Slow light deflection in Gaussian pumped atomic medium

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We present the moments formalism theory to study the deflection of the slow signal light in the cold atomic media, which is under the condition of the Gaussian control laser and electromagnetically induced transparency. Deflection, the interesting phenomenon on quantum coherence, is testified by analytic and numerical methods. Results show that, as the signal light propagating in the medium, there would be an observable deflection before the general diffraction. Influences of the coupling intensity on deflection phenomenon and the beam waist of the signal light in the medium are also investigated.

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The characterization of the spatial behavior of laser profiles constitutes a field of current interest[1−8]. As is well known, the key shape parameters of every laser source are the minimum beam waist and the far-field beam divergence, which can be expressed in terms of the so-called second-order moments of the intensity and radiant intensity of the field. Moments formalism is a powerful tool to derive the propagation laws. In this letter, we use the moments formalism to study the deflection phenomena in the cold atomic media.

As an optical phenomenon, deflection has been studied for ages. Deflection always takes place where there is an inhomogeneous medium[9], and it can be easily observed in our daily lives. As a result, researchers take much interest in it and do much work about it. Recently, many researchers found that, under the condition of external field, the atomic media could be induced to the inhomogeneous media[10−18]. The gradient refractive index (GRIN) is induced by magnetic-field-modified optical pumping[11], while the spatial inhomogeneity of the refractive index is induced by an expanded Gaussian-profile pump beam[15−18]. It should be mentioned that, in Refs. [15-18], since the conditions for electromagnetically induced transparency (EIT)[19,20] are satisfied, the probe light transmitting in the cold atomic media is very slow. The deflection of the slow light in atomic media has been observed experimentally[21,22] and studied in theory[23]. In Ref. [23], the relations between the deflection angle, the injection position, and the probe detuning were illustrated with the semiclassical theory.

In this letter, we investigate the deflection of the slow light in the cold atomic media by the classical theory. With the help of the Fresnel propagation program[9], we can easily gain the analytical expression of propagation equation, by which the ray path of the signal light and the real beam-waist can be induced by the moments formalism. Results show that, as the signal light propagating in the medium, there would be an observable deflection before the general diffraction. Influences of the coupling intensity on deflection phenomenon and the beam waist of the signal light in the medium are also investigated.

In our recent work[15−18], we consider the cold atomic gas cell composed of Λ-type three-level atoms (Fig. 1). Under the condition of EIT, the linear susceptibility of the incident field can be written as[19]

$$\chi(\Delta_{\rm p},r)=-\frac{|\mu_{13}|^2\rho}{\varepsilon_0\hbar}\times\left\{\frac{4\Delta_{\rm p}(\Omega_{\rm c}^2-4\Delta_{\rm p}^2)-4\Delta_{\rm p}\Gamma_2^2}{(\Omega_{\rm c}^2+\Gamma_2\Gamma_3-4\Delta_{\rm p}^2)^2+4\Delta_{\rm p}^2(\Gamma_2+\Gamma_3)^2}+{\rm i}\frac{8\Delta_{\rm p}^2\Gamma_3+2\Gamma_2(\Omega_{\rm c}^2+\Gamma_2\Gamma_3)}{(\Omega_{\rm c}^2+\Gamma_2\Gamma_3-4\Delta_{\rm p}^2)^2+4\Delta_{\rm p}^2(\Gamma_2+\Gamma_3)^2}\right\}, \quad (1)$$

where ρ is the atomic density, Γ_2 and Γ_3 are the decay rates of the meta-stable state $|2\rangle$ and $|3\rangle$, $\Omega_{\rm c}$ and $\Omega_{\rm p}$ are Rabi frequencies of control and probe beams, respectively, μ is the electric dipole moment of transition $|1\rangle\rightarrow|3\rangle$. Under the control of the Gaussian beam with Rabi frequency $\Omega_{\rm c}(r)=\Omega_0\exp(-r^2/\sigma^2)$, where σ is the waist of the control beam, if $r^2\ll\sigma^2$, the index distribution of an EIT medium with a negative frequency detuning ($\Delta_{\rm p}<0$) can be approximated by a GRIN medium as

$$n^2(x,y)=n_0^2[1-g^2(x^2+y^2)], \quad (2)$$

where

$$n_0=\frac{1}{\Omega_0}\sqrt{(\Omega_0^2+\eta\Delta_{\rm p})}, \quad (3)$$

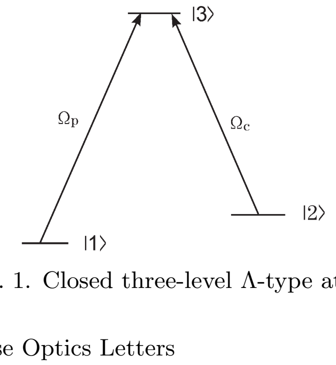

Fig. 1. Closed three-level Λ-type atom.

$$g = \frac{1}{\sigma}\sqrt{\frac{-2\eta\Delta_{\rm p}}{\Omega_0^2 + \eta\Delta_{\rm p}}}. \quad (4)$$

In Eqs. (3) and (4), $\Delta_{\rm p}$ is the probe detuning, and $\eta = 4\rho\mu^2/\varepsilon_0\hbar$.

