A qualitative approach to aberrations of optical elements

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A qualitative method to analyze wavefront aberrations is presented. Aberrations of the primary type, expressed in their matrix forms, are used to write the generalized ray transfer matrix of an optical component. The aberrations were treated collectively by examining the pseudospectra of an augmented matrix constructed from the aberration matrices. Results show that aberrations can be distinguished and relative strengths pronounced using this qualitative method.

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Analytic expressions for the aberrations introduced into an incident wave by an optical element can be obtained by a series expansion of Hamilton's characteristic function in an appropriate coordinate system[1]. An advantage of this approach is that each term in the expansion corresponds to an aberration and is treated independently. Recently, the introduction of aberration ray transfer matrices extended the analysis of aberrations to the paraxial regime[2]. In this formalism, each aberration is represented by a 2×2 matrix, essentially retaining aberration independence.

The interdependence of aberrations allows the performance of more meaningful analyses using a unified or collective treatment. In this letter, we introduce a method that takes this into account by examining the pseudospectra of aberration transfer matrices. The class of aberrations associated with the effects of propagation[3], however, is beyond the scope of this study.

Aberrations serve to deform and degrade images. They occur when light fails to terminate at a prescribed point after interacting with an optical element or imaging system. The arbitrary aberrations of an optical element have been discussed using ray transfer matrices (RTMs) by associating an aberration with a 2×2 matrix[2]. This formalism allows the description of the action of one or more aberrations using products of the appropriate matrices. For an optical element with known RTM $\mathbf{M}$, the equation

$$\mathbf{M}_\mathrm{a} = \mathbf{M}\mathbf{T}_\mathrm{a} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \prod_i \begin{pmatrix} 1 & 0 \\ p_i & 1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \sum_i p_i & 1 \end{pmatrix} \quad (1)$$

expresses the generalized tangential RTM in the presence of aberrations, where $\mathbf{T}_\mathrm{a}$ is a product of the tangential aberration matrices[2]. The matrix $\mathbf{M}_\mathrm{a}$ is general in that it expresses every possible aberration, its respective strengths, and the action on an optical element. A similar matrix can be constructed for the sagittal plane using products of the corresponding aberration matrices[2]. The elements p_i (i = 1, 2, 3, 4, respectively representing defocus, astigmatism, coma, and spherical aberration) in $\mathbf{T}_\mathrm{a}$ are weighted Zernike polynomials with the weights determining the relative strength of each aberration. In the absence of aberrations, the elements p_i vanish, reducing $\mathbf{T}_\mathrm{a}$ to the identity matrix; consequently, $\mathbf{M}_\mathrm{a}$ reverts to $\mathbf{M}$. For this reason, one may consider $\mathbf{M}$ as the aberration-free RTM, a result consistent with the literature.

Equation (1) expresses two facts about the optical element, represented by the matrix $\mathbf{M}$, and the aberrations contained in $\mathbf{T}_\mathrm{a}$. Firstly, the aberrations and the optical element are separable and distinct. This suggests that the aberrations can be studied independent of the optical element. Secondly, the aberrations are linearly independent and act on the emergent beam. This is mathematically similar to the results of the power series treatment[1].

In order to describe the aberrations and their effects in a collective, unified manner, the construction of an augmented matrix whose blocks are comprised of the aberration transfer matrices is proposed. By doing so, the properties of each transfer matrix and hence the effect of each aberration are accounted for while allowing for the relative aberration to be expressed. Using the aberration RTMs[2], one has

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ P_1 & 1 & P_2 & 1 \\ 1 & 0 & 1 & 0 \\ P_3 & 1 & P_4 & 1 \end{pmatrix} \quad (2)$$

as the augmented matrix, where P_i are the aberration terms normalized with respect to the strongest aberration. The aberrations are analyzed qualitatively when one or more of the aberrations change using the pseudospectra of $\mathbf{A}$[4]. To increase the distinction between the pseudospectra, the block corresponding to the aberration being analyzed is transposed, and the pseudospectra of

