Light propagation and reflection-refraction event in absorbing media

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The light propagation within an absorbing medium and the reflection and refraction at the interface of two absorbing media are studied. By using the unit vectors denoting the planes of constant field amplitude and constant phase respectively, the light propagation and attenuation are described by the effective refractive indices which depend on both the complex refractive index of the medium and the angle between the unit vectors. With the expression for the light propagation, the corresponding Snell's law and the expression of Fresnel coefficients are obtained, which can be applied to describe the reflection-refraction event at the interface between an arbitrary combination of transparent and absorbing media.

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Geometrical optics approximation (GOA) of light scattering by large particles with the size much larger than the wavelength of the incident light is considered as a good method by avoiding difficulties in Lorenz-Mie calculation. Studies on the GOA of light scattering have been of interest to many researchers during the past several decades[1−12]. Most of the publications treat the light scattering by transparent particles. As far as we know, the GOA calculations of light scattering by absorbing particles are rare due to the difficulties in calculating the propagation of light in the absorbing medium and its Fresnel coefficients at the interface of the media. This problem was recently investigated by Yang *et al.*[13,14] by introducing the effective refractive index, and they presented the formulae for calculating the Fresnel coefficients at the interface of the transparent-absorbing media, which was based on the assumption of the plane of constant amplitude being parallel to the interface.

The assumption of Yang *et al.* is reasonable for the reflection-refraction event while the incident light hits the absorbing particle suspended in air, as $p = 0$ in Fig. 1. However, for the high order reflection-refraction events (i.e., $p = 1,2,\cdots$) in which the light within the particle is reflected and refracted at the interface, the plane of constant amplitude is usually unparallel to the interface. Besides, if one would calculate the coated particles containing absorbing core and shell, he has to account for the interface between absorbing-absorbing media.

In this letter, we study the light propagation in an absorbing medium and the reflection-refraction event of the light at the interface of absorbing-absorbing media. The media are assumed to be uniform and isotropic. The parameters such as the permeability ε, the permittivity μ, and the conductivity σ are time-independent. The time dependence of the light wave is expressed in the form exp $(i\omega t)$, where ω is the angular frequency and t is the time.

Substituting the time-harmonic wave (i.e., $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\exp(\mathrm{i}\omega t)$ and $\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})\exp(\mathrm{i}\omega t)$) into the Maxwell equations (in SI units)[15,16], we may obtain

$$\begin{cases} \nabla\times\mathbf{E}(\mathbf{r}) = -\mathrm{i}\omega\mu\mathbf{H}(\mathbf{r}) \\ \nabla\times\mathbf{H}(\mathbf{r}) = \mathrm{i}\omega\hat{\varepsilon}\mathbf{E}(\mathbf{r}) \end{cases}, \tag{1}$$

$$\begin{cases} \nabla^2\mathbf{E}(\mathbf{r}) + k_0^2\hat{m}^2\mathbf{E}(\mathbf{r}) = 0 \\ \nabla^2\mathbf{H}(\mathbf{r}) + k_0^2\hat{m}^2\mathbf{H}(\mathbf{r}) = 0 \end{cases}, \tag{2}$$

where $\hat{\varepsilon} = \varepsilon - \mathrm{i}\frac{\sigma}{\omega}$ is the equivalent complex permeability, k_0 is the wave number in vacuum. The complex refractive index $\hat{m}$ is defined by $\hat{m}^2 = (\omega/k_0)^2\hat{\varepsilon}\mu = c^2\hat{\varepsilon}\mu$ (c is the light speed in vacuum), whose real and imaginary parts m_{re} and m_{im} are obtained as

$$\begin{cases} \hat{m} = m_{\mathrm{re}} - \mathrm{i}\cdot m_{\mathrm{im}}, \\ m_{\mathrm{re}} = c\sqrt{\dfrac{1}{2}\left(\sqrt{\mu^2\varepsilon^2 + \dfrac{\mu^2\sigma^2}{\omega^2}} + \mu\varepsilon\right)} \\ m_{\mathrm{im}} = c\sqrt{\dfrac{1}{2}\left(\sqrt{\mu^2\varepsilon^2 + \dfrac{\mu^2\sigma^2}{\omega^2}} - \mu\varepsilon\right)} \end{cases}. \tag{3}$$