Given the light field $E_0(x_0, y_0)$ at a transverse plane $z = 0$, the field $E(x, y, z)$ at any plane $z > 0$ can be calculated via an integral formula[24,25]:

$$E_z(x, y, z) = \int_S {\rm d}x_0{\rm d}y_0 E_0(x_0, y_0)G(x, y, x_0, y_0, z), \quad (5)$$

where $G(x, y, x_0, y_0, z)$ is the propagator,

$$G(x, y, x_0, y_0, z) = \frac{kn_0 g\exp({\rm i}kn_0 z)}{i2\pi\sin(gz)}\exp\left[\frac{{\rm i}kn_0 g}{2\tan(gz)}\left(x^2 + y^2 - 2\frac{xx_0 + yy_0}{\cos(gz)} + x_0^2 + y_0^2\right)\right], \quad (6)$$

where $k = \omega/c$ is the wave number in free space. In free space, $n_0 = 1$ and $g = 0$, the integral formula becomes the usual Fresnel integral.

Considering the signal lights as the Gaussian beam

$$E_0(x_0) = {\rm e}^{-\alpha(x_0 - a)^2}, \quad (7)$$

for the signal lights incident onto the medium at $\vec{r}_{\rm i} = (x_{\rm i}, 0, 0)$ and along the positive z-axis within the region $\sqrt{x^2 + y^2} \ll \sigma^2$, Eq. (5) can be integrated as

$$E_z(x) = A\sqrt{\frac{\pi}{\alpha - {\rm i}\beta}}\exp\left[\frac{-\alpha(\gamma x - \beta a)^2}{\alpha^2 + \beta^2} + {\rm i}\frac{(\alpha^2 + \beta^2 - \gamma^2)\beta x^2 - 2\alpha^2\gamma a x + \alpha^2\beta a^2}{\alpha^2 + \beta^2}\right], \quad (8)$$

where $A = \dfrac{kn_0 g{\rm e}^{{\rm i}kn_0 z}}{{\rm i}2\pi\sin(gz)}$, $\beta = \dfrac{{\rm i}kn_0 g}{2\tan(gz)}$, and $\gamma = \dfrac{kn_0 g}{2\sin(gz)}$. In the derivation process, the integral $\int_{-\infty}^{\infty}{\rm e}^{-ax^2}{\rm d}x = \sqrt{\pi/a}$ $(a > 0)$ is used. As a result, the intensity is

$$\begin{aligned} I_z(x) &= E_z(x)\cdot E_z^*(x) \\ &= |A|^2\frac{\pi}{\sqrt{\alpha^2 + \beta^2}}\exp\left[\frac{-2\alpha(\gamma x - \beta a)^2}{\alpha^2 + \beta^2}\right]. \end{aligned} \quad (9)$$

In order to describe the beam deflection, we obtain the beam center by

$$\begin{aligned} x_{\rm c}(z) &= \frac{\int xI_z(x){\rm d}x}{\int I_z(x){\rm d}x} \\ &= \beta a/\gamma = a\cos(gz) \\ &\approx a(1 - g^2z^2/2). \end{aligned} \quad (10)$$

As to the original position $x_0 = a$, the deflection angle is

$$\theta = \partial x/\partial z = -g^2az. \quad (11)$$

Another parameter, the beam waist, can be expressed as[4]

$$\begin{aligned} \omega_{\rm c}^2(z) &= <(x - x_{\rm c})^2> \\ &= \frac{\int(x - x_{\rm c})^2 I_z(x){\rm d}x}{\int I_z(x){\rm d}x} = \frac{\alpha^2 + \beta^2}{4\alpha\gamma^2} \\ &= \frac{4\alpha^2\sin^2(gz) + k^2n_0^2g^2\cos^2(gz)}{4\alpha k^2n_0^2g^2}. \end{aligned} \quad (12)$$

