

# Generation of millimeter-wave sub-carrier optical pulse by using cascaded all-pass cavities

Qinfeng Xu (徐钦峰)\*, Qing Ye (叶青), Zhengqing Pan (潘政清), Zujie Fang (方祖捷),  
Haiwen Cai (蔡海文), and Ronghui Qu (瞿荣辉)

Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China

\*E-mail: xuqf5678@163.com

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A novel scheme is proposed to transform an ultra-short optical pulse to a millimeter-wave frequency-modulated pulse by using the cascaded all-pass cavities (APCs). The envelope waveform of the generated pulse train is calculated, showing effective improvement by APC cascading. The extinction ratio is analyzed with different input pulses, different cavity reflectivities, and different cascading numbers. It is shown that the cascading does not introduce much effect on the extinction ratio. Two designs by using Gires-Tournois cavity and waveguide ring resonators are proposed to realize the cascaded APC.

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Demand on bandwidth of telecommunication system is increasing tremendously in recent years, especially for mobile communications, for which the carrier is transferred from micrometer to millimeter wave band. Radio over fiber (RoF) is regarded as a promising technology, and generation of high-frequency radio sub-carrier in optical domain is one of the key technologies<sup>[1]</sup>. Researchers have proposed varieties of schemes to obtain sub-carrier optical sources, such as by means of fiber Bragg grating structures<sup>[2-4]</sup>, double-sideband modulation<sup>[5]</sup>, and time-domain Talbot effect<sup>[6]</sup>. Multiplication of optical pulse repetition rate is also an attractive research topic. A design of coupled Fabry-Perot (F-P) resonator to multiply the repetition rate was proposed in Ref. [7]. Characteristics of Gires-Tournois (G-T) cavity and ring resonator for pulse repetition rate multiplication (PRRM) were analyzed in Refs. [8, 9]. An ordinary F-P cavity was taken as a pulse multiplication device to generate a pulse train, which showed merits of simplicity and low cost as a millimeter wave sub-carrier signal<sup>[10]</sup>. However, the envelope of generated millimeter pulse is a decayed profile. Another disadvantage is its low energy efficiency, since only the transmitted pulses are used while the reflected pulses are left. To overcome the shortcoming, G-T cavity and ring resonator are considered to be a good candidate. In this letter, a new scheme using cascaded G-T cavity and ring resonator is proposed to generate a pulse train from a single optical pulse, and the basic properties are analyzed. It is shown that the combination of multiple all-pass filters can improve the envelope shape and provide more parameters for optimization of the millimeter signals.

As pointed out in Refs. [7] and [10], if an ultra-short optical pulse is incident into a F-P cavity with a round-trip time larger than its pulse width, a pulse train will be generated due to the multiple reflections and transmissions on the two mirrors of the F-P cavity. For a F-P cavity with low-reflection mirrors, pulse amplitudes in the train will decay fast; when high-reflection mirrors are used, the pulse train can sustain for a longer time,

but its amplitude is low since most energy of the incident pulse will be reflected by the F-P cavity at the first reflection, resulting in low energy efficiency. It is conjectured that G-T cavity may help to solve the problem. An ideal G-T cavity consists of two lossless mirrors with different reflectivities: a low-reflection front mirror and a 100% reflective rear mirror. It is an all-pass cavity (APC) since the incident light beam will be totally reflected. Another example is the ring resonator, including fiber ring and waveguide ring resonators. When the loss in the optical path is low enough to be neglected, it is also an all-pass cavity. Figure 1(a) shows the basic structure of a G-T cavity; Fig. 1(b) is a ring resonator, where  $r$  and  $i(1-r^2)^{1/2}$  are the amplitude ratios of the coupler, and  $i$  is the imaginary symbol to indicate  $\pi/2$  phase shift between two beams. The transfer functions of G-T cavity and ring resonator have the same form written as

$$H(\omega) = \frac{r - e^{i\phi}}{1 - re^{i\phi}} = \exp(i\alpha), \quad (1)$$

$$\alpha = \tan^{-1} \frac{p \sin \phi}{\cos \phi - q}, \quad (2)$$

where  $R = r^2$  and  $\phi = 2nkdcos\theta$  for G-T cavity, and  $\phi = \beta L = n_{eff}kL$  for ring resonator with the effective index  $n_{eff}$ , cavity length  $L$ , and wave vector  $k$ . In the expression of phase shift  $\alpha$ , it is defined that  $p = (1-r^2)/(1+r^2)$  and  $q = 2r/(1+r^2)$  with property of  $p^2 + q^2 = 1$ . The repetition rate  $f$  of the pulse train can be deduced to be