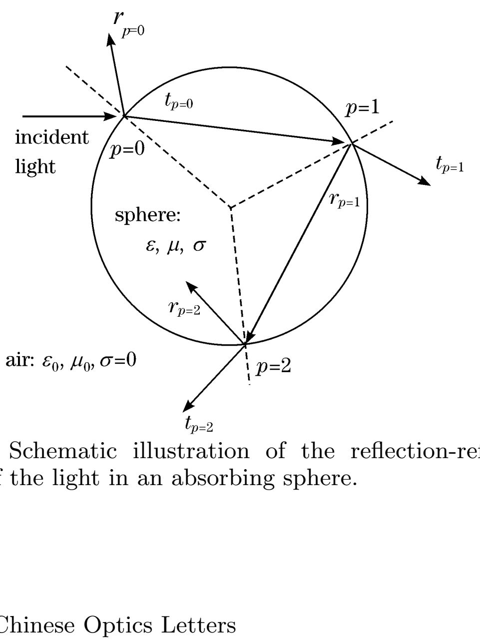

Fig. 1. Schematic illustration of the reflection-refraction events of the light in an absorbing sphere.

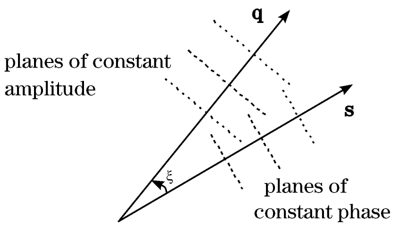

Fig. 2. Schematic illustration of the wave propagation in an absorbing medium.

Figure 2 schematically illustrates the light propagation in an absorbing medium, wherein the planes of constant amplitude are unparallel to the planes of constant phase. The unit vector **q** is normal to the planes of constant amplitude and the unit vector **s** is normal to the planes of constant phase. The angle between these two vectors is $\xi = \cos^{-1}(\mathbf{q}\cdot\mathbf{s})$. Let the electromagnetic field be

$$\begin{cases} \mathbf{E}(\mathbf{r}) = \mathbf{E}_0 \cdot \exp\left[-\mathrm{i}\left(k_s\mathbf{s} - \mathrm{i}k_q\mathbf{q}\right)\cdot\mathbf{r}\right] \\ \mathbf{H}(\mathbf{r}) = \mathbf{H}_0 \cdot \exp\left[-\mathrm{i}\left(k_s\mathbf{s} - \mathrm{i}k_q\mathbf{q}\right)\cdot\mathbf{r}\right] \end{cases}, \tag{4}$$

where $(\mathbf{E}_0, \mathbf{H}_0)$ is the electromagnetic field at the origin of the system, k_s and k_q are the parameters to be determined. Combining Eqs. (2) and (4), we find

$$\begin{cases} N_s = \dfrac{k_s}{k_0} = \sqrt{\dfrac{1}{2}\left\{\sqrt{\left(m_{\mathrm{re}}^2 - m_{\mathrm{im}}^2\right)^2 + \left(\dfrac{2m_{\mathrm{re}}m_{\mathrm{im}}}{\cos\xi}\right)^2} + \left(m_{\mathrm{re}}^2 - m_{\mathrm{im}}^2\right)\right\}} \\ N_q = \dfrac{k_q}{k_0} = \sqrt{\dfrac{1}{2}\left\{\sqrt{\left(m_{\mathrm{re}}^2 - m_{\mathrm{im}}^2\right)^2 + \left(\dfrac{2m_{\mathrm{re}}m_{\mathrm{im}}}{\cos\xi}\right)^2} - \left(m_{\mathrm{re}}^2 - m_{\mathrm{im}}^2\right)\right\}} \end{cases}. \tag{5}$$

The parameters N_s and N_q are the so-called effective refractive indices and are relative to the propagation and attenuation of light in the medium. Their values depend not only on the complex refractive index but also on the angle between the planes of constant amplitude and of constant phase.

By substituting Eq. (4) into Eq. (1), we have

$$\begin{cases} \mathbf{H}(\mathbf{r}) = \dfrac{1}{\omega\mu}k_0\left(N_s\mathbf{s} - \mathrm{i}N_q\mathbf{q}\right)\times\mathbf{E}(\mathbf{r}) \\ \mathbf{E}(\mathbf{r}) = -\dfrac{1}{\omega\hat{\varepsilon}}k_0\left(N_s\mathbf{s} - \mathrm{i}N_q\mathbf{q}\right)\times\mathbf{H}(\mathbf{r}) \end{cases}. \tag{6}$$

This means that, if $N_q \neq 0$ and $\xi \neq 0$, the conditions for transversality are broken and the waves are inhomogeneous.