In the case of $\Delta_{\rm p} > 0$, based on Eq. (4), $g \to {\rm i}g$. Accordingly, $\cos(gz) \to \cosh(gz)$, $\sin(gz) \to {\rm i}\ \sinh(gz)$, the beam center and the deflection angle turn to

$$x_{\rm c}(z) \approx a(1 + g^2z^2/2), \quad (13)$$

and

$$\theta = g^2az. \quad (14)$$

The beam waist comes with the form of

$$\begin{aligned} \omega_{\rm c}^2(z) &= <(x - x_{\rm c})^2> \\ &= \frac{4\alpha^2\sinh^2(gz) + k^2n_0^2g^2\cosh^2(gz)}{4\alpha k^2n_0^2g^2}. \end{aligned} \quad (15)$$

It should be noted that, in order to make the deflection meaningful, the propagation distance z should not be too long. So we need $gz \ll \pi/2$, that is

$$z \ll \frac{\pi}{2g}. \quad (16)$$

From Eqs. (11) and (14), it is obvious that the deflection angle increases with the increase of incident position a. The sign relations of the deflection angle, incident positions, and probe detuning are shown in Table 1. The corresponding deflection phenomena were illustrated in Fig. 2. For $a \neq 0$, the probe light with red detuning ($\Delta_{\rm p} < 0$) feels an "attractive potential" toward z axis, while a blue detuned light ($\Delta_{\rm p} > 0$) experiences a "repulsive potential". In the case of $\Delta_{\rm p} = 0$ or $a = 0$, no deflection appears. The above qualitative analysis shows the same results in Ref. [23].

Table 1. Probe Detuning and Beam Deflection

Deflection Angle	$a < 0$	$a > 0$
$\Delta_{\rm p} < 0$	$\theta > 0$	$\theta < 0$
$\Delta_{\rm p} > 0$	$\theta < 0$	$\theta > 0$

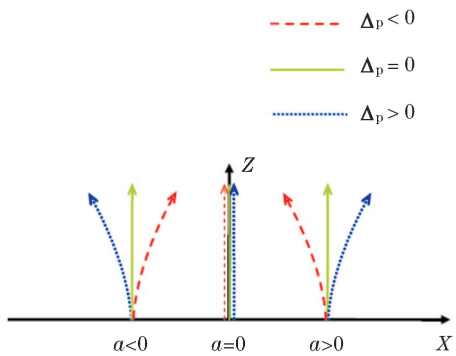

Fig. 2. Deflection phenomena of probe light in the presence of control light with Gaussian profile. Three cases ($\Delta_p < 0$, $\Delta_p = 0$, and $\Delta_p > 0$) are shown in different incident positions $a < 0$, $a = 0$, and $a > 0$.

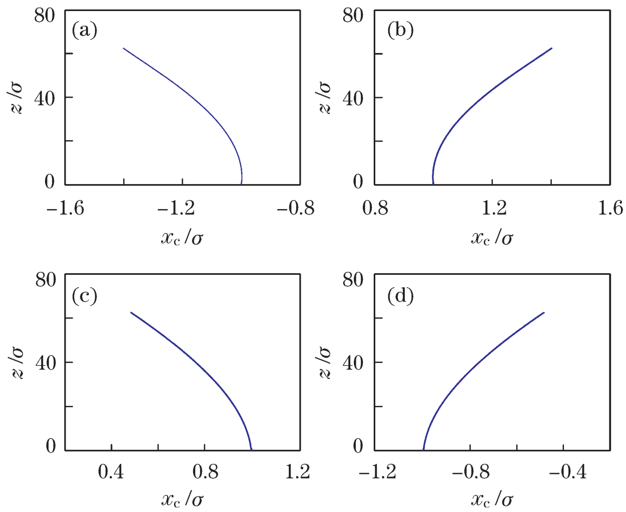

Fig. 3. Numerical calculation of the deflection phenomena. (a) $\Delta_p = 0.5 \times 10^{-6}$ s^{-1}, $a = -\sigma$; (b) $\Delta_p = 0.5 \times 10^{-6}$ s^{-1}, $a = \sigma$; (c) $\Delta_p = -0.5 \times 10^{-6}$ s^{-1}, $a = \sigma$; (d) $\Delta_p = -0.5 \times 10^{-6}$ s^{-1}, $a = -\sigma$. Other parameters are $\Omega_0 = 4 \times 10^7$ s^{-1}, $\eta = 4 \times 10^5$ s^{-1}, $\lambda = 800$ nm, $\Gamma_2 = 3 \times 10^3$ s^{-1}, $\Gamma_3 = 3 \times 10^7$ s^{-1}, and $\alpha = 1$.