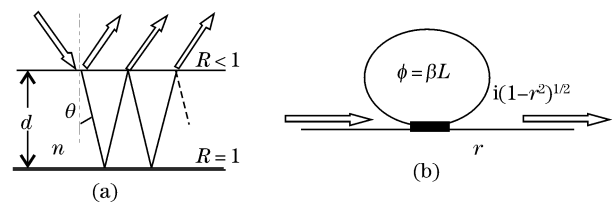


Fig. 1. Schematic diagrams of (a) G-T cavity and (b) ring resonator.

$f = c/2nd\cos\theta$  for G-T cavity, or  $f = c/n_{\text{eff}}L$  for ring resonator, which correspond to the free spectral range of cavity. Both G-T cavity and ring resonator are phase filters with periodical resonances, and are used for dispersion compensation in fiber communications. When the pulse width of input optical pulse is shorter than the cavity round-trip time, a pulse train will be generated by multiply traveling the cavity, which can be used in RoF technology and pulse rate multiplication. For RoF applications,  $f$  should be designed to be the required millimeter wave frequency. For example,  $f = 60$  GHz can be obtained from a G-T cavity with  $n = 1.45$ ,  $\theta \sim 0$ , and  $d = 1.72$  mm.

Being different from F-P scheme, all energy of the incident optical pulse can be utilized in G-T cavity and ring resonator. To describe the pulse train envelope, considering an ultra-short input pulse with pulse width neglected, the normalized intensity of the envelope can be described as

$$A_1 = R, A_{j>1} = T^2 R^{j-2}, \quad (3)$$

where  $T = 1 - R$ ,  $j$  is the series number of the pulses,  $A_j = I_j/I_0$ ,  $I_j$  is the intensity of the  $j$ th pulse, and  $I_0$  is the input pulse intensity. Except the first pulse, amplitudes of the pulse train will decay as a function of  $\exp(-t/t_d)$ , where  $t_d = (f \ln R^{-1})^{-1}$ ; it is the same as that in F-P scheme. The amplitude of the first pulse is simply  $R$  of the incident pulse. Figure 2 shows the amplitude distribution in the envelope of pulse trains for  $R = 0.3, 0.382, 0.5$ , and  $0.6$ , where  $R = 0.382$  gives equal amplitude of the first and second pulses. For  $R = 0.5$ , we get  $R = T$ , and  $A_j = R^j$ . It is shown that for higher reflectivity the amplitudes of most pulses are smaller and decay more slowly except for the first pulse; while for lower reflectivity the amplitude of the first pulse may even be lower than that of the second one, but the following pulses decay fast. It is indicated that the envelope of the pulse train generated by APC is not much superior to the scheme of F-P cavity except for the energy efficiency.

It is conjectured that if several identical G-T cavities or identical ring resonators are connected in a sequence, the envelope distribution can be adjusted, just as proposed for PRRM in Refs. [8,9]. When the pulse train generated by the first APC is incident into the second APC, each pulse will generate its own pulse train with the same repetition rate and a delay of one period; two pulse trains can then be added together into one pulse train, resulting in a modified envelope distribution. Consequently, more cavities cascaded in a sequence will further modify the envelope with  $R = 0.5$ , the amplitude profile is  $A_j = jR^{j+1}$  for 2-cascaded APC,  $A_j = (1+2+\dots+j)R^{j+2}$  for 3-cascaded APC, and a little more complicated expressions can be written for more APCs. Figure 3(a) shows the modified envelope for cases of 1–6 cascaded APCs with  $R = 0.5$ . It is shown that the envelope profile is obviously improved towards a symmetric millimeter wave signal from a single one to 6 cascaded APCs. Figure 3(b) is for  $R = 0.382$  and the number of cascaded APCs from one to four, showing that similar effect can be obtained with cascaded APCs less than that for  $R = 0.5$ . Profile of pulse train envelopes for different  $R$  and cascaded numbers can be calculated similarly.

