Analytical self-similar solutions of the Ginzburg-Landau equation with three-order dispersion effect

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Based on the technique of the symmetry reduction, we find the asymptotic self-similarity analytical resolutions from the constant coefficient Ginzburg-Landau equation considering both influences of the third-order dispersion and gain dispersion on the evolution of pulses. We have obtained the self-similar pulse amplitude function, phase function, strict linear chirp function, and the effective temporal pulse width. Numerical simulations show qualitative agreement with these theoretical results.

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The latest progress of linear-chirp self-similar pulses of parabolic asymptotic evolution has been obtained in analytic theories, numerical simulations, and experimental results at rare-earth ions doped fibers amplifiers and lasers for a decade. These results show that the self-similar pulses have three significant properties[1−3] and may be widely used in fibers communications, nonlinear optics, ultra-fast optics, transient optics, and laser process[4−8]. Because of self-similar features of linear chirp and robust resisting pulse broken, the evolution pulse energy is gradually increasing with pulse transmission. This can result in the nonlinearity and high-order dispersion effects. Especially when the second group velocity dispersion (GVD) is small in dispersion management optic fiber amplifiers, the third-order dispersion (TOD, β_3) will play an importance role. At present, the experimental studies of self-similar parabolic asymptotic pulse evolution have transformed to measure the self-similar amplitude shape, strict linear chirp feature, and effective temporal width. Theoretical analyses have focused on nonlinear Schrödinger equation (NLSE)[8−14] only considering the infinite frequency bandwidth, and on constant and varying coefficients Ginzburg-Landau equation (G-LE) considering the realistic influence of doped elements effect, i.e., effect of gain media finite frequency bandwidth[15,16]. There are only numerical simulations results for high-order dispersion in NLSE and G-LE[17−22] but no report on study of the analytical resolutions of self-similar pulses with high-order dispersion in G-LE to date.

In this letter, we will further theoretically investigate self-similar pulse evolution features with the third-order dispersion of G-LE under affection of high-order dispersion and gain media finite frequency bandwidth in doped fibers. Especially, we will discuss the dynamic mechanism of self-similar pulse of analytical results, and then compare the analytical results with the simulations in high-order dispersion of G-LE.

Suppose that the incident pulse has larger energy (but smaller than the gain saturation energy) in the rare-earth ions doped gained fibers, the evolution of the pulse can be described by general G-LE[1,3,22] expressed as

$$\begin{aligned}\frac{\partial \Psi}{\partial z} &= \mathrm{i}\gamma\left|\Psi\right|^2\Psi - \mathrm{i}\frac{\beta_2}{2}\frac{\partial^2\Psi}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3\Psi}{\partial T^3} \\ &\quad + \frac{g(T)}{2}\Psi + \frac{g(T)}{2\Omega^2}\frac{\partial^2\Psi}{\partial T^2},\end{aligned} \tag{1}$$

where $\Psi = \Psi\ (z,\ T)$ is a slowly varying amplitude of the pulse envelope in a co-moving frame, $T = t-\beta_1 z$ is retarded time, z is the propagation distance. $\beta_2(>0)$, β_3, γ, and $g(T)$ are GVD, TOD, nonlinearity parameter, and gain coefficient, respectively. $\frac{g(T)}{2\Omega^2}\frac{\partial^2\Psi}{\partial T^2}$ is called gain dispersion factor which originates from the frequency dependence of the gain, where Ω is the bandwidth of the doped gained fibers.

We suppose Eq. (1) has the self-similar probe solution of[1−3]

$$\left\{\begin{array}{l}\Psi(z,T) = A(z,T)\exp \mathrm{i}\Phi(z,T) \\ \Phi(z,T) = B(z) + CT^2\end{array}\right., \tag{2}$$

where $A(z,\ T)$, $\Phi(z,\ T)$, B(z), and C are amplitude function, function phase, phase offset function and chirp parameter, respectively. Substituting Eq. (2) into Eq. (1) and comparing the real with the image part, we can get

