New method for calibration of sun photometers

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A new method for calibration of sun photometers based on Bouguer-Beer law is proposed. The developed basic equation of calibration makes it possible to formulate the derivative methods of calibration on the basis of photometric measurements upon optical air masses, the ratio of which is an integer number.

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It is obvious that the development of new space and ground remote sensing devices leads to the increase of requirements for more perfect calibration of these sensors. One can state that till now the method of Langley diagrams remains as a basic method for the calibration of ground sets of sun photometers[1−4]. This method is based on Bouguer-Beer law, according to which the intensity of solar radiation at the input of photometer $I(\lambda)$ may be determined as

$$I(\lambda) = I_0(\lambda)\mathrm{e}^{-m\tau_{\mathrm{atm}}}, \tag{1}$$

where λ is the wavelength, $I_0(\lambda)$ is the solar constant, i.e., the intensity of sun radiation at the upper border of atmosphere; m is the optical air mass; τ_{atm} is the optical thickness of atmosphere, which is defined in ultraviolet (UV) band as

$$\tau_{\mathrm{atm}} = \tau_{\mathrm{oz}} + \tau_{\mathrm{Ray}} + \tau_{\mathrm{aer}}, \tag{2}$$

where τ_{oz}, τ_{Ray}, and τ_{aer} are optical thicknesses of atmospheric ozone, Rayleigh scattering, and aerosol, respectively.

According to the method of Langley, Eq. (1) should be transformed to

$$\ln I(\lambda) = \ln I_0(\lambda) - m\tau_{\mathrm{a_{tm}}}. \tag{3}$$

Then we can draw the linear diagram of dependence of $\ln I(\lambda)$ on m, as shown in Fig. 1.

The common rule for drawing Langley diagrams consists of the following steps.

1) Calculation of $\ln I(\lambda)$ for two values of m, i.e., m_1 and m_2, which correspond to appropriate angles of observation of the Sun, θ_1 and θ_2.

2) Drawing the graphic model of function $\ln I(\lambda) = f(m)$ using two values of m.

3) The linear type graphics of aforesaid function is extrapolated as far as $m = 0$, where this line crosses the ordinate axis.

4) Taking into consideration the linearity of "input-output" functional dependence of the photometer. It should be assumed that the output signal is proportional to $I_0(\lambda)$.

The main shortage of the Langley diagram method is that temporal variations of optical depth of atmosphere may lead to mistakes in the calibration of photometers. There are some modifications of this method. For example, it is proposed to use the formula[5]

$$\frac{\ln I}{m} = \frac{\ln I_0}{m} - \tau,$$

i.e., to draw similar diagrams of

$$\frac{\ln I}{m} = f\left(\frac{1}{m}\right).$$

It is stated that in the case of "short" diagrams ($m_{\max} = 3$), the Langley method and the alternative method proposed in Ref. [2] are quivalent, but in the case of "long" diagrams ($m_{\max} = 8$), the difference between these two methods becomes more significant. When the high-frequency atmospheric fluctuation occurs, the alternative method is better than the former one; but if the low-frequency atmospheric fluctuation presents, the classic Langley method is preferable.

The interesting idea suggested in Ref. [6] is that similar diagrams for three-wavelength photometer should have an argument not for the optical air mass, but for the adjustable combination of optical thicknesses in three wavelengths. Adjusting the value of this combination as far as zero, one can reach the wanted combination of solar constants in three wavelengths. But this idea is significant only for multi-wavelength methods of sun photometry.

We describe a new calibration method for sun photometers which is also based on Bouguer-Beer Law. It is assumed that the photometric ground measurements are carried out at the wavelength λ by air mass m. In this case, the intensity of solar radiation at the input of photometer may be determined as

$$I\,(\lambda,\, m) \,=\, I_0\,(\lambda)\; \mathrm{e}^{-m\tau}, \tag{4}$$

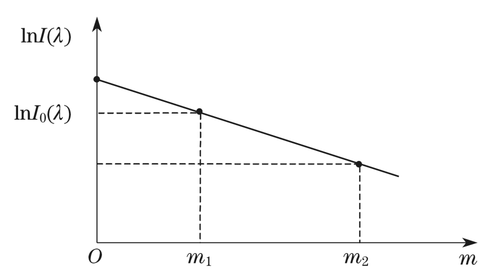

Fig. 1. Langley diagram for calibration of sun photometers.