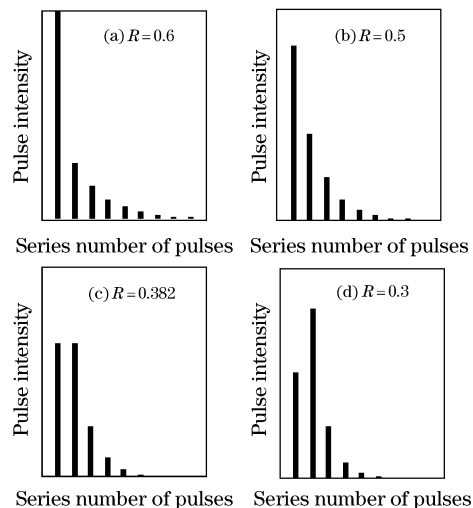


Fig. 2. Pulse trains generated by single APCs with different reflectivities. (a)  $R = 0.6$ ; (b)  $R = 0.5$ ; (c)  $R = 0.382$ ; (d)  $R = 0.3$ .

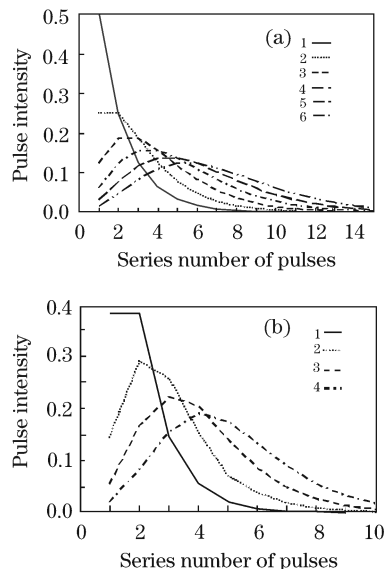


Fig. 3. Envelope profiles of cascaded APCs with different cascading numbers. (a)  $R = 0.5$ ; (b)  $R = 0.382$ .

Cascaded G-T filters have been used in fiber dispersion compensations, as analyzed in Ref. [11]. For RoF applications, similar configurations of cascaded APCs can be designed. Two identical G-T cavities placed parallel and opposite with each other can provide a cascaded APC, as shown in Figs. 4(a) and (b), where the second G-T cavity (G-T2) can move horizontally or vertically to change the cascading number. If a 60-GHz repetition rate of the pulse train is needed, a glass plate with  $nd = 2.5$  mm can be used to fabricate the G-T cavity, with the reflectivity of front mirror designed as analyzed above. An ultra-short optical pulse incident to the G-T pair can be reflected several times, corresponding to the cascaded G-T cavity. By changing the spacing between two G-T cavities, or changing the displacement, the cascade number can be adjusted, as depicted by dotted lines in Figs. 4(a) and (b). Figure 4(c) is a configuration of cascaded waveguide rings. Since the resonance frequency is in the needed millimeter

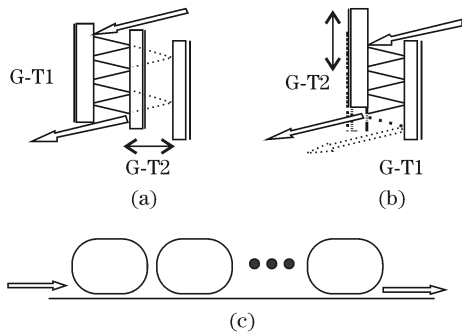


Fig. 4. Cascaded G-T cavities with (a) horizontally and (b) vertically movable configurations; (c) cascaded waveguide ring configuration.

