Large negative Goos-Hänchen shift from a wedge-shaped thin film

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The analytical expression for the complex amplitude of light reflected from a wedge-shaped thin film is derived. For plane wave incidence, a simple ray tracing approach is used to calculate Goos-Hänchen (GH) shifts; and for non-plane wave incidence, for example, a Gaussian beam, the angular spectrum approach of plane wave is used in simulation. The two approaches predict that a wedge-shaped thin film can produce large negative longitudinal GH shifts. Although the reflectivity is small near the condition of resonance, the large negative GH shifts can be more easily detected in comparison with the shift from a plane-parallel film in vacuum.

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Goos-Hänchen (GH) effect describes the spatial shift of the reflected ray with respect to a point where the ray of incidence intersects the boundary, and the shift is called the GH shift[1]. This phenomenon has been widely analyzed in theory[2−13] and verified in experiments[14−17]. Recently, the GH shifts from plane-parallel dielectric films with the positive[7−9] and negative[10−13] refractive indices have attracted a great deal of attention. However, to the best of our knowledge, the GH effect from a wedge-shaped dielectric film has not been studied. The wedge-shaped film is a fundamental optical element, which can be used in lasers, interferometers, and some experiments[18−22]. Some electromagnetic properties of the wedge-shaped film have been analyzed[23,24]. In this letter, we analyze the GH shift from a wedge-shaped thin film.

Consider a unit-amplitude and monochromatic plane wave incident at an angle θ on a nonmagnetic wedge-shaped thin film with a small apex angle α (see Fig. 1(a)). The film's thickness at the point of incidence is h and its refractive index is n. Because of multiple reflections at the surfaces, the reflected light consists of a set of plane waves propagating in different directions. If the angle of refraction for the first time is denoted as θ', the pth wave of the reflected set emerges at an angle θ_p where, from the law of refraction and reflection, n sin $\theta_p = n'$sin $[\theta' + 2(p-1)\alpha]$, with n' being the refractive index of the wedge-shaped film, and n being the refractive index of the surrounding medium. The virtual reflected wave-fronts W_1, W_2, $\ldots$, W_p, $\ldots$, which contain the wedge apex, would be cophasal if there is no phase change at reflection. At a point P on the first surface with a distance ρ from the apex O, the difference of optical path of the pth wave and the wave which is directly reflected (p=1) is therefore $\Delta S_p^0 = n(PN_p - PN_1) = n\rho(\sin\theta_p - \sin\theta_1)$, where N_p and N_1 are the feet of perpendicular from P to W_p and W_1, respectively. In terms of the geometric relations, the difference of optical path between two successively-reflected rays is $\Delta s = n'(AB + BC) - nCD$ (see Fig. 1(b)). Thus, the total phase difference δ_p between the pth wave and the directly reflected wave is

$$\delta_p = \frac{2\pi}{\lambda}[\Delta S_p + (p-1)\Delta s] = \frac{4\pi}{\lambda}n'h\cos\theta'\left\{\frac{\sin(p-1)\alpha}{\tan\alpha}\{\cos(p-1)\alpha - \tan\theta'\sin(p-1)\alpha\} + (p-1)\beta\right\}, \quad (1)$$

where λ is the wavelength in vacuum, h (= ρtan α) is the thickness of the film at P, θ' is determined from the law of refraction $n\sin\theta = n'\sin\theta'$, and

$$\beta = \frac{1+\tan\theta'\tan(\theta'+\alpha)}{2} \times \left[1 + \frac{\cos\theta' - \sin\theta'\sin2(\theta'+\alpha)}{\cos(\theta'+2\alpha)}\right]. \quad (2)$$

Let r be the Fresnel reflection coefficient, t be the Fresnel transmission coefficient, and r', t' be the

