Theoretical and experimental investigation of the mode-spacing of fiber Bragg grating Fabry-Perot cavity

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The mode-spacing of the fiber Bragg grating Fabry-Perot (FBG F-P) cavity is calculated by using the effective cavity length which contains the effective length of the FBG. The expression of the effective length, defined by using the phase-time delay, is obtained and simplified as a function of the peak reflectivity at the Bragg wavelength, the band edges, and the first zero-reflectivity wavelength. The effective length is discussed from the energy penetration depth point of view. Three FBG F-P cavities are fabricated in order to validate the effective length approach. The experimental data fits well with the theoretical predictions. The limitation of this method is also pointed out and the improved approach is proposed.

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Fiber Bragg grating Fabry-Perot (FBG F-P) cavities have been used as mode selecting devices in the linear or ring cavity fiber lasers[1−4], of which the mode-spacing is a key parameter. In such configurations, the total length of the FBG F-P cavity should be short enough in order to obtain a large mode-spacing, thereby achieving a single-longitudinal-mode laser. So the length of the FBG is usually of the same order of magnitude with the cavity length, and the effect of the FBG length has to be considered. Normally the mode-spacing can be calculated by using the effective cavity length, which contains the effective length of the FBG. The effective length can be defined by using the phase-time delay of the FBG[5], or using the approximate linearity of the phase curve of the reflection coefficient[6].

In this letter, we focus on the mode-spacing of the FBG F-P cavity calculated by the effective cavity length. The effective length of the FBG is defined by the phase-time delay, which has been brought forward and discussed in Ref. [5]. The effective length approach for calculating mode-spacing of the FBG F-P cavity is given. The analytical expression of the effective length is obtained. The effective length is discussed from the energy penetration depth point of view. The experiment is carried out to verify the effective length approach for calculating the mode-spacing of FBG F-P cavity.

Figure 1 shows the schematic of a FBG F-P cavity formed by two uniform FBGs, i.e., FBG1 and FBG2. The lengths of FBG1 and FBG2 and their spatial interval are L_{g1}, L_{g2}, and L, respectively. Supposing the amplitude reflection coefficients of FBG1 and FBG2 are $r_1 = |r_1|\exp(\mathrm{i}\varphi_1)$ and $r_2 = |r_2|\exp(\mathrm{i}\varphi_2)$, the mode-spacing of the FBG F-P cavity satisfies[7]

$$4\pi\bar{n}L\frac{|\Delta\lambda|}{\lambda^2} + |\varphi_1(\lambda) - \varphi_1(\lambda+\Delta\lambda)| + |\varphi_2(\lambda) - \varphi_2(\lambda+\Delta\lambda)| = 2\pi, \quad (1)$$

where $\bar{n}$ is the effective refractive index of the FBG region, $\Delta\lambda$ is the mode-spacing between two adjacent resonant peaks.

By using the phase-time delay which is defined as[8]

$$\tau(\lambda) = -\frac{\lambda^2}{2\pi c}\frac{\mathrm{d}\varphi_r}{\mathrm{d}\lambda}, \quad (2)$$

where c is the speed of light in vacuum and φ_{r} is the phase of the amplitude reflection coefficient, and Eq. (1) can be expressed as

$$|\Delta\lambda| = \frac{\lambda^2}{2\bar{n}(L + L_{\mathrm{eff\ a1}} + L_{\mathrm{eff\ a2}})} = \frac{\lambda^2}{2\bar{n}L_{\mathrm{FBG\,F-P}}}, \quad (3)$$

where $L_{\mathrm{FBG\,F-P}}$ is the effective cavity length of the FBG F-P cavity and

$$L_{\mathrm{eff\ a1,2}} = \left|\frac{1}{\Delta\lambda}\int_{\lambda}^{\lambda+\Delta\lambda} L_{\mathrm{eff\ 1,2}}(\lambda)\mathrm{d}\lambda\right| \quad (4)$$

is defined as the average effective length of the corresponding FBG for the wavelength range from λ to $\lambda+\Delta\lambda$.

$$L_{\mathrm{eff\ 1,2}}(\lambda) = \frac{c\tau_{1,2}(\lambda)}{2\bar{n}} \quad (5)$$

is defined as the effective length at the wavelength λ.