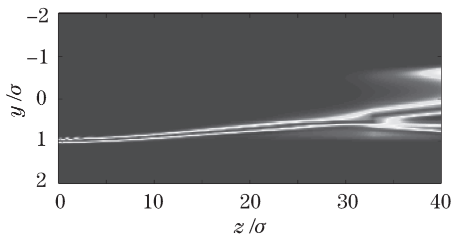

Fig. 4. Normalized intensity distribution of $I(x = 0; y; z)$. Parameters are $\sigma = 5.0 \times 10^{-4}$, $\alpha = \sqrt{2}$, and $a = 1.4\,\sigma$, the others are the same those in Fig. 3.

Figure 3 gives the qualitative results of the deflection. As a comparison, the deflection is numerically calculated with the help of Eq. (1) and the split-step-Fourier method[26]. The parameters are $\Omega_0 = 4 \times 10^7$ s^{-1}, $\eta = 4 \times 10^5$ s^{-1}, $\lambda = 800$ nm, $\Gamma_2 = 3 \times 10^3$ s^{-1}, $\Gamma_3 = 3 \times 10^7$ s^{-1}, and $\alpha = 1$. From Fig. 3, we can obviously see that, the numerical calculation results are well consistent with the qualitative results.

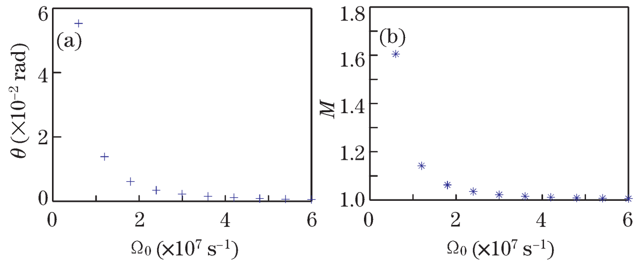

Fig. 5. (a) Deflection angle of the probe light varies with the intensity of the Rabi frequency of the control beam; (b) magnification of the probe light waist as the function of the intensity of the Rabi frequency of the control beam. Parameters are $\alpha = 1$, $z = 10\sigma$, and $a = 0.5\sigma$, the others are the same as those in Fig. 3.

As mentioned above, the observable deflection appears just when $z \ll \pi/2g$ is satisfied. In order to investigate the general propagation of the slow signal light in the medium, the normalized intensity distribution of $I(x = 0; y; z)$ is plotted in Fig. 4. Most parameters are the same as those in Fig. 3 except for $\sigma = 5.0 \times 10^{-4}$, $\alpha = \sqrt{2.5}$, and $a = 1.0\,\sigma$. From Fig. 4, we can see that, in the area of $z \ll \pi/2g$ (about $z \ll 100\sigma$), the signal light is deflected by the Gaussian pumped medium. On the plane of $z = 10\sigma$, the deflection angle is about $\theta = 3.0 \times 10^{-2}$ rad, which is approximate to the analytic result $\theta = 2.5 \times 10^{-2}$ rad (calculated by Eq. (11)). While beyond $z \ll \pi/2g$, with the increase of propagation distance z, the deflection phenomenon disappears, and it is replaced by diffraction.

We also do the numerical calculation about the deflection angle and the waist of the propagating signal light, as shown in Fig. 5. In Fig. 5, the parameters are $\alpha = 1$, $z = 10\sigma$, $a = 0.5\sigma$, and the others are the same as those in Fig. 3. In this case, the deflection angle and the waist of the propagating signal light (expressed as $\omega_c(z) = (\alpha^2 + \beta^2)/4\alpha\gamma^2 \simeq \beta^2/4\gamma^2$ $(\beta \gg \alpha)$) are independent of the waist of the control beam. If the ratio of the beam waist at the position of z to the waist at $z = 0$ is defined as

$$M = \frac{\omega_c(z)}{\omega_c(0)}, \tag{17}$$

the deflection angle and the magnification of the probe light waist varying with the intensity of the Rabi frequency Ω_0 of the control beam are illustrated in Fig. 5. From Fig. 5, we can see that, during the propagating process, with the increase of Ω_0, the deflection becomes weaker, and the beam waist of the probe gets narrower, which even can be the same wide as the waist of the probe ($M = 1$) at the original position $z = 0$.

In conclusion, we firstly assume the cold atomic medium is under the control of the coupling Gaussian laser and the condition of the EIT is satisfied. With the moments formalism theory, the ray path of the slow light is obtained, as well as the beam waist. Results show that, as the signal light propagating in the medium, there would be an observable deflection before the general diffraction. The influences of the coupling intensity on deflection phenomenon and the beam waist of the signal light in the medium are also investigated in this work. As a potential application, the study on the deflection in the coherent atomic medium in this letter will motivate some innovations in quantum storage and quantum information processing.

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