Reflection-refraction event of the light wave is illustrated in Fig. 3, wherein the absorbing media are characterized with $(\varepsilon_i, \mu_i, \sigma_i)$ and $(\varepsilon_t, \mu_t, \sigma_t)$ in which the subscripts i and t represent the incident and refracted sides, respectively. The interface is aligned in the xy plane. The incident, reflected, and refracted light can thus be expressed as

$$\begin{cases} \mathbf{E}_j(\mathbf{r}) = \mathbf{E}_{0j}\cdot\exp\left[-\mathrm{i}k_0\left(N_{sj}\mathbf{s}_j - \mathrm{i}N_{qj}\mathbf{q}_j\right)\cdot\mathbf{r}\right] \\ \mathbf{H}_j(\mathbf{r}) = \mathbf{H}_{0j}\cdot\exp\left[-\mathrm{i}k_0\left(N_{sj}\mathbf{s}_j - \mathrm{i}N_{qj}\mathbf{q}_j\right)\cdot\mathbf{r}\right] \end{cases}, \tag{7}$$

where the subscript j can be replaced by i, r, and t, denoting the incident, reflected, and refracted light, respectively.

The boundary conditions, i.e., the spatial (and temporal) variation of all fields must be the same at the interface, mean that the phase factors all equal at $z = 0$. This results are shown in the following expressions:

$$\begin{cases} N_{si}\mathbf{s}_i\cdot\mathbf{r}\big|_{z=0} = N_{sr}\mathbf{s}_r\cdot\mathbf{r}\big|_{z=0} = N_{st}\mathbf{s}_t\cdot\mathbf{r}\big|_{z=0} \\ N_{qi}\mathbf{q}_i\cdot\mathbf{r}\big|_{z=0} = N_{ri}\mathbf{q}_r\cdot\mathbf{r}\big|_{z=0} = N_{qt}\mathbf{q}_t\cdot\mathbf{r}\big|_{z=0} \end{cases}. \tag{8}$$

It may be immediately found that all these three wave vectors (i.e., $\mathbf{s}_i$, $\mathbf{s}_r$, and $\mathbf{s}_t$) must lie in a plane and so do the vectors $\mathbf{q}_i$, $\mathbf{q}_r$, and $\mathbf{q}_t$. For simplicity, our study is limited on the special case that all the abovementioned unit vectors are coplanar. Therefore, from Eq. (8) we may find that

$$\begin{cases} k_{si} = k_{sr}, \theta_i = \pi - \theta_r \\ k_{qi} = k_{qr}, \zeta_i = \pi - \zeta_r \\ \xi_i = \left|\theta_i - \zeta\right|_i = \xi_r = \left|\theta_r - \zeta_r\right| \end{cases}, \tag{9}$$

$$N_{si}\sin\theta_i = N_{st}\sin\theta_t,\ N_{qi}\sin\zeta_i = N_{qt}\sin\zeta_t. \tag{10}$$

Equation (9) denotes that the angle of incidence equals the angle of reflection (i.e., $\theta_i = \pi - \theta_r$) and the same relationship holds between the angles ζ_i and ζ_r. A combination of Eq. (10) with Eq. (5) for the refracted light leads to

$$\begin{cases} N_{st} = \sqrt{\dfrac{1}{2}\left\{\begin{array}{l} \sqrt{\begin{array}{l}\left(m_{t,\mathrm{re}}^2 - m_{t,\mathrm{im}}^2 + N_{qi}^2\sin^2\zeta_i - N_{si}^2\sin^2\theta_i\right)^2 \\ +4\left(m_{t,\mathrm{re}}m_{t,\mathrm{im}} - N_{si}N_{qi}\sin\theta_i\sin\zeta_i\right)^2\end{array}} \\ +\left(m_{t,\mathrm{re}}^2 - m_{t,\mathrm{im}}^2 + N_{qi}^2\sin^2\zeta_i + N_{si}^2\sin^2\theta_i\right) \end{array}\right\}} \\ N_{qt} = \sqrt{\dfrac{1}{2}\left\{\begin{array}{l} \sqrt{\begin{array}{l}\left(m_{t,\mathrm{re}}^2 - m_{t,\mathrm{im}}^2 + N_{qi}^2\sin^2\zeta_i - N_{si}^2\sin^2\theta_i\right)^2 \\ +4\left(m_{t,\mathrm{re}}m_{t,\mathrm{im}} - N_{si}N_{qi}\sin\theta_i\sin\zeta_i\right)^2\end{array}} \\ +\left(m_{t,\mathrm{im}}^2 - m_{t,\mathrm{re}}^2 + N_{qi}^2\sin^2\zeta_i + N_{si}^2\sin^2\theta_i\right) \end{array}\right\}} \end{cases}. \tag{11}$$