The phase-time delay can be shown to be [9]

$$\tau = \frac{\mathrm{d}\varphi_{\mathrm{r}}}{\mathrm{d}\omega} = \frac{\bar{n}L_{\mathrm{g}}}{c}\frac{1}{\gamma_{\mathrm{g}}^2 + \hat{\sigma}^2\tanh^2(\gamma_{\mathrm{g}}L_{\mathrm{g}})} \cdot\left[\hat{\sigma}^2\tanh^2(\gamma_{\mathrm{g}}L_{\mathrm{g}}) + \frac{\kappa^2}{\gamma_{\mathrm{g}}L_{\mathrm{g}}}\tanh(\gamma_{\mathrm{g}}L_{\mathrm{g}}) - \hat{\sigma}^2\right], \quad (6)$$

where L_{g} is the grating length, $\hat{\sigma}$ is the general "direct current" (DC, period averaged) self-coupling coefficient,

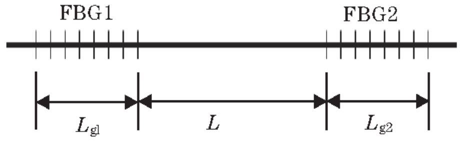

Fig. 1. Schematic of a FBG F-P cavity.

κ is the "alternating current" (AC) coupling coefficient, $\gamma_g = \sqrt{\kappa^2 - \hat{\sigma}^2}$. So the effective length can be obtained as

$$L_{\text{eff}}(\lambda) = \frac{L_g}{2} \frac{1}{\gamma_g^2 + \hat{\sigma}^2 \tanh^2(\gamma_g L_g)} \cdot \left[\hat{\sigma}^2 \tanh^2(\gamma_g L_g) + \frac{\kappa^2}{\gamma_g L_g} \tanh(\gamma_g L_g) - \hat{\sigma}^2 \right]. \quad (7)$$

The effective length can be simplified as

$$L_{\text{eff}}(\lambda_B) = \frac{\sqrt{R_B}}{2\text{atanh}\left(\sqrt{R_B}\right)} L_g, \quad (8)$$

$$L_{\text{eff}}(\lambda_{b-e}) = \frac{1 + \frac{2}{3}\text{atanh}^2\left(\sqrt{R_B}\right)}{2\left[1 + \text{atanh}^2\left(\sqrt{R_B}\right)\right]} L_g, \quad (9)$$

and

$$L_{\text{eff}}(\lambda_0) = \left[\frac{\text{atanh}^2(\sqrt{R_B})}{2\pi^2} + \frac{1}{2} \right] L_g \quad (10)$$

at the Bragg wavelength, the band edges, and the first zero-reflectivity wavelength, respectively. In fact, Eq. (8) has been obtained in Ref. [5]. Here the AC coupling coefficient κ is considered as a constant across the grating wavelength and $R_B = \tanh^2(\kappa L_g)$ is the peak reflectivity of the FBG.

Figure 2 shows the three effective lengths of a FBG versus its peak reflectivity. When the peak reflectivity is zero, the effective length is half of the physical length of the FBG. The effective length decreases to $1/(2k)$ with the increasing peak reflectivity at the Bragg wavelength, decreases to $L_g/3$ with the increasing peak reflectivity at the band edges, and increases to more than L_g with the increasing peak reflectivity at the first zero-reflectivity wavelength.

The effective length of a FBG also gives the approximate optical energy penetration depth into the grating[10]. The optical power is the square of the optical field, so the effective length starts from half of the physical length. For the light inside the stop band, the reflection is primary and the light cannot penetrate deep into the FBG, so the effective length is less than half of the physical length and decreases with the increasing peak reflectivity. The minimum effective length appears at the Bragg wavelength where the peak reflectivity appears. However, for the light near the first zero-reflected wavelength, the effective length is more than half of the physical length and increases with the peak reflectivity, because the transmission is a resonant effect which involves many round trips inside the grating at these wavelengths[11].

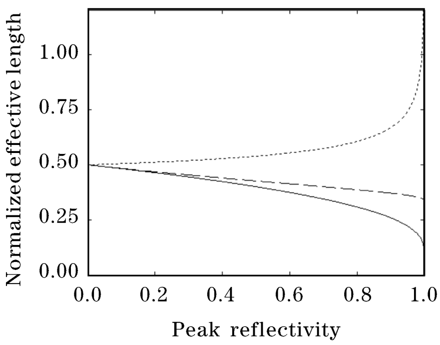

Fig. 2. Effective lengths of the FBG at the Bragg wavelength (solid curve), the band edges (dashed curve), and the first zero-reflectivity wavelength (dotted curve).

