

# A novel accelerometer based on microring resonator

Wenqin Mo (莫文琴)<sup>1,2</sup>, Huaming Wu (吴华明)<sup>1</sup>, Dingshan Gao (郜定山)<sup>1</sup>,  
and Zhiping Zhou (周治平)<sup>3,4\*</sup>

<sup>1</sup>Wuhan National Laboratory for Optoelectronics, College of Optoelectronic Science and Engineering,  
Huazhong University of Science and Technology, Wuhan 430074, China

<sup>2</sup>Faculty of Mechanical and Electronic Information,  
China University of Geosciences, Wuhan 430074, China

<sup>3</sup>State Key Laboratory on Advanced Optical Communication Systems and Networks,  
Peking University, Beijing 100871, China

<sup>4</sup>School of Electrical and Computer Engineering, Georgia Institute of Technology,  
Atlanta, Georgia 30332, USA

\*E-mail: zjzhou@pku.edu.cn

Received December 16, 2008

A novel microring resonator accelerometer is proposed. It is realized by a suspended straight waveguide coupled with a microring resonator. Under the external acceleration, the coupling coefficient is a function of gap spacing between the two waveguides. The mathematical model of the sensing element is established. Both the finite element method and coupled mode theory are adopted to analyze and optimize the proposed structure. Simulation results show that the mechanical sensitivity is  $0.015 \mu\text{m}/g$  with the working frequency below 500 Hz and cross-axis sensitivity less than 0.001%, which is promising in seismic related applications.

OCIS codes: 130.6010, 230.5750, 280.4798.

doi: 10.3788/COL20090709.0798.

Accelerometer for vibration measurement plays a critical role in the seismic prospecting<sup>[1]</sup>. Various accelerometers based on different techniques and principles have been designed, which can be classified as electrical and optical accelerometers. The electrical accelerometers such as capacitive and piezoresistive ones have inertial limitations from electromagnetic interference. With the advantages of light weight, passivity, high sensitivity, and environmental ruggedness, the optical methods based on optic fiber, interferometer, or silicon microstructures have been reported<sup>[2-7]</sup>. Although these systems have good performances, optical fiber and Mach-Zehnder interferometer (MZI) cannot suffer the vibration directly. Also, the hybrid-integrated method makes the fabrication more complicated. Besides, the frequency information below 10 Hz is valuable for identification of signal source in oil and gas seismic prospecting applications<sup>[8]</sup>.

Recently, microring resonator has been proposed for sensing applications. The resonance effect can provide effective long interaction length to achieve a sufficient phase shift. Such a property can significantly reduce the amount of samples needed for detection. Therefore, microring resonator is more sensitive compared with straight waveguide sensors<sup>[9,10]</sup>.

In this letter, we propose an optical-intensity-modulated accelerometer composed by suspended straight waveguide and microring resonator. The sensor configuration is described followed by the principle of coupling effect between the straight waveguide and microring. When suffering the external acceleration, the modulated optical intensity is analyzed as a function of the gap between them. The structural and optical parameters for high performance are analyzed. The optimization process is performed by using the finite el-

ement method (FEM) and coupled mode theory. The working frequency of the novel accelerometer is designed below 500 Hz, which is enough in seismic prospecting applications<sup>[11]</sup>.

The coupling coefficient is very sensitive to the gap changes between the straight waveguide and microring<sup>[12]</sup>. The mechanism of a microring resonator accelerometer designed for seismic prospecting is depicted in Fig. 1. The accelerometer consists of a suspended straight waveguide acting as a vibration membrane and a microring waveguide. The waveguides are both made of  $\text{Si}_3\text{N}_4$ . Air and  $\text{SiO}_2$  constitute the top and bottom claddings of the microring waveguide. As it is subject to acceleration, the straight waveguide makes corresponding movement by the inertial force applied on it. Therefore, the acceleration is converted to the gap change between the straight waveguide and microring. The coupling coefficient  $k$  for the two waveguides is determined by the major physical parameters: gap spacing  $d$ , ring radius  $R$ , input wavelength  $\lambda_0$ , and refractive index  $n$ . When the others are kept constant,  $k$  is a function of the gap spacing. So the external vibration will modulate the input optical power by the gap changes.

The equation of motion for the suspended waveguide subject to the acceleration force  $f$  is written as

$$M\ddot{\delta} + C\dot{\delta} + K\delta = -f, \quad (1)$$

where  $\delta$ ,  $\dot{\delta}$ ,  $\ddot{\delta}$  are the  $x$ -direction displacement, velocity, and acceleration of the membrane, respectively;  $M$  is the global mass matrix and  $K$  is the global stiffness matrix;  $C$  is the damp coefficient determined by the Rayleigh damping model. The numerical results for  $\delta$  are easy to obtain through FEM.