Equation (11) means that the effective refractive indices N_{st} and N_{qt} to determine the refracted light depend on the refractive index of the refracted medium ($m_{t,\mathrm{re}}$ and $m_{t,\mathrm{im}}$) and the parameters of incident wave (θ_i, ζ_i, N_{si}, and N_{qi}). Once the effective refractive indices N_{st} and N_{qt} are determined, Snell's law, as given in Eq. (10) can be solved, wherein the equation $N_{si}\sin\theta_i = N_{st}\sin\theta_t$ expresses the direction of the refracted wave and $N_{qi}\sin\zeta_i = N_{qt}\sin\zeta_t$ denotes the planes of constant amplitude of the refracted wave.

Finally, we study the Fresnel coefficients of the electromagnetic wave at the interface of two absorbing media. Due to its inhomogeneity characteristics within the absorbing medium, the electromagnetic wave is decomposed into a transverse electric (TE) mode and a transverse magnetic (TM) mode. In the TE mode the electric vector is perpendicular to the direction of wave propagation whereas in the TM mode the magnetic vector is perpendicular to the direction of wave propagation.

For the TE mode, the fields are given as

$$\left\{\begin{array}{l}\mathbf{E}_j(\mathbf{r}) = E_{0j,x}\mathbf{e}_x \cdot \exp\left[-\mathrm{i}k_0\left(N_{sj}\mathbf{s}_j - \mathrm{i}N_{qj}\mathbf{q}_j\right)\cdot\mathbf{r}\right] \\ \mathbf{H}_j(\mathbf{r}) = \left[\left(N_{sj}s_{j,z} - \mathrm{i}N_{qj}q_{j,z}\right)\mathbf{e}_y - \left(N_{sj}s_{j,y} - \mathrm{i}N_{qj}q_{j,y}\right)\mathbf{e}_z\right]\cdot\dfrac{1}{c\mu_j}E_{0j,x}\cdot\exp\left[-\mathrm{i}k_0\left(N_{sj}\mathbf{s}_j - \mathrm{i}N_{qj}\mathbf{q}_j\right)\cdot\mathbf{r}\right]\end{array}\right., \tag{12}$$

where $s_{j,y}$ and $s_{j,z}$ are the components of the unit vector $\mathbf{s}_j$ on y and z axes, $q_{j,y}$ and $q_{j,z}$ are those of the unit vector $\mathbf{q}_j$. However, the direct use of Eq. (12) may lead to an unconservation of energy. This is because that, for the specular reflection, the phase propagation and the absorption are mirror-symmetric about the interface. Considering that the incident and refracted fields propagate along the positive z direction whereas the reflected field propagates along the negative z direction and the media are absorptive, the refracted field for the TE mode is expressed in a different form:

$$\left\{\begin{array}{l}\mathbf{E}_r(\mathbf{r}) = E_{0r,x}\mathbf{e}_x \cdot \exp\left[-\mathrm{i}k_0\left(N_{sr}\mathbf{s}_r - \mathrm{i}N_{qr}\mathbf{q}_r\right)\cdot\mathbf{r}\right] \\ \mathbf{H}_r(\mathbf{r}) = \left[\left(N_{sr}s_{r,z} + \mathrm{i}N_{qr}q_{r,z}\right)\mathbf{e}_y - \left(N_{sr}s_{r,y} + \mathrm{i}N_{qr}q_{r,y}\right)\mathbf{e}_z\right]\cdot\dfrac{1}{c\mu_r}E_{0r,x}\cdot\exp\left[-\mathrm{i}k_0\left(N_{sr}\mathbf{s}_r - \mathrm{i}N_{qr}\mathbf{q}_r\right)\cdot\mathbf{r}\right]\end{array}\right.. \tag{13}$$