In order to investigate the mode-spacing of the FBG F-P cavity, three FBG F-P cavities were fabricated. The FBG F-P cavity was composed of two equal uniform 1-cm-long FBGs which were written in the hydrogen-loaded Corning SMF-28 fiber by using the KrF excimer laser operated at 240 nm and had a spatial interval of 1 cm. The positions and lengths of the FBGs are controlled by a high-precision translation stage. An optical spectrum analyzer (OSA, ANDO AQ6317C) with a resolution of 0.01 nm was used to monitor the transmission spectra when the FBG F-P cavity was fabricated.

Figure 3 shows the spectra of the three FBG F-P cavities, which are named as A, B, and C. The dashed curves show the transmission spectra of the corresponding FBGs. The experimental results (ER) and corresponding analytical results (AR) by using Eqs. (8) and (9) are summarized in Table 1. The AR1 are calculated by replacing $L_{\text{eff a}}$ with $L_{\text{eff}}(\lambda_B)$ and AR2 are calculated by replacing $L_{\text{eff a}}$ with $L_{\text{eff}}(\lambda_{b-e})$. AR3 are the average values of AR1 and AR2.

As can be seen from Table 1, with the increase of peak reflectivity, all of the results increase and the difference among them becomes larger and larger. The AR1 calculated by using $L_{\text{eff}}(\lambda_B)$ are larger than the ER. This is because that the effective length at the Bragg wavelength is the smallest one for a FBG, by using which we will get the largest mode-spacing. Also we can see that the AR2 calculated by using $L_{\text{eff}}(\lambda_{b-e})$ are normally smaller than the ER, especially when the peak reflectivity is high. The AR3 always correspond well with the ER.

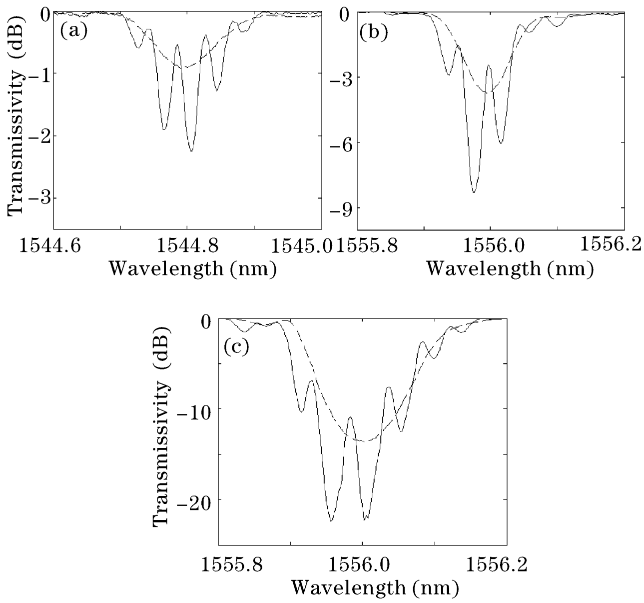

Fig. 3. Experimental transmission spectra of the FBG F-P cavity composed of two equal uniform 1-cm-long FBGs with a spatial interval of 1 cm. The dashed curves show the transmission spectra of the corresponding FBGs, of which the peak reflectivities are (a) 20.57%, (b) 59.73%, (c) 95.75%.

Table 1. Comparison of Experimental Result ER and Analytical Results AR1−AR3

	R_B(%)	Mode Spacing (nm)			
		AR1	AR2	AR3	ER
A	20.57	0.0427	0.0425	0.0426	0.042
B	59.73	0.0476	0.0457	0.0466	0.046
C	95.75	0.0583	0.0485	0.0534	0.054

Usually, the mode-spacing of the FBG F-P cavity should be large enough in order to achieve a single-longitudinal-mode laser. So it will be better if we take the average value of the two ARs, i.e., the ARs calculated by using the effective lengths of the FBG at the Bragg wavelength and band edges, as the prediction value.

In conclusion, we have shown both the theoretical formulation and experimental results of the mode-spacing of the FBG F-P cavity by using an effective cavity length which contains the effective length of the FBG. The expression of the effective length defined by using the phase-time delay is obtained and simplified as a function of the peak reflectivity at the Bragg wavelength, the band edges, and the first zero-reflectivity wavelength. The effective length is discussed from the energy penetration depth point of view. Three FBG F-P cavities formed by two equal uniform FBGs with different peak reflectivities are fabricated to validate the effective length approach. It is better to take the average value of the two ARs calculated by using the effective lengths of the FBG at the Bragg wavelength and band edges as the prediction value in practice.

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