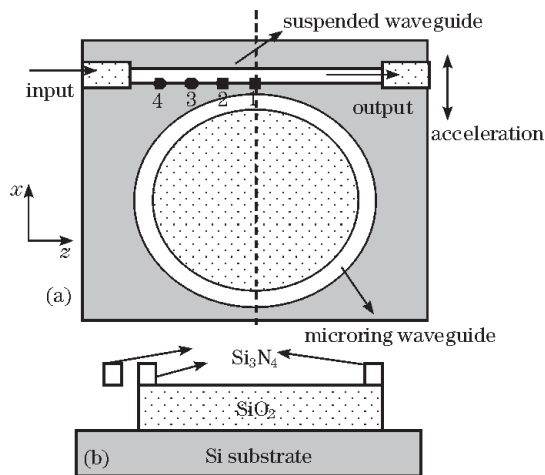


Fig. 1. Structure of a microring resonator accelerometer. (a) Top view; (b) cross section at the dotted line in (a).

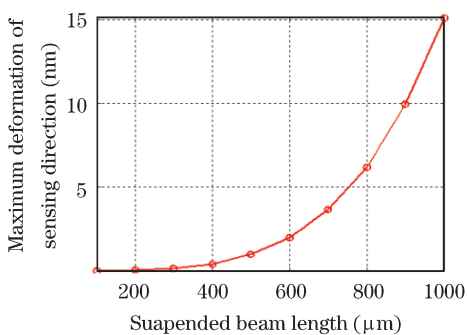


Fig. 2. Maximum deflection of the suspended waveguide versus beam length.

In a practical situation, it is usually desirable to obtain the optimal parameters to maximize the sensitivity of the accelerometer within a large frequency range. To analyze the dynamic response of the accelerometer, FEM is conducted to the model based on different dimensions and accelerations. The waveguide width  $w$  and thickness  $h$  are selected as  $0.5 \mu\text{m}$  to guarantee the single-mode propagation. Then the length  $l$  is determined by analyzing the deformation of the waveguide under an acceleration of  $1g$  ( $9.8 \text{ m/s}^2$ ).

Figure 2 shows the  $x$ -direction deformation of the suspended waveguide with the length varying from 100 to  $1000 \mu\text{m}$ . It is obvious that the larger length-to-width ratio can increase the amplitude of the beam vibration, resulting in relatively high sensitivity. Based on the theory of vibrational mechanics, the deformation of the waveguide can be expressed as

$$\delta = \frac{4Pl^3}{Ewh^3}, \quad (2)$$

where  $E$  is the Young's modulus of the waveguide and  $P$  is an external inertial force induced by acceleration. However, the natural frequency of the accelerometer is defined as

$$\omega_n = \sqrt{\frac{Ewh^3}{4l^3m_e}}, \quad (3)$$

where  $m_e$  is the equivalent mass index. In other words, with a larger length, a higher sensitivity can be ob-

tained but with a lower natural frequency of the structure. Therefore, suitable length is very significant to improve the vibration sensitivity within enough working frequency range.

In the structure depicted in Fig. 1, the coupling coefficient is a very important parameter, which transfers the vibration of the suspended waveguide to the change of output optical power. Considering the TE mode of the equivalent slab in  $x$ - $z$  plane, the expression of coupling coefficient  $k$  is given by<sup>[13,14]</sup>

$$k = K(2s_0) \times \int_{-\infty}^{\infty} \exp \left[ -\frac{\alpha_2}{2R} z^2 - j \left( \beta_1 z - \beta_2 z + \frac{z^3}{2R^2} \beta_2 \right) \right] dz, \quad (4)$$

where  $K(2s_0)$  is the coupling coefficient for two parallel waveguides which are separated by a distance  $2s_0 = d + w$ ;  $\beta_1$  and  $\beta_2$  are the  $x$ - $z$  components of propagation constants inside the straight and microring waveguides, respectively; the transverse decay constant  $\alpha_2$  is given by  $\alpha_2 = \sqrt{\beta_2^2 - n_0^2 k_0^2}$ , where  $n_0$  and  $k_0$  are the substrate refractive index and the free-space wave vector, respectively. By use of this equation, we are able to analyze the physical parameters affecting the coupling coefficient: microring radius, refractive index, and gap spacing.