The boundary conditions for the TE mode require that

$$\left\{\begin{array}{l}E_{0i,x} + E_{0r,x} = E_{0t,x} \\ H_{0i,y} + H_{0r,y} = H_{0t,y}\end{array}\right., \tag{14}$$

which leads to the Fresnel coefficients of reflection and refraction for the TE-mode electric fields:

$$\left\{\begin{array}{l}r_{E,\mathrm{TE}} = \dfrac{E_{0r,x}}{E_{0i,x}} = \dfrac{\mu_t\left(N_{si}\cos\theta_i - \mathrm{i}N_{qi}\cos\zeta_i\right) - \mu_i\left(N_{st}\cos\theta_t - \mathrm{i}N_{qt}\cos\zeta_t\right)}{\mu_t\left(N_{si}\cos\theta_\mathrm{i} + \mathrm{i}N_{qi}\cos\zeta_i\right) + \mu_i\left(N_{st}\cos\theta_t - \mathrm{i}N_{qt}\cos\zeta_t\right)} \\ t_{E,\mathrm{TE}} = \dfrac{E_{0t,x}}{E_{0i,x}} = \dfrac{2\mu_t N_{si}\cos\theta_i}{\mu_t\left(N_{si}\cos\theta_\mathrm{i} + \mathrm{i}N_{qi}\cos\zeta_i\right) + \mu_i\left(N_{st}\cos\theta_t - \mathrm{i}N_{qt}\cos\zeta_t\right)}\end{array}\right.. \tag{15}$$

Similarly, for the TM mode we have

$$\left\{\begin{array}{l}\mathbf{H}_j(\mathbf{r}) = H_{0j,x}\mathbf{e}_x \cdot \exp\left[-\mathrm{i}k_0\left(N_{sj}\mathbf{s}_j - \mathrm{i}N_{qj}\mathbf{q}_j\right)\cdot\mathbf{r}\right] \\ \mathbf{E}_j(\mathbf{r}) = \left[\left(N_{sj}s_{j,y} - \mathrm{i}N_{qj}q_{j,y}\right)\mathbf{e}_z - \left(N_{sj}s_{j,z} - \mathrm{i}N_{qj}q_{j,z}\right)\mathbf{e}_y\right]\cdot\dfrac{1}{c\hat{\varepsilon}_j}H_{0j,x}\cdot\exp\left[-\mathrm{i}k_0\left(N_{sj}\mathbf{s}_j - \mathrm{i}N_{qj}\mathbf{q}_j\right)\cdot\mathbf{r}\right]\end{array}\right. \tag{16}$$

for the incident and refracted fields and

$$\left\{\begin{array}{l}\mathbf{H}_r(\mathbf{r}) = H_{0r,x}\mathbf{e}_x \cdot \exp\left[-\mathrm{i}k_0\left(N_{sr}\mathbf{s}_r - \mathrm{i}N_{qr}\mathbf{q}_r\right)\cdot\mathbf{r}\right] \\ \mathbf{E}_r(\mathbf{r}) = \left[\left(N_{sr}s_{r,y} + \mathrm{i}N_{qr}q_{r,y}\right)\mathbf{e}_z - \left(N_{sr}s_{r,z} + \mathrm{i}N_{qr}q_{r,z}\right)\mathbf{e}_y\right]\cdot\dfrac{1}{c\hat{\varepsilon}_r}H_{0r,x}\cdot\exp\left[-\mathrm{i}k_0\left(N_{sr}\mathbf{s}_r - \mathrm{i}N_{qr}\mathbf{q}_r\right)\cdot\mathbf{r}\right]\end{array}\right. \tag{17}$$

for the reflected one. The corresponding boundary conditions for the TM mode are

$$\left\{\begin{array}{l}H_{0i,x} + H_{0r,x} = H_{0t,x} \\ E_{0i,y} + E_{0r,y} = E_{0t,y}\end{array}\right.. \tag{18}$$

The Fresnel coefficients of reflection and refraction for the TM mode magnetic fields are obtained as

$$\left\{\begin{array}{l}r_{H,\mathrm{TM}} = \dfrac{H_{0r,x}}{H_{0i,x}} = \dfrac{\hat{\varepsilon}_t\left(N_{si}\cos\theta_i - \mathrm{i}N_{qi}\cos\zeta_i\right) - \hat{\varepsilon}_i\left(N_{st}\cos\theta_t - \mathrm{i}N_{qt}\cos\zeta_t\right)}{\hat{\varepsilon}_t\left(N_{si}\cos\theta_i + \mathrm{i}N_{qi}\cos\zeta_i\right) + \hat{\varepsilon}_i\left(N_{st}\cos\theta_t - \mathrm{i}N_{qt}\cos\zeta_t\right)} \\ t_{H,\mathrm{TM}} = \dfrac{H_{0t,x}}{H_{0i,x}} = \dfrac{2\hat{\varepsilon}_t N_{si}\cos\theta_i}{\hat{\varepsilon}_t\left(N_{si}\cos\theta_i + \mathrm{i}N_{qi}\cos\zeta_i\right) + \hat{\varepsilon}_i\left(N_{st}\cos\theta_t - \mathrm{i}N_{qt}\cos\zeta_t\right)}\end{array}\right.. \tag{19}$$

