## Simulation and optimization of spatial light modulation of twisted-nematic liquid crystal display

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The approaches to obtaining desired intensity or phase modulation by twisted-nematic liquid crystal display (TN-LCD) have been extensively studied based on the knowledge of the LCD's internal structure parameters. Generally, the TN-LCD placed between two linear polarizers (P) produces coupled intensity and phase modulation. To obtain the commonly used phase-only modulation, quarter wave plates (QWPs) are often used in front of and/or behind the LCD. Here we present a method to optimize the optical modulation properties of the TN-LCD to obtain phase-only modulation in the configuration of P-QWP-LCD-QWP-P each with proper orientation. Our method is based on the macroscopical Jones matrix descriptions for the LCD, the QWPs, and the linear polarizers. Through Jones matrix calculations, the orientations of the polarizers and QWPs can be optimized to satisfy differently desired modulation demands. In contrast to the traditional method, which requires knowledge of the LCD's internal structure parameters, our method simplified the complicated theory analysis and can work in the absence of information on the LCD's internal structure parameters, which are usually not available for the commercial products.

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Twisted-nematic liquid crystal display (TN-LCD), working as a controllable spatial light modulator (SLM), is of increasing attentions recently due to its universality and ease of availability. It is widely used in the fields of holographic optics<sup>[1]</sup>, diffractive optics<sup>[2]</sup>, optical information processing<sup>[3]</sup>, adaptive optics<sup>[4]</sup>, and so on. Consequently, the approaches to obtaining desired intensity or phase modulation by TN-LCD have been extensively studied. Generally, the TN-LCD placed between two linear polarizers produces coupled intensity and phase modulation<sup>[5]</sup>. However, phase-only modulation is of increasing potentials in many fields, e.g., holographic optical tweezers<sup>[6]</sup>, wavefront correction<sup>[7,8]</sup>, beam deflection<sup>[9]</sup>, and optical correlator<sup>[10]</sup>. Therefore, many efforts were made to acquire phase-only modulation by TN-LCD. Konforti et al. obtained phase-only modulation by simply placing the LCD between two polarizers properly oriented and keeping the applied voltage below a certain level, namely optical threshold<sup>[5]</sup>. But this method is only suitable for early thick TN-LCD, which can offer sufficient phase shift in such a small voltage range. Pezzaniti et al. utilized eigenpolarization states to minimize the polarization state modulation inherent in the twisted nematic cell and obtained phase-only modulation<sup>[11,12]</sup>. Unfortunately, these eigenpolarization states are dependable of the applied gray level (gl), so they defined an average eigenvector throughout the range of applied gray level as an approximation and compromise. Durán et al. designed a generator composed of a linear polarizer (P) and a quarter wave plate (QWP) to generate equi-azimuth polarization states and proved that by adding a properly oriented analyzer, it was possible to achieve phase-only modulation by these polarization states<sup>[13]</sup>.

An alternative way to get desired modulation property is to find a proper configuration by numerical simulation, which requires the Jones matrix description for the TN-LCD. In the early work of Lu *et al.*<sup>[14]</sup>, they simply supposed that the twisted angle of liquid crystal (LC) molecules changed linearly with the distance from the input face and kept a constant tilt angle which varied with the applied voltage. After that, several modified models<sup>[15-18]</sup> are presented successively based on the knowledge of the LCD's internal structure parameters, e.g., the orientation of LC molecules at the surfaces, the twist angle, the thickness of the LC layer, and the birefringence of the material. However, these parameters are usually not available for the commercial products, resulting in the inconvenience in practical use. To avoid this drawback, Moreno et al. proposed an approach to describing the optical modulation property of TN-LCD macroscopically by calibrating the Jones matrix of TN-LCD without knowing its internal parameters<sup>[19]</sup>. We improved this method much simpler and more accurate<sup>[20]</sup>. When the Jones matrix for the LCD is determined, its optical modulation properties can be totally predicted



Fig. 1. Schematic of the LCD placed between the PSG and the PSD.



Fig. 2. Optical modulation curves of the configuration of P-LCD-P. (a) Phase modulation; (b) normalized intensity modulation.  $\chi_1 = -23^{\circ}, \chi_2 = 12^{\circ}, \phi_1 = \phi_2 = 0^{\circ}$ .



Fig. 3. Optical modulation curves of the configuration of P-QWP-LCD-P. (a) Phase modulation; (b) normalized intensity modulation.  $\chi_1 = 36^\circ, \chi_2 = 23^\circ, \phi_1 = 90^\circ, \phi_2 = 0^\circ$ .

and optimized by Jones matrix calculation.

In this letter, based on the Jones matrix theory, we calculate and compare the modulation properties of the TN-LCD in three configurations, of which null, one, and



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Fig. 4. Optical modulation curves of the configuration of P-QWP-LCD-QWP-P. (a) Phase modulation; (b) normalized intensity modulation.  $\chi_1 = 38^{\circ}, \chi_2 = -35^{\circ}, \phi_1 = 90^{\circ}, \phi_2 = 48^{\circ}.$ 

two QWPs are used, respectively, besides the two polarizers. The proper orientations for each device are optimized for phase-only modulation in each configuration, and the best results are achieved in the configuration of P-QWP-LCD-QWP-P.