Figures 3 and 4 show the dependences of coupling coefficient on the ring radius and refractive index, respectively. Firstly, at given gap spacing and refractive index, the coupling coefficient increases with the ring radius increasing because of the increased effective coupling length. Secondly, at fixed ring radius and gap spacing, the coupling coefficient decreases when the refractive index increases, resulting in the strong confined optical field.

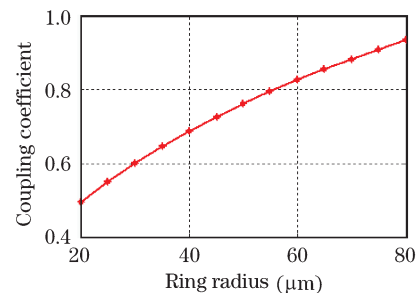


Fig. 3. Coupling coefficient as a function of ring radius. Gap spacing  $d=0.1 \mu\text{m}$ , refractive indices of waveguides  $n_1=n_2=2.0$ .

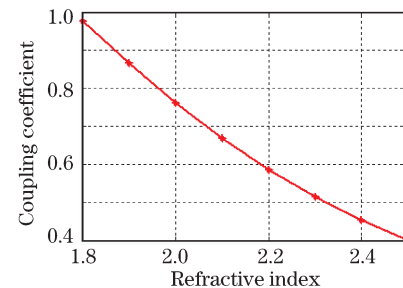


Fig. 4. Coupling coefficient as a function of refractive index of waveguides. Ring radius  $R=50 \mu\text{m}$ ,  $d=0.1 \mu\text{m}$ .

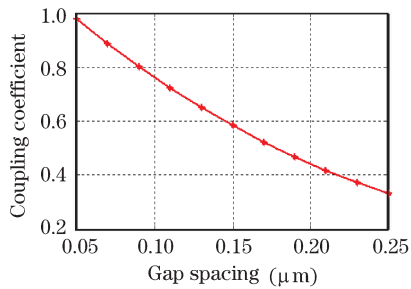


Fig. 5. Coupling coefficient as a function of gap spacing.  $R=50 \mu\text{m}$ ,  $n_1=n_2=2.0$ .

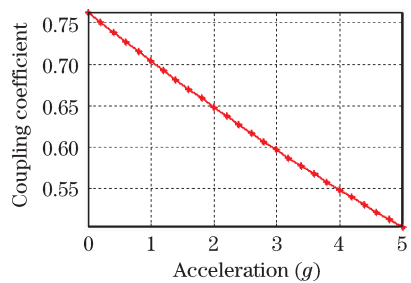


Fig. 6. Acceleration versus coupling coefficient.  $R=50 \mu\text{m}$ , width  $w=0.5 \mu\text{m}$ ,  $n_1=n_2=2.0$ .

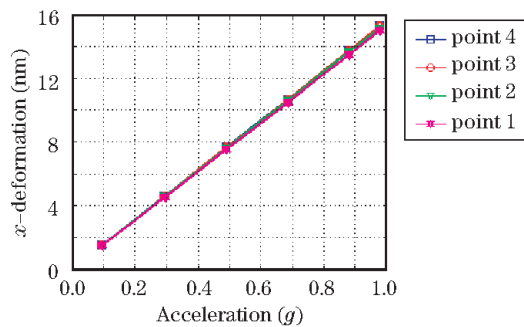


Fig. 7. Change of  $x$ -deformation with acceleration.

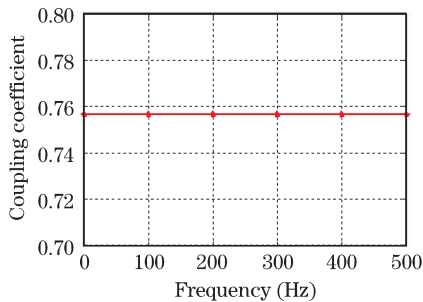


Fig. 8. Coupling coefficient versus vibration frequency. Acceleration  $a=0.1g$ .

Therefore, we consider a  $50\text{-}\mu\text{m}$ -diameter microring coupled to a straight waveguide, both with the refractive index of 2.0. Next we focus on the gap spacing between two waveguides.

Figure 5 shows that the coupling coefficient decreases by almost 64% with the gap increasing from 0.05 to 0.25  $\mu\text{m}$ . We choose 0.1  $\mu\text{m}$  as the initial gap spacing between the two waveguides to ensure enough optical power coupled into the microring.