$$\left\{\begin{aligned}\frac{\partial A}{\partial z} &= \beta_2\frac{\partial A}{\partial T}\frac{\partial \Phi}{\partial T} + \frac{\beta_2 A}{2}\frac{\partial^2\Phi}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3} + \frac{\beta_3}{6}\frac{\partial A}{\partial T}\left(\frac{\partial \Phi}{\partial T}\right)^2 \\ &\quad - \frac{\beta_3}{2}A\frac{\partial^2\Phi}{\partial T^2}\frac{\partial\Phi}{\partial T} + \frac{g(T)A}{2} + \frac{g(T)}{2\Omega^2}\frac{\partial^2 A}{\partial T^2} \\ &\quad - \frac{g(T)A}{2\Omega^2}\left(\frac{\partial\Phi}{\partial T}\right)^2 \quad , \\ A\frac{\partial\Phi}{\partial z} &= -\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \frac{\beta_2 A}{2}\left(\frac{\partial\Phi}{\partial T}\right)^2 + \frac{\beta_3}{2}\frac{\partial^2 A}{\partial T^2}\frac{\partial\Phi}{\partial T} + \frac{\beta_3}{2}\frac{\partial A}{\partial T}\frac{\partial^2\Phi}{\partial T^2} \\ &\quad - \frac{\beta_3 A}{6}\left(\frac{\partial\Phi}{\partial T}\right)^3 + \gamma A^3 + \frac{g(T)}{\Omega^2}\frac{\partial A}{\partial T}\frac{\partial\Phi}{\partial T} \\ &\quad + \frac{g(T)A}{2\Omega^2}\frac{\partial^2\Phi}{\partial T^2} \quad .\end{aligned}\right. \tag{3}$$

In order to separate variation further, we write the am-

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plitude $A(z,T)$ as

$$A(z,T) = f(z)F(T), \tag{4}$$

where $f(z)$ and $F(T)$ are spatial evolution function and instantaneous amplitude envelop function, respectively. Supposing $g(T) = g$ is a constant, we have

$$\begin{cases} \frac{\partial A}{\partial z} = F\frac{\partial f}{\partial z} \\ \frac{\partial A}{\partial T} = f\frac{\partial F}{\partial T} \\ \frac{\partial^2 A}{\partial T^2} = f\frac{\partial^2 F}{\partial T^2} \\ \frac{\partial^3 A}{\partial T^3} = f\frac{\partial^3 F}{\partial T^3} \end{cases}, \quad \begin{cases} \frac{\partial \Phi}{\partial z} = \frac{\partial B}{\partial z} \\ \frac{\partial \Phi}{\partial T} = 2CT \\ \frac{\partial^2 \Phi}{\partial T^2} = 2C \\ \frac{\partial^3 \Phi}{\partial T^3} = 0 \end{cases}. \tag{5}$$

Substituting Eq. (5) into the first one of Eq. (3), it can be stated as

$$\begin{aligned} \frac{1}{f}\frac{\partial f}{\partial z} &= 2\beta_2 CT\frac{1}{F}\frac{\partial F}{\partial T} \\ &+\beta_2 C + \frac{\beta_3}{6}\frac{1}{F}\frac{\partial^3 F}{\partial T^3} + \frac{2C^2\beta_3}{3}T^2\frac{1}{F}\frac{\partial F}{\partial T} \\ &-2\beta_3 C^2 T + \frac{g}{2} + \frac{g}{2\Omega^2}\frac{1}{F}\frac{\partial^2 F}{\partial T^2} - \frac{2gC^2}{\Omega^2}T^2. \end{aligned} \tag{6}$$

Analyzing variation properties of spatial and temporal in Eq. (6), we can get

$$\begin{cases} \frac{1}{f}\frac{\partial f}{\partial z} = \beta_2 C + \frac{g}{2}, \\ \frac{\beta_3}{6}\frac{1}{F}\frac{\partial^3 F}{\partial T^3} + \frac{g}{2\Omega^2}\frac{1}{F}\frac{\partial^2 F}{\partial T^2} + (2\beta_2 CT + \frac{2\beta_3 C^2}{3}T^2)\frac{1}{F}\frac{\partial F}{\partial T} \\ -(2\beta_3 C^2 T + \frac{2gC^2}{\Omega^2}T^2) = 0. \end{cases} \tag{7}$$

From the first one of Eq. (7), we can obtain $f(z) = f_0\exp[(\beta_2 C + \frac{g}{2})z]$ immediately. The chirp parameter is[15,16]

$$C = -\frac{20\beta_2\gamma f_0^2}{6\beta_2^2 + \beta_3^2\Omega^2}. \tag{8}$$

The spatial amplitude function can be stated as

$$\begin{aligned} f(z) &= f_0\exp[(\beta_2 C + \frac{g}{2})z] \\ &= f_0\exp[(-\frac{20\beta_2^2\gamma f_0^2}{6\beta_2^2 + \beta_3^2\Omega^2} + \frac{g}{2})z], \end{aligned} \tag{9}$$

where f_0 is the initial amplitude of the evolution pulse.