TN-LCD, as a polarization device, is generally described by the Jones matrix as

$$M = c \exp\left(-j\beta\right) \begin{pmatrix} X - jY & Z - jW \\ -Z - jW & X + jY \end{pmatrix}, \quad (1)$$

where c represents intensity loss caused by surface reflections etc.,  $\beta$  is an external phase shift, and the parameters, X, Y, Z, and W, are real values in the range [-1, 1] satisfying the normalization condition of  $X^2 + Y^2 + Z^2 + W^2 = 1$ . All the parameters change with the gray level addressed to the TN-LCD changes but c. Because the intensity obtained after the second polarizer is normalized to the total intensity at the output beam, the parameter c is ignored.

The vector

$$|J_1\rangle = |\chi_1, \phi_1\rangle = \begin{pmatrix} \cos \chi_1 \\ \sin \chi_1 \exp (j\phi_1) \end{pmatrix}$$
(2)

is employed to describe a light beam with arbitrary polarization state and normalized intensity, which can be generated by a polarization state generator (PSG) composed of a linear polarizer (P<sub>1</sub>) and a quarter wave plate (QWP<sub>1</sub>), where  $\chi_1$  stands for the angle of the linear polarizer's transmission axis with respect to the x axis of the frame, and  $\phi_1$  indicates the phase difference between the horizontal and vertical components of incident field introduced by the wave plate. The transmitted light is detected by a polarization state detector (PSD) composed of another QWP<sub>2</sub> and P<sub>2</sub>, which is described by  $\langle \chi_2, \phi_2 |$  correspondingly.

We derive the Jones coefficients X, Y, Z, and W from three sets of intensity curves versus gray level signal of three specific configurations, and measure the external phase shift parameter  $\beta$  by interferometric method in Ref. [20]. The TN-LCD we used is a product of Sony company, which has  $1024 \times 768$  pixels with  $25.8 \times 25.8$ ( $\mu$ m) pixel size. When the TN-LCD is placed between the PSG and PSD, as shown in Fig. 1, the Jones vector of the emerging light can be described by

$$|J\rangle = |\chi_2, 0\rangle \langle \chi_2, \phi_2 | M | \chi_1, \phi_1 \rangle$$
  
=  $\sqrt{I} \exp(-j\delta) | \chi_2, 0 \rangle.$  (3)

Assuming  $c_1 = \cos \chi_1$ ,  $s_1 = \sin \chi_1$ ,  $c_2 = \cos \chi_2$ ,  $s_2 = \cos \chi_2$ , and  $\phi = \phi_1 + \phi_2$ , we have the light intensity of

$$I = \{c_1 c_2 X - c_1 s_2 (Z \cos \phi_2 - W \sin \phi_2) + s_1 c_2 (Z \cos \phi_1 + W \sin \phi_1) + s_1 s_2 (X \cos \phi - Y \sin \phi)\}^2 + \{c_1 c_2 Y + c_1 s_2 (Z \sin \phi_2 + W \cos \phi_2) - s_1 c_2 (Z \sin \phi_1 - W \cos \phi_1) - s_1 s_2 (X \sin \phi + Y \cos \phi)\}^2,$$
(4)

and the phase of

$$\delta = \beta + \arctan \frac{c_1 c_2 Y + c_1 s_2 \left(Z \sin \phi_2 + W \cos \phi_2\right) - s_1 c_2 \left(Z \sin \phi_1 - W \cos \phi_1\right) - s_1 s_2 \left(X \sin \phi + Y \cos \phi\right)}{c_1 c_2 X - c_1 s_2 \left(Z \cos \phi_2 - W \sin \phi_2\right) + s_1 c_2 \left(Z \cos \phi_1 + W \sin \phi_1\right) + s_1 s_2 \left(X \cos \phi - Y \sin \phi\right)}.$$
 (5)

As all the parameters of the LCD's Jones matrix are known, the optimization of the optical modulation properties of the TN-LCD means to find a proper configuration ( $\chi_1$ ,  $\phi_1$ ,  $\chi_2$ ,  $\phi_2$ ) to satisfy the specific demand according to Eqs. (4) and (5).