Under the condition of acceleration  $a$  from 0 to 5g along

negative  $x$ -axis, the coupling coefficient varies with the acceleration, as shown in Fig. 6. The sensitivity of the accelerometer,  $s$ , can be calculated via  $S = \frac{\Delta p}{a}$ , which is decided by the input optical power and power change  $\Delta p$  at the output of the straight waveguide.

The suspended waveguide is a continuous system. Under the harmonic vibration, every element of the suspended waveguide suffers different deformation. Figure 7 describes the  $x$ -deformation of the middle part at the suspended waveguide. The lines in the figure present the deformations of the points 1–4 belonging to the waveguide (see Fig. 1), respectively. It is obvious that the deformations are almost uniform within the 50- $\mu\text{m}$  range. That is to say, the  $x$ -deformations of the straight waveguide can be taken as the same in the coupling region, because the differences of  $x$ -deformation among the points are very small compared with the  $x$ -deformation value. Moreover, the  $z$ -deformation is almost less than 0.001% of the  $x$ -deformation. Therefore, the cross-axis sensitivity can be ignored.

Frequency response is an important performance of an accelerometer. If the frequency of the external acceleration is the same with the natural frequency of the structure, the accelerometer will be destroyed as a result of resonance. Therefore, the maximum working frequency should be lower than the first natural frequency, which is given by

$$\omega_1 = \frac{9}{4}\pi^2 \sqrt{\frac{Eh^2}{12\rho l^4}}, \quad (5)$$

where  $\rho$  is described as mass density. We can get the first natural frequency of 4605 Hz via numerical calculation.

As the acceleration frequency increases from 0 Hz to the first natural frequency, the deformation of the suspended waveguide decreases, resulting in the change of coupling coefficient. An ideal accelerometer is expected to have a flat frequency response and allow undistorted measurement of the acceleration at all frequencies. Figure 8 shows that the coupling coefficient varies with the acceleration frequency for  $a=0.1g$ . The calculated coupling coefficient obtained through Eqs. (1) and (4) changes by only 0.013% in the range of 0–500 Hz. Therefore, it can be approximately considered as a constant.

In conclusion, we present the design and analysis of a novel accelerometer based on the microring resonator. The coupling coefficient related to the acceleration is analyzed as a function of the gap spacing between the two waveguides. The structural parameters, such as ring radius, refractive index, and initial gap spacing, are optimized to get better coupling coefficient. The simulation results show that the mechanical sensitivity is 0.015  $\mu\text{m}/g$  and cross-axis sensitivity is less than 0.001%. The coupling coefficient changes only 0.013% below 500 Hz, which is well above the working frequency for seismic prospecting applications.

This work was partially supported by the National Natural Science Foundation of China under Grant No. 60578048.

## References

1. A. Mita and I. Yokoi, Proc. SPIE **4330**, 479 (2001).

2. W. Zhang, L. Liu, F. Li, and Y. Liu, *Chin. Opt. Lett.* **5**, 507 (2007).
3. H. Y. Au, S. K. Khijwania, and H. Y. Tam, *Proc. SPIE* **7004**, 70042S (2008).
4. B. Wu, C. Chen, G. Ding, D. Zhang, and Y. Cui, *Opt. Eng.* **43**, 313 (2004).
5. W. Zhang, Y. Liu, and F. Li, *Chin. Opt. Lett.* **6**, 631 (2008).
6. J. Nayak, T. Srinivas, A. Selvarajan, and D. V. K. Sastry, *J. Microlith. Microfab. Microsyst.* **5**, 043012 (2006).
7. E. Ollier, P. Philippe, C. Chabrol, and P. Mottier, *J. Lightwave Technol.* **17**, 26 (1999).
8. K. E. Speller and D. Yu **5417**, 63 (2004).
9. C.-Y. Chao and L. J. Guo, *J. Lightwave Technol.* **24**, 1395 (2006).
10. Z. Xia, Y. Chen, and Z. Zhou, *IEEE J. Quantum Electron.* **44**, 100 (2008).
11. Y. Zhang, (ed.) *Theory and Data Solutions in Reflection Seismic Exploration* (in Chinese) (Geological Publishing House, Beijing, 2007).
12. S. C. Hagness, D. Rafizadeh, S. T. Ho, and A. Taflove, *J. Lightwave Technol.* **15**, 2154 (1997).
13. Y. Liu, T. Chang, and A. E. Craig, *Opt. Eng.* **44**, 084601 (2005).
14. K. Okamoto, *Fundamentals of Optical Waveguides* (Academic Press, San Diego, 2000).