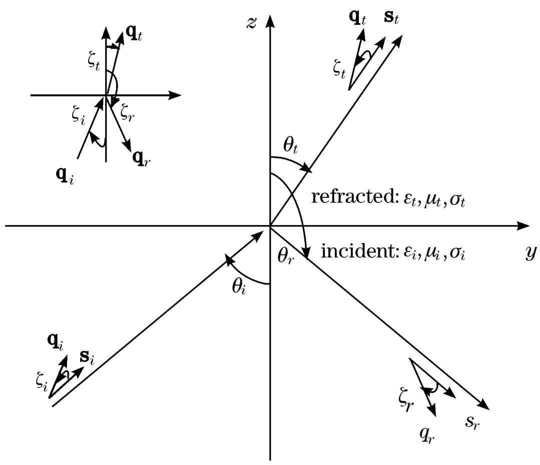

Fig. 3. Geometric configuration for the reflection and refraction of the electromagnetic wave at the interface of the absorbing media.

The Fresnel coefficients expressed in Eqs. (15) and (19) depend on the permeabilities, permittivities, and conductivities of the absorbing media, the incident (or reflected) and refracted angles. In addition, they are related to the planes of constant amplitude of the incident (or reflected) and refracted waves. The Fresnel formula satisfies the conservation law naturally.

It can be proved that, when the medium is transparent ($\sigma = 0$), the refractive index given in Eq. (3) is real ($m_{\rm re} = c\sqrt{\mu\varepsilon}$ and $m_{\rm im} = 0$) and hence the effective refractive indices in Eq. (5) are $N_s = m_{\rm re}$ and $N_q = 0$. As a result, Eq. (6) is simplified to

$$\begin{cases} \mathbf{H}(\mathbf{r}) = \dfrac{1}{\omega\mu} k_0 m_{\rm re} \mathbf{s} \times \mathbf{E}(\mathbf{r}) \\ \mathbf{E}(\mathbf{r}) = -\dfrac{1}{\omega\varepsilon} k_0 m_{\rm re} \mathbf{s} \times \mathbf{H}(\mathbf{r}) \end{cases}. \tag{20}$$

Therefore, the electromagnetic field in the unbound transparent medium meets the conditions for transversality and is homogeneous.

For the reflection-refraction event of the electromagnetic wave at the interface of transparent-absorbing media, we may simply assume one of the conductivities to be zero, i.e., $\sigma_i = 0$ or $\sigma_t = 0$. This is very useful while one calculates light scattering by an absorbing particle within the geometric optics regime. As shown in Fig. 1, $\sigma_i = 0$ corresponds to the case of the incident light hitting the particle ($p = 0$) whereas $\sigma_t = 0$ corresponds to the high order reflection-refraction events when the ray transmission is from the absorbing particle into air ($p = 1, 2, \cdots$). The most special case is the reflection and refraction at the interface of transparent media, in which both the conductivities are zero and expressions of the Fresnel coefficients are simplified into the form given in Ref. [16].

In conclusion, the wave propagation in an absorbing medium and the reflection-refraction event at the interface between absorbing media are studied. The wave propagation in the absorbing medium is characterized by the so-called effective refractive indices N_s and N_q together with their corresponding unit vectors $\mathbf{q}$ and $\mathbf{s}$. The light propagates along the unit vector $\mathbf{q}$ and the field amplitude attenuates along the unit vector $\mathbf{s}$. The effective refractive indices depend on both the complex refractive index of the medium and the angle between $\mathbf{q}$ and $\mathbf{s}$. By using these parameters, Snell's law is expressed in a real form which determines not only the directions for the propagation of the reflected and refracted waves but also the directions for the decay of these waves. The Fresnel coefficients at the interface of the absorbing media are obtained. Snell's law and the Fresnel coefficients obtained in this work can be applied to the reflection-refraction event at the interface of the media, wherein either/both of them is/are absorbing or transparent. The results may find applications in GOA of light scattering and other ray-tracking calculations.

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