It has been demonstrated in the numerical researches that the evolution pulse of amplitude envelop will deviate parabolic asymptotic model under the conditions of the high order dispersion. However, the amplitude envelop profile of the evolution pulse is still self-similarity[15−18]. Therefore, in order to get instantaneous amplitude envelop function $F(T)$, we suppose

$$F(T) = (1 + MT + NT^2 + LT^3), \tag{10}$$

where M, N, and L are coefficients related to the doped fibers respectively. equation (10) is the result of the third-order power series(as we know that the parabolic asymptotic model of instantaneous amplitude envelop function $F(T)$ is just the second-order power series). In fact, considering nth-order dispersion, we can suppose that $F(T) = (1 + MT + NT^2 + LT^3 + \cdots) = \sum_{i=0}^{n} P_i T^i$, where the coefficients P_i relate to the doped fibers. There are $F'(T) = (M + 2NT + 3LT^2)$, $F''(T) = (2N + 6LT)$, and $F'''(T) = 6L$ in Eq. (10). Substituting these together with Eq. (10) into the second one of Eq. (7), comparing the coefficients of T and calculating those by determinant, we can get

$$\begin{cases} M = \frac{3C\beta_3}{2}\frac{12\beta_2^2\Omega^4 - g^2}{18\beta_2^3\Omega^4 + \beta_3^2 Cg\Omega^2 + 9\beta_2 g^2} \\ N = \frac{3C}{2}\frac{6\beta_2^2 g\Omega^4 + 3g^3 + 4\beta_3^2\beta_2 C\Omega^6}{18\beta_2^3\Omega^6 + \beta_3^2 Cg\Omega^4 + 9\beta_2 g^2\Omega^2} \\ L = \frac{\beta_3 C^2\Omega^2}{3}\frac{2\beta_3^2 C\Omega^2 + 21\beta_2 g}{18\beta_2^3\Omega^4 + \beta_3^2 Cg\Omega^2 + 9\beta_2 g^2} \end{cases}. \tag{11}$$

Substituting Eq. (11) into Eq. (10), the instantaneous amplitude envelop function is in the form as

$$\begin{aligned} F(T) = 1 &+ \frac{3C\beta_3}{2}\frac{12\beta_2^2\Omega^4 - g^2}{18\beta_2^3\Omega^4 + \beta_3^2 Cg\Omega^2 + 9\beta_2 g^2}T \\ &+ \frac{3C}{2}\frac{6\beta_2^2 g\Omega^4 + 3g^3 + 4\beta_3^2\beta_2 C\Omega^6}{18\beta_2^3\Omega^6 + \beta_3^2 Cg\Omega^4 + 9\beta_2 g^2\Omega^2}T^2 \\ &+ \frac{\beta_3 C^2\Omega^2}{3}\frac{2\beta_3^2 C\Omega^2 + 21\beta_2 g}{18\beta_2^3\Omega^4 + \beta_3^2 Cg\Omega^2 + 9\beta_2 g^2}T^3. \end{aligned} \tag{12}$$

Equation (12) shows that the temporal of evolution pulse is completely decided by parameters of the doped fibers and initial energy of the incident pulses, which is an important propriety of self-similar asymptotic pulse. Under the conditions of the third-order dispersion, there are other two features in Eq. (12). One is that the gain dispersion factor has a significance influence on the temporal amplitude function. The other one is that there is the third-order time factor T^3 in Eq. (12) because of TOD effect, which results in the shape of evolution pulse deviating from the parabolic asymptotic. M in Eq. (11) shows that the evolution pulse of amplitude envelop has deviated parabolic asymptotic model, which is consistent with conclusions in Refs. [15–21]. Combining Eqs. (9) and (12), the magnitude of pulse amplitudes grow exponentially with z, so does $T_p(z)$. Thus, self-similar pulses can reach the very high energy during its evolution.