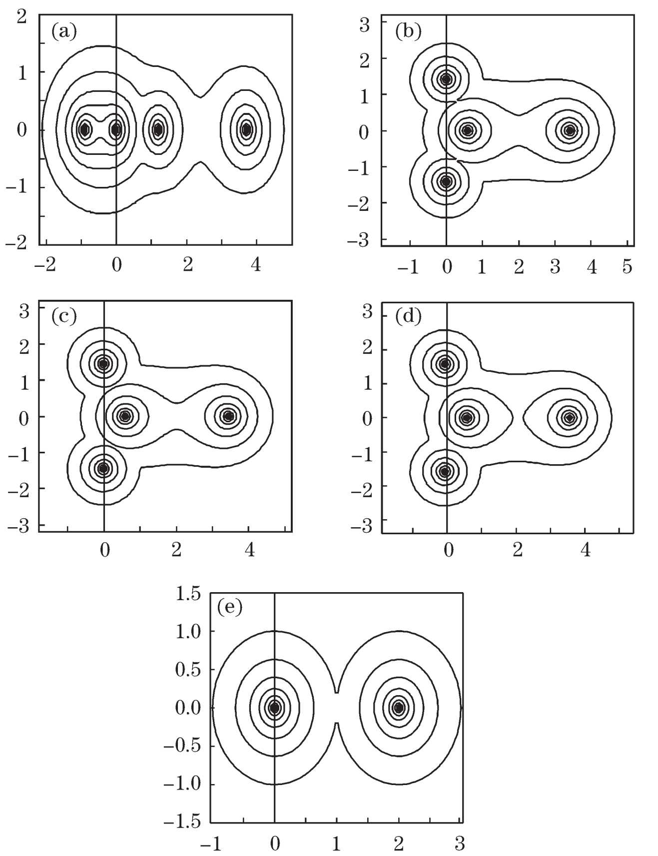

Fig. 1. (a) Pseudospectra of spherical aberration; the pseudospectra of coma are (b) twice, (c) 2.1 times, and (d) 2.5 times as large as other aberrations; (e) pseudospectra in the absence of any aberrations.

the augmented matrix are analyzed. The order of the 2×2 blocks is immaterial. Using pseudospectra, identifying which aberrations are present and determining their relative strengths are possible.

As proof of the principle, we plot the pseudospectra of $\mathbf{A}$ under different aberration strengths using "EigTool"[5]. The pseudospectra, in the absence of coma, astigmatism, and distortion, are shown in Fig. 1(a). In this plot, where spherical aberration is the only aberration, the eigenvalues are represented by black dots bounded by level curves. The eigenvalues and level curves characterize the augmented matrix and allow for the analysis of aberrations. A qualitatively different plot is obtained when coma dominates the other aberrations (see Figs. 1(b)−(d)). There are still four eigenvalues, but their magnitudes are now different. The differences in eigenvalues between the different analyzed aberrations are easily seen in the pseudospectra. Relative aberration strengths, in terms of pseudospectra, are illustrated in Figs. 1(b)−(d). In these figures, the relative strength of coma increases with respect to the other aberrations, which are kept constant. As coma increases, for example, the eigenvalues remain constant, but the number and nature of the level curves that bound the eigenvalues are observed to change. For comparison, the pseudospectra in the absence of aberrations are shown in Fig. 1(e). There are four eigenvalues, each of an algebraic multiplicity of 2, which explains why there are only two distinguishable eigenvalues shown.

These results show that by analyzing pseudospectra, aberrations can be distinguished and their strengths resolved qualitatively. The primary advantage of pseudospectra is that aberrations and their relative strengths, treated collectively, can be immediately identified. The pseudospectra presented were generated on an IBM compatible PC (AthlonXP, 1.9 GHz, 128 MB RAM) in less than 5 min. As such, the computing time is not detrimental to the practicality of this method. Furthermore, by requiring only the aberration transfer matrices, this method is not restricted to particular optical elements.

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