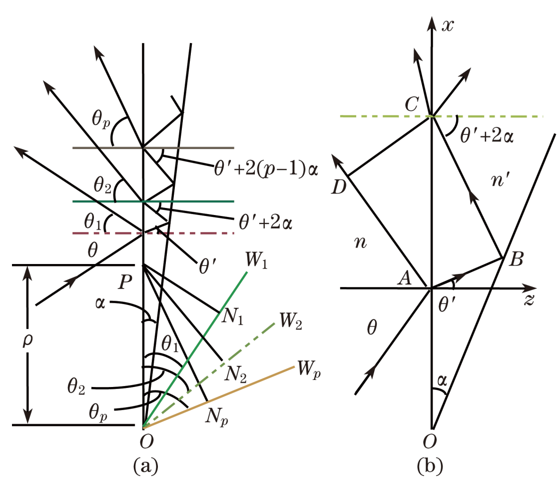

Fig. 1. Illustration for multiple reflections in a wedge.

corresponding coefficients for a wave travelling from the wedge to the surrounding medium. When the apex angle α is sufficiently small, the effect of the apex angle on r, r', t, t' can be neglected, i.e., r, r', t, t' are replaced with the Fresnel coefficients of the corresponding plane-parallel film. Under this circumstance, the complex amplitudes of the waves reflected from the wedge are $r\exp(\mathrm{i}\delta_1)$, $tt'r'\exp(\mathrm{i}\delta_2)$, $tt'r'^3\exp(\mathrm{i}\delta_3), \cdots$, $tt'r'^{(2j-3)}\exp(\mathrm{i}\delta_j), \cdots$, where $j = 2, 3, \cdots$. The amplitude of the electric vector of the reflected wave at P is the superposition of the infinity of such waves, which is expressed as

$$A(\theta) = r + tt' \sum_{p=2}^{M} r'^{(2p-3)} \exp(\mathrm{i}\delta_p). \tag{3}$$

For the incident beam with a sufficiently large beam waist (i.e., the beam has a very narrow angular spectrum), the GH shift of the reflected beam can be calculated analytically by[8,10]

$$\Delta = -(\lambda/2n\pi\cos\theta)\mathrm{d}\phi(\rho,\theta)/\mathrm{d}\theta, \tag{4}$$

where $\phi(\rho,\theta)$ is the phase of $A(\rho,\theta)$, and λ is the wavelength of the wave in the air. Equation (4) was proposed by Artmann using the stationary-phase method[2]. In the following calculations, only the TE-polarized waves are considered. The results for the TM-polarized waves could be obtained similarly.

Figure 2 shows the dependences of the GH shift, Δ, and magnitude of reflection coefficient, $|A|$, on the incident angle θ, where the wedge-shaped thin film is in vacuum and its thickness, apex angle, and refractive index are h=1.8λ, $\alpha = 5°$, and $n' = \sqrt{2}$, respectively. For comparison, the results of the corresponding plane-parallel film ($\alpha = 0°$) are compiled together. Under these conditions of structure parameters, it is found from Fig. 2(a) that all of the GH shifts of the light reflected from the plane-parallel film in vacuum are positive. For a plane-parallel film with arbitrary structure parameters, Li has shown that the longitudinal GH shifts are always positive, except that the incident angle is quite large (for example, $\theta = 80°$)[7]. However, for the wedge-shaped film, negative GH shifts appear in small ranges of incident angle. As θ tends to the resonant condition, the negative GH shift tends to assume its largest values, followed by a decrease in the magnitude of reflection coefficient. In Fig. 2(b), the absolute value of the maximum negative GH shift from the wedge-shaped film is one order of magnitude larger than that of the GH shift from the corresponding plane-parallel film. Meanwhile, the positive GH shift can be enhanced or suppressed under certain conditions. Generally, the larger the GH shift is, the smaller the magnitude of reflection coefficient is. However, although the magnitude of reflection coefficient is small near resonance, large negative GH shifts reflected from the wedge-shaped film are detectable. For example, it can be found from Fig. 2(a) that the magnitude of reflection coefficient of the wedge-shaped film is $|A|$=0.07 at $\theta = 61°$, which is much larger than $|A|$=0.01 of the plane-parallel film.

Figure 3 shows the dependences of the GH shift and magnitude of reflection coefficient on the thickness of the film, where θ=60° and the other parameters are the same as those in Fig. 2. From calculations, we find that the oscillation of the GH shift with respect to h is closely related to the periodical occurrence of transmission resonance approximately at $kn'h\sin\theta = m\pi$ (k is the wave number, $m = 1, 2, 3, \cdots$). At the resonance points, the GH shift is large and negative for the wedge film but is positive for the plane film. We also find that the spatial period of the GH shift is independent of the apex angle of the wedge.

