parallax image and the unrectified right parallax image. A rectified stereoscopic image was also synthesized using the left parallax image and the rectified right parallax image. The unrectified and rectified images are
shown in Figs. 5(a) and (b), respectively.
Both the unrectified and rectified stereoscopic images were displayed on a 22 -inch autostereoscopic display, and several individuals were invited for viewing. Experimental results showed that the viewers had less visual fatigue after being shown the rectified stereoscopic image.
In conclusion, we propose a method to eliminate vertical parallaxes in multi-view parallax images. The method is based on the principle that the homologous points in different parallax images correspond to the same object point. The coordinate mapping relationship between a standard parallax image and an awaiting rectification parallax image has been established based on the coordinates of the image points of the rectangular calibration board vertices. Experimental results show that the rectified parallax images have little vertical parallax. Viewers claim less visual fatigue when viewing the rectified stereoscopic image compared with the unrectified one. The proposed method can be applied to all parallax images with vertical parallaxes induced by either a toedin stereo camera or the misalignment of a stereo camera.


Fig. 3. Setup of the stereo camera for shooting the parallax images.

Table 1. Vertical Coordinate and Vertical Parallax of Homologous Points in Parallax Images

| Homologous <br> Points | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Coordinates <br> in Fig. 4(a) <br> (pixel) | 252 | 360 | 307 | 309 | 200 |
| Coordinates <br> in Fig. 4(b) <br> (pixel) | 259 | 369 | 318 | 319 | 210 |
| Coordinates <br> in Fig. 4(c) <br> (pixel) | 252 | 362 | 309 | 311 | 202 |
| Original <br> Vertical | 7 | 9 | 11 | 10 | 10 |
| Parallax (pixel) | 2 | 2 | 1 | 2 |  |
| Rectified <br> Vertical <br> Parallax (pixel) | 0 | 2 |  |  |  |



Fig. 4. Left parallax image and right parallax image before and after rectification. (a) Left parallax image; (b) unrectified right parallax image; (c) rectified right parallax image.


Fig. 5. (a) Unrectified stereoscopic image synthesized using the left and unrectified right parallax images and (b) rectified stereoscopic image synthesized using the left and rectified right parallax images.

This work was supported by the National Natural Science Foundation of China under Grant No. 60877004.

## References

1. H. Zheng, Y. Yu, T. Wang, and L. Dai, Chin. Opt. Lett. 7, 1151 (2009).
2. H. Yoshikawa and T. Yamaguchi, Chin. Opt. Lett. 7, 1079 (2009).
3. Y.-H. Tao, Q.-H. Wang, J. Gu, W.-X. Zhao, and D.-H. Li, Opt. Lett. 34, 3220 (2009).
4. L. Zhou, Q. Wang, Y. Tao, D. Li, X. Zhao, and T. Jiao, Acta Opt. Sin. (in Chinese) 29, 3506 (2009).
5. H. Yamanoue, in Proceedings of IEEE International Conference on Multimedia and Expo 20061701 (2006).
6. R. S. Allison, Proc. SPIE 5291, 167 (2004).
7. A. Woods, T. Docherty, and R. Koch, Proc. SPIE 1915, 36 (1993).
8. F. L. Kooi and A. Toet, Displays 25, 99 (2004).
9. M. Zhu, Y. J. Ge, S. F. Huang, and W. B. Chen, in Proceedings of IEEE International Integration Technology 2007133 (2007).
10. N. A. Dodgson, Proc. SPIE 3295, 100 (1998).
11. H. Kang, N. Hur, S. Lee, and H. Yoshikawa, Opt. Commun. 281, 1430 (2008).