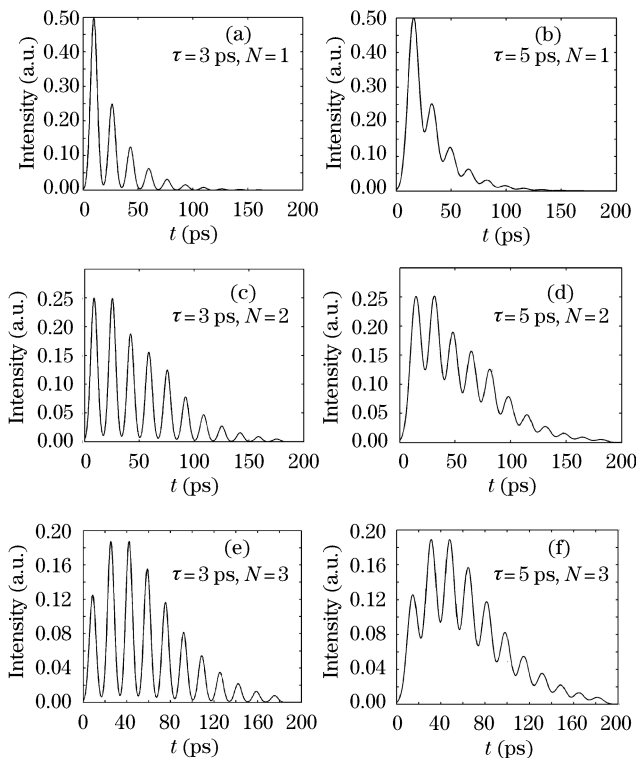


Fig. 5. Pulse train profiles with different input pulse widths  $\tau$  and cascaded numbers  $N$  ( $R = 0.5$ ). (a)  $\tau = 3$  ps,  $N = 1$ ; (b)  $\tau = 5$  ps,  $N = 1$ ; (c)  $\tau = 3$  ps,  $N = 2$ ; (d)  $\tau = 5$  ps,  $N = 2$ ; (e)  $\tau = 3$  ps,  $N = 3$ ; (f)  $\tau = 5$  ps,  $N = 3$ .

wave band, the cavity length should be designed at  $n_{\text{eff}} L = 5$  mm, which is too small to make a practical fiber ring. However, a ring fabricated on a planar waveguide is practical and suitable. The waveguide scheme has the advantages of smaller volume and higher stability; while G-T cavity pair scheme provides more adjustable parameters, such as incident angle and displacement of two G-T cavities. It is noticeable that the time delay in optical path between adjacent APCs does not affect the cascaded result so long as the input pulse is short enough compared with the required millimeter wave period, and the dispersion of the path is negligible, which gives a freedom for the device design.

The above analyses hold under a condition of very short incident pulses, where no mutual influence exists between two pulses in succession. When the pulse width of incident beam is not short enough, the above results have to

be revised. Considering a pulse with Gaussian waveform as  $E_0(t) \sim \exp(-t^2/2\tau^2 + i\omega_0 t)$ , where  $\tau$  is half-width of  $1/e$  time of the maximum intensity, the extinction ratio of the pulse train will decrease; except for the first pulse, the extinction ratio can be derived by a method similar to Ref. [10] as

$$\varepsilon = \frac{I_j + I_{j+1}}{2I_{\text{mid}}} = \frac{1 + R}{2(1 + R + 2r \cos \Delta\phi)} \exp \frac{1}{4f^2\tau^2}, \quad (4)$$

where  $\Delta\phi = \phi + \delta\phi$  is the phase shift between two successive pulses with phase difference  $\delta\phi$  between the front edge and rear edge of the pulse at interval of one repetition period  $1/f$ ,  $I_j$  and  $I_{j+1}$  stand for intensities of the  $j$ th and  $(j+1)$ th pulses, and  $I_{\text{mid}}$  is the intensity at the middle between them<sup>[3]</sup>. At resonance of the cavity, we have  $\phi = 2m\pi$ , and for non-dispersive cavity,  $\delta\phi = 0$ . In practice, the phase shift mismatch of the output pulse in the cascaded APC is inevitable and complex. However, the input pulse that we discuss is an ultra-short pulse. Therefore, the phase shift brings little effect on the extinction ratio of the output pulse. So now we only discuss the non-dispersive cavity. For the cascaded APC, digital calculation has to be used to show the waveform. Figure 5 gives the calculated output pulse trains for cascaded APC numbers  $N = 1, 2$ , and  $3$  with  $R = 0.5$ , when the Gaussian pulses with  $\tau = 3$  and  $5$  ps are taken as input. It is shown that APC cascading has little influence on the extinction ratio, and even gives improvement for wider input pulse, as can be seen from the comparison between Figs. 5(b) and (d). The improvement is attributed to the repeated use of the first pulse with the highest amplitude.