From Eq. (12), we can define the effective width of the self-similar pulse as

$$\begin{aligned} T_p(z) &= \frac{1}{\sqrt{|N|}} \\ &= \sqrt{\frac{2}{3C}\frac{18\beta_2^3\Omega^6 + \beta_3^2 Cg\Omega^4 + 9\beta_2 g^2\Omega^2}{6\beta_2^2 g\Omega^4 + 3g^3 + 4\beta_3^2\beta_2 C\Omega^6}}. \end{aligned} \tag{13}$$

When $\beta_3 = 0$, $T_p(z) = \sqrt{\frac{2\Omega^2\beta_2}{Cg}\frac{2\beta_2^2\Omega^4 + g^2}{2\beta_2^2\Omega^4 + g^2}}$. When $M = 0$ and $L = 0$, there is $F(T) = (1+NT^2)$ in Eq. (11), which is a typical example of parabolic asymptotic self-similar pulse temporal amplitude[15,16].

Substituting Eqs. (4) and (5) into the second one of Eq. (3), we get

$$\begin{aligned} \frac{\partial B}{\partial z} &= -\frac{\beta_2}{2}\frac{1}{F}\frac{\partial^2 F}{\partial T^2} + 2\beta_2 C^2 T^2 + \beta_3 CT\frac{1}{F}\frac{\partial^2 F}{\partial T^2} \\ &+\beta_3 C\frac{1}{F}\frac{\partial F}{\partial T} - \frac{4}{3}\beta_3 C^3 T^3 + \frac{2CgT}{\Omega^2 F}\frac{\partial F}{\partial T} \\ &+\frac{Cg}{\Omega^2} + \gamma A^2. \end{aligned} \tag{14}$$

Analyzing variation properties of spatial and temporal in

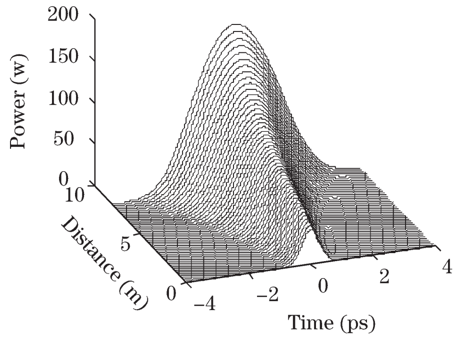

Fig. 1. Evolution of self-similar pulse in Eq. (1).

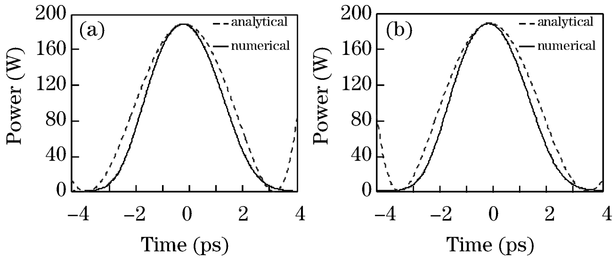

Fig. 2. Comparison of profiles of self-similar pulse with TOD affection. (a) $\beta_3>0$; (b) $\beta_3<0$.

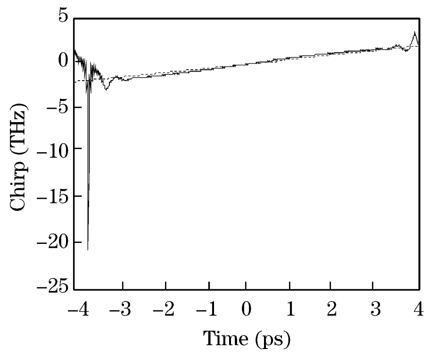

Fig. 3. Comparison of the chirp of self-similar pulse with TOD affection.

Eq. (14), we can get

$$\left\{\begin{array}{l} \frac{\mathrm{d}B}{\mathrm{d}z}=\frac{Cg}{\Omega^2} \\ (\beta_3 CT-\frac{\beta_2}{2})\frac{1}{F}\frac{\partial^2 F}{\partial T^2}+(\beta_3 C+\frac{2CgT}{\Omega^2})\frac{1}{F}\frac{\partial F}{\partial T} \\ +2\beta_2 C^2T^2-\frac{4}{3}\beta_3 C^3 T^3+\gamma f^2F^2=0. \end{array}\right. \quad (15)$$

We can get the self-similar phase function from the first one in Eqs. (15), (8), and (2)

$$\Phi(z)=B_0(0,T_0)-\frac{20\beta_2\gamma zgf_0^2}{\Omega^2(6\beta_2^2+\beta_3^2\Omega^2)} -\frac{20\beta_2\gamma f_0^2}{6\beta_2^2+\beta_3^2\Omega^2}T^2, \quad (16)$$

where $B_0(0,\ T_0)$ is an arbitrary initial phase constant. From Eq. (16), we can get the self-similar chirp function as

$$\omega_c(T)=-\frac{\partial\Phi}{\partial T}=\frac{40\beta_2\gamma f_0^2}{6\beta_2^2+\beta_3^2\Omega^2}T. \quad (17)$$

Equation (17) shows that linear chirp of the self-similar evolution pulse is only decided by the parameters of doped fibers and initial energy of the incident pulses. The linear chirp is the outcome of the balance among gain, nonlinearity, gain dispersion, and self-phase modulation (SPM), while self-similar profile of intensity comes from the high gain in doped ion fibers.