However, the plane wave incidence assumed above is an approximation, since the GH shift is the physical effect of real finite-sized beam rather than plane wave. Now, we calculate the lateral shift of the finite Gaussian beam incident upon the wedge using the angular spectrum method of plane wave. The electric field of the incident

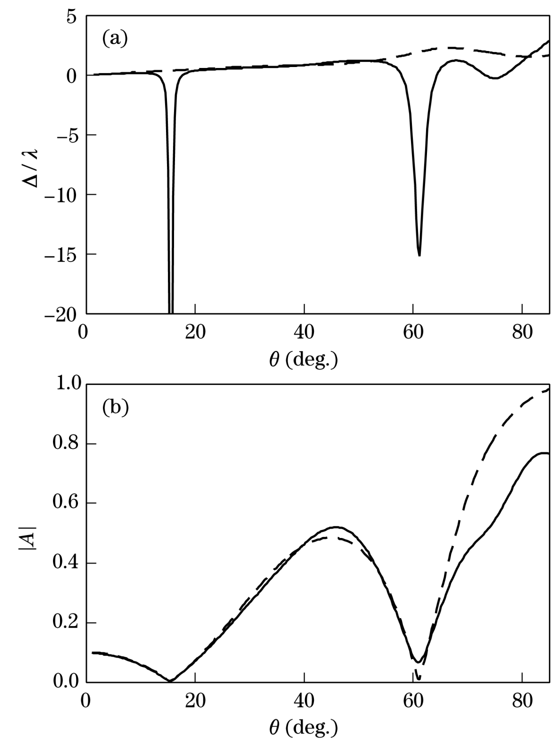

Fig. 2. Dependences of (a) Δ (in units of λ) and (b) $|A|$ on the incident angle θ with h=1.8λ and $n' = \sqrt{2}$. Solid curves: wedge-shaped film ($\alpha = 5°$); dashed curves: plane-parallel film ($\alpha = 0°$).

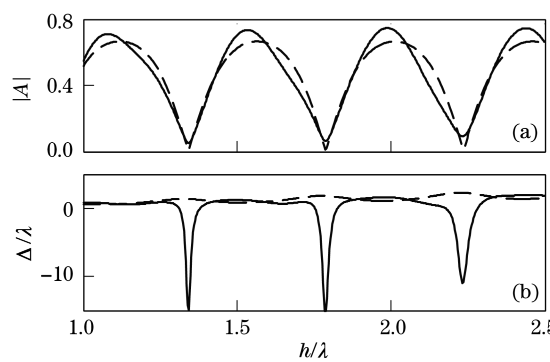

Fig. 3. Dependences of (a) $|A|$ and (b) Δ (in units of λ) on the thickness h of the film with $\theta = 60°$ and $n' = \sqrt{2}$. Solid curves: wedge-shaped film ($\alpha = 5°$); dashed curves: plane-parallel film ($\alpha = 0°$).

beam at the plane of z=0 is given by

$$E_{\mathrm{iy}}(x, z=0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp(\mathrm{i}k_x x)\psi(k_x)\mathrm{d}k_x, \quad (5)$$

where E_{iy} denotes the incident electric field in the y-polarized TE mode, and

$$\psi(k_x) = \frac{w_{x0}}{\sqrt{2}} \exp\left[-\frac{w_{x0}^2(k_x - k_{x0})}{4}^2\right] \quad (6)$$

is the angular spectrum of the Gaussian beam with the incident angle θ, $k_{x0} = k_0 \sin\theta$, $w_{x0} = w \sec\theta$, w is the radius of the beam waist. Then the electric field in the reflected plane can be written as

$$E_y(x, z=0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} [1 + A(k_x)]\exp(\mathrm{i}k_x x)\psi(k_x)\mathrm{d}k_x, \quad (7)$$

where $A(k_x)$ is the reflection coefficient of the wedge after the multiple-reflection effect is considered, which can be obtained from Eq. (3). From the view point of angular spectrum, the Artmann's formula (Eq. (4)) calculates only the GH shift for a given k_x, which is valid for an incident plane wave. However, k_x conversion has to be considered for an incident Gaussian beam. The peak position difference between the incident and reflected beams is denoted by the lateral shift Δ. In the following calculations, we take $w = 12\lambda > \lambda$ (the incident beam is well collimated and $\psi(k_x)$ is sharply distributed around k_{x0}), and hence, it is expected that the reflected beam is still the Gaussian beam without obvious distortion.