It is noticed that the dispersion compensation function of APC is neglected in the above analysis, because for an incident short enough optical pulse, its spectrum covers a wide band over many resonance peaks, and the all-pass filter will not change its spectrum much. But for a longer pulse its spectrum does not cover many resonance peaks, and cavity dispersion will affect the performances of output pulse train. For longer incident pulses, it may be better to make simulation in spectral domain, as done in Refs. [7-9]. However, it is necessary to point out that the transfer function, Eq. (1) in this letter, or similar expressions in Refs. [8, 9], cannot be used to the first reflected pulse, which does not travel inside the cavity. In other words, the dispersion of cavity gives no influence on the first pulse. Figure 6 shows a calculated waveform of a 2-cascaded APC with  $R = 0.5$ , when a linear chirped input pulse is used with  $E_0(t) \sim \exp[-(1+iC)t^2/2\tau^2 + i\omega_0 t]$ . It is noticed that linearly chirp induces little effect on the extinction

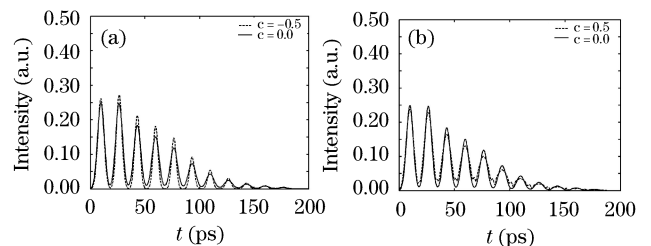


Fig. 6. Pulse train profile with different chirp coefficients of input pulse with  $N = 2$ ,  $R = 0.5$ , and  $\tau = 3$  ps; (a)  $C = -0.5$ ; (b)  $C = 0.5$ .

ratio when  $\tau \ll 1/f$ , and the sign of chirp coefficient  $C$  gives some noticeable effect. The extinction ratio seems to be improved for  $C = -0.5$ , which is believed an effect of the phase factor  $\Delta\phi$  in Eq. (4), if  $\cos(\Delta\phi)$  reaches  $-1$  for example. However, we cannot conclude that negative  $C$  is always better than positive  $C$ . Figure 6 shows a calculated example. The result may depend on the combination of the related parameters. Detailed calculation is needed for a practical configuration. It is shown by the simulation that the dispersion of APC gives less influence on the proposed scheme so long as the input pulse width is much shorter than the period of required millimeter wave.

In practical APC, loss exists more or less inevitably. Transfer function of APC with loss can be deduced by the same method of Eq. (1) and the characteristics of cascaded APC can be obtained by the same simulation method. The main effect of loss is believed to be faster decays of the pulse train. The concrete analysis on loss influence will be given in the future.

In conclusion, a novel and simple method to achieve millimeter wave modulated optical pulse by using cascaded APC is proposed and analyzed. It is shown that the scheme has better energy efficiency than F-P cavity, and APC cascading can improve the generated pulse envelopes effectively by choosing proper reflectivity  $R$  and cascading number  $N$ . The extinction ratio of the proposed scheme is analyzed for different input pulses and cascading numbers. It is also shown that APC cascading induces not much influence on the extinction ratio if the input pulse width is much shorter than the period of the

required millimeter wave. Two designs of APC with G-T cavity and waveguide ring resonators are proposed to realize the cascaded APC. The scheme provides hopefully a new solution for future RoF communication network sub-carriers.

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