In the following, we try to find proper orientations for each device for three different configurations to obtain approximate phase-only modulation. We introduce the parameters  $\delta_{dep}$ ,  $\overline{I}$ , and F to represent the phase modulation depth, average intensity transmission, and the fluctuation of the transmission, respectively, among which  $\overline{I}$ and F are respectively defined as

$$\overline{I} = \frac{1}{256} \sum_{\rm gl=0}^{255} I_{\rm gl},\tag{6}$$

and

$$F = \frac{1}{256} \sum_{\rm gl=0}^{255} |I_{\rm gl} - \overline{I}|.$$
 (7)

These parameters are criterion in the procedure of optimization. The phase-only modulation means a large phase modulation depth and a flat and high intensity transmission throughout the gray level range. In other words, we are trying to find a configuration to make the phase modulation depth  $\delta_{dep}$  as large as possible and the intensity fluctuation F as small as possible. In addition, the intensity of the desired configuration should not be too low in practice. It is of heavy workloads to calculate every possible configuration, so we simplify this procedure by firstly calculating for every  $10^{\circ}$  for the devices' orientations to find the potential range, and then minish the step to  $1^{\circ}$  to obtain the optimal results.

At first, we analyze the simple configuration of P-LCD-P. In the absence of QWP, both  $\phi_1$  and  $\phi_2$  are 0, so I and  $\delta$  can be simplified as

$$I = [X \cos(\chi_1 - \chi_2) + Z \sin(\chi_1 - \chi_2)]^2 + [Y \cos(\chi_1 + \chi_2) + W \sin(\chi_1 + \chi_2)]^2, \quad (8)$$

and

$$\delta = \beta + \arctan\left[\frac{Y\cos\left(\chi_1 + \chi_2\right) + W\sin\left(\chi_1 + \chi_2\right)}{X\cos\left(\chi_1 - \chi_2\right) + Z\sin\left(\chi_1 - \chi_2\right)}\right].$$
 (9)

Figures 2(a) and (b) show the phase modulation and the normalized intensity transmission modulation, where  $\chi_1$  and  $\chi_2$  are  $-23^{\circ}$  and  $12^{\circ}$ , respectively. It is found that this configuration offers a phase modulation depth of  $0.7\pi$  with obvious intensity fluctuation of F = 5.7%, so this simple configuration is not suitable for phase-only modulation.

Then we examine the optical modulation properties of the LCD when a QWP is placed in front of or behind the LCD, which is expected to have better results for phaseonly modulation<sup>[19]</sup>. In the case of the polarizer placed in front of the LCD, i.e.,  $\phi_2 = 0$ , the normalized intensity and phase shift can be described as

$$I = \{c_1 c_2 X - c_1 s_2 \left( Z \cos \phi_2 - W \sin \phi_2 \right) + s_1 c_2 Z + s_1 s_2 \left( X \cos \phi_2 - Y \sin \phi_2 \right) \}^2 + \{c_1 c_2 Y + c_1 s_2 \left( Z \sin \phi_2 + W \cos \phi_2 \right) + s_1 c_2 W - s_1 s_2 \left( X \sin \phi_2 + Y \cos \phi_2 \right) \}^2,$$
(10)

$$\delta = \beta + \arctan \frac{c_1 c_2 Y + c_1 s_2 \left(Z \sin \phi_2 + W \cos \phi_2\right) + s_1 c_2 W - s_1 s_2 \left(X \sin \phi_2 + Y \cos \phi_2\right)}{c_1 c_2 X - c_1 s_2 \left(Z \cos \phi_2 - W \sin \phi_2\right) + s_1 c_2 Z + s_1 s_2 \left(X \cos \phi_2 - Y \sin \phi_2\right)}.$$
(11)

In this configuration, the best results are obtained when  $\chi_1$ ,  $\chi_2$ , and  $\phi_1$  are 36°, 23°, and 90°, respectively, and the corresponding modulation curves are shown in Fig. 3. The phase modulation depth is enlarged to be about  $1.4\pi$  and the intensity fluctuation F is reduced to 1.5%. Unfortunately, the average intensity transmission  $\overline{I}$  is only about 0.5. Finally, when two QWPs are placed in front of and behind the LCD respectively, the normalized intensity and phase shift can be described by Eqs. (4) and (5), respectively. In this configuration, the optimized results are shown in Fig. 4. It can be seen that the phase modulation depth is  $1.3\pi$ , the average intensity transmission is increased to 0.9, and the intensity fluctuation is reduced in an appropriate range with F = 1%. Thus, the good phase-only modulation with flat and high intensity transmission throughout the gray level range is obtained in the configuration of P-QWP-LCD-QWP-P with  $\chi_1$ ,  $\chi_2$ ,  $\phi_1$ , and  $\phi_2$  being 38°,  $-35^\circ$ , 90°, and 48°, respectively.

In conclusion, we have proposed a simple method to predict and optimize the optical modulation properties of the TN-LCD based on the macroscopical Jones matrix description of the LCD. We verify the method in seeking for the proper parameters for phase-only modulation in the configurations of P-LCD-P, P-QWP-LCD-P, and P-QWP-LCD-QWP-P, among which the P-QWP-LCD-QWP-P with proper orientations offers the best phaseonly modulation. Compared with the traditional method which requires knowledge of the LCD's internal structure parameters, our method is much simpler in analysis and calculation, and can work in the absence of information on the LCD's internal structure parameters which are usually not available for the commercial products.

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