The analytical results of the Eqs. (4), (9), (12), (15), and (16) present a general description of the self-similar evolution with TOD in doped fiber amplifiers. The amplitude function is

$$\begin{aligned} A(z,T)=&\ f(z)F(T)=f_0\exp[(\beta_2 C+\frac{g}{2})z] \\ &(1+MT+NT^2+LT^3) \\ &=f_0\exp[(\beta_2 C+\frac{g}{2})z] \\ &[1+\frac{3C\beta_3}{2}\frac{12\beta_2^2\Omega^4-g^2}{18\beta_2^3\Omega^4+\beta_3^2 Cg\Omega^2+9\beta_2 g^2}T \\ &+\frac{3C}{2}\frac{6\beta_2^2 g\Omega^4+3g^3+4\beta_3^2\beta_2 C\Omega^6}{18\beta_2^3\Omega^6+\beta_3^2 Cg\Omega^4+9\beta_2 g^2\Omega^2}T^2 \\ &+\frac{\beta_3 C^2\Omega^2}{3}\frac{2\beta_3^2 C\Omega^2+21\beta_2 g}{18\beta_2^3\Omega^4+\beta_3^2 Cg\Omega^2+9\beta_2 g^2}T^3], \end{aligned} \quad (18)$$

and the phase function is Eq. (16).

In order to confirm our analytical results, we have numerically calculated Eq.(1) and compared the analytic solutions in intensity profiles and chirp properties. Here, we assume that the doped fibers parametric conditions are $\Omega = 960\ \mathrm{ps}^{-1}$, $\beta_2 = 69\times10^{-3}\ \mathrm{ps^2m^{-1}}$, $\beta_3 = \pm1.85\times10^{-3}\ \mathrm{ps^3m^{-1}}$, $\gamma = 1.5\times10^{-3}\ \mathrm{W^{-1}m^{-1}}$, and $g = 0.378\ \mathrm{m}^{-1}$. Gaussian input pulse incident energy is $U_{\mathrm{in}}= 30$ pJ, its initial pulse width is $T_0 = 400$ fs, and the center wavelength is $\lambda_0 = 1550$ nm. Figure 1 is the power profile of evolution pulse from Gaussian input pulse to the self-similar pulse of Eq. (1) in 0−8 m propagation distance. Figures 2(a) and (b) are power and chirp curves for self-similar pulse of analytical and numerical solution at 8-m location of the propagation distance, respectively. There are obvious differences between the right part of Fig. 2(a) and left part of Fig. 2(b) originated from the TOD affection in Eq. (18). That is just what the TOD results in the asymmetry of the curves in Fig. 2. It shows that the asymmetry of the curves originates from the TOD affection[17,21].

Figure 3 shows that the linear chirp of the evolution pulse center is all valid either in analytical solutions or in numerical solution of high-order dispersion affect of Eq. (1). We also note that there is a little difference between the analytical and simulation solutions in Fig. 2. The reasons can be deduced that the analytical solution of self-similar pulse evolution is based on the symmetry reduction method which has been approximated in Eq. (10)[15,16]. The results in Refs. [17, 21] have showed that the third-order dispersion can change the profiles of power (delaying pulses peak) and spectrum distortion(causing oscillation in pulses leading and rearing). In the linear chirp of self-similar evolution scope (about 6 ps), our theoretical results are consistent with numerical simulations. We have noted that the vibration and intense peak of the simulation in the right part of Fig. 3 may originate in numerical calculating from computer simulation, which have exceeded the scope of linear chirp of self-similar evolution.

In conclusion, based on analytic method of symmetry reduction in normal GVD region with gain dispersion and the third- order dispersion, we get the self-similar pulse evolution analytical solutions of G-LE. To our knowledge, this is the first time to provide the general analytic forms describing self-similar pulse in high-order dispersion influence limited by the effect of transition in realistic doped fibers, which will give strong theoretical backing to study the high energy and the high power self-similar pulses in nonlinear optics, ultra-fast optics, and transient optics fields.

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