Figures 4 and 5 show the dependences of the lateral shift Δ on the incident angle θ when$h$ =4λ and on the thickness h of the wedge when θ=60°, respectively, where the incident beam is the Gaussian beam with $w = 12\lambda$. From Fig. 4, it is found that in order to obtain large negative GH shift, a large resonance incident angle is chosen to be $\theta = 70.2°$. From Fig. 5, it is found that the GH shift presents a periodical oscillation with the film thickness, and the spatial period is independent of the apex angle of the wedge, but the positions of peaks slightly shift towards the edge of the wedge. The GH shift is large and negative at the resonance points of transmission. These behaviors are similar to those for the plane wave incident on the wedge. However, under the condition of the same apex angle of the wedge, the maximum GH shift generated from the Gaussian beam incidence is smaller than that generated from the plane wave incidence. This is due to the divergence characteristic of Gaussian beams.

It is seen from the above calculation results that a wedge-shaped film can generate large negative GH shift around the resonance points. Our explanation for this is as follows. From Eq. (3), we can obtain the dispersion relation of a wedge-shaped structure numerically. When the dispersion relation is satisfied, the reflection coefficient presents a pole. A leaky mode corresponds to a pole of the reflection coefficient[24]. We find that the phase of $A(\theta)$ in a wedge varies near the poles more quickly than the case in a parallel film and the varying direction is opposite. Thus a large negative GH shift can be obtained from a wedge-shaped film.

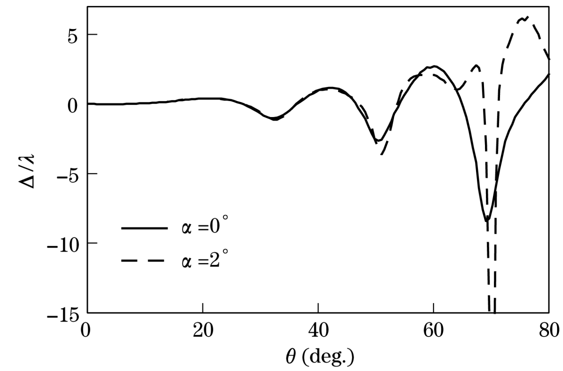

Fig. 4. Dependence of the lateral shift Δ (in units of λ) on the incident angle θ with h=4λ for two different values of α. The refractive index of the wedge in air is $n' = \sqrt{2}$ and the waist of the Gaussian beam is $w = 12\lambda$.

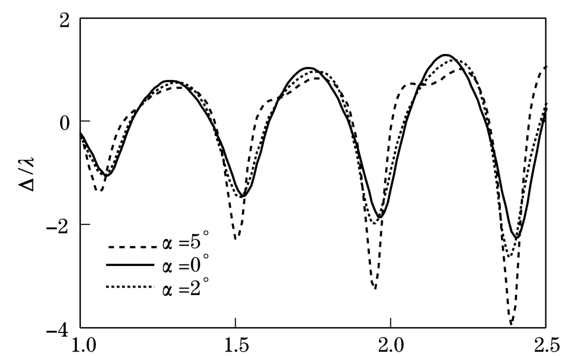

Fig. 5. Dependence of the lateral shift Δ (in units of λ) on the thickness h of the film with $\theta = 60°$ for three different values of α.

In conclusion, the analytical expression for the reflection coefficient from a wedge-shaped thin film is presented and the GH shift of the reflected beam is calculated. For plane wave the ray tracing approach is used, and for non-plane wave the angle spectrum approach is used to calculate the reflected field. The numerical results based on the two different methods show that the light reflected from a wedge-shaped thin film can produce large negative GH shifts near the resonant condition. Although the reflectivity is small near resonance, the large negative GH shift from the wedge-shaped film can be more easily detected in comparison with the case from the parallel plane film.

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