which can also be written as

$$I^k(\lambda, m) = I_0^k(\lambda)\, \mathrm{e}^{-k m \tau}. \tag{5}$$

Introducing the new value of air mass $m_1 = km$, we have

$$I^k(\lambda, m) = I_0^k(\lambda)\, \mathrm{e}^{-m_1 \tau}. \tag{6}$$

Now we assume that the photometric measurements by air mass m_1 are carried out, i.e.,

$$I(\lambda, m_1) = I_0(\lambda)\, \mathrm{e}^{-m_1 \tau}. \tag{7}$$

From Eq. (6) we have

$$\mathrm{e}^{-m_1\tau} = \frac{I^k(\lambda, m)}{I_0^k(\lambda)}. \tag{8}$$

From Eq. (7) we have

$$\mathrm{e}^{-m_1\tau} = \frac{I(\lambda, m_1)}{I_0(\lambda)}. \tag{9}$$

Comparing Eqs. (8) and (9), we can get

$$\frac{I^k(\lambda, m)}{I_0^k(\lambda)} = \frac{I(\lambda, m_1)}{I_0(\lambda)}. \tag{10}$$

Then

$$I_0(\lambda) = \sqrt[k-1]{\frac{I^k(\lambda, m)}{I(\lambda, m)}} \tag{11}$$

or

$$I^k(\lambda, m) = I_0^{k-1}(\lambda) \cdot I(\lambda, k m). \tag{12}$$

Equation (12) is the basis of the suggested calibration method. Using this equation, we can deduce the following formulas for calibration purposes.

1) If $k = 1$, we have the trivial result $I(\lambda, m) = I(\lambda, m)$.

2) If $k = 2$, we have

$$I^2(\lambda, m) = I_0(\lambda) \cdot I(\lambda, 2m). \tag{13}$$

3) If $k = 3$, we have

$$I^3(\lambda, m) = I_0^2(\lambda) \cdot I(\lambda, 3m). \tag{14}$$

Equations (13) and (14) allow us to suggest the following practical methods for calibration of sun photometers.

In line with Eq. (13), one would carry out the photometric measurements at air masses m and $2m$. It should be noted that the classic Langley diagram method also requires to carry out photometric measurements at two different air masses, but the ratio is not required. In the proposed method, we use two measured diagrams, axes of which represent the physical parameters, homogenous with $I(\lambda, m)$, $I_0(\lambda)$, and $I(\lambda, 2m)$ (Fig. 2). First of all, we should designate the quadrate with area equal to $I^2(\lambda, 2m)$ to be marked. The main requirement of this method is to find out such a point $I_0(\lambda)$ on the other axis, upon which the area of rectangle $OI(\lambda, 2m)B_2(\lambda, m)$ has the area of quadrate $OI(\lambda, m)B_1I(\lambda, m)$. The geometric interpretation of this method is shown in Fig. 2.

According to this method, in line with Eq. (14) the photometric measurements on air masses m and $3m$ may be carried out. Then the three measured diagrams should be used, the axes of which represent the physical parameter homogenous with $I(\lambda, m), I_0(\lambda)$, and $I(\lambda, 3m)$. In one axis, the value of $I(\lambda, 3m)$ should be marked. On the plane which is perpendicular to this axis, we should place the quadrate with area equal to $I_0^2(\lambda)$. The main condition of this contraction is that the volume of the parallelepiped with height equal to $I(\lambda, 3m)$ and basis area equal to $I_0^2(\lambda)$ would be equal to the volume of the cube with its side equal to $I(\lambda, m)$. The geometric interpretation of this method is shown in Fig. 3.

In order to describe the major advantages of suggested method in comparison with Langley method, the error analysis is carried out. The relevant graphical explanation is given in Fig. 4. The function of initial Langley plot diagram (line 1 in Fig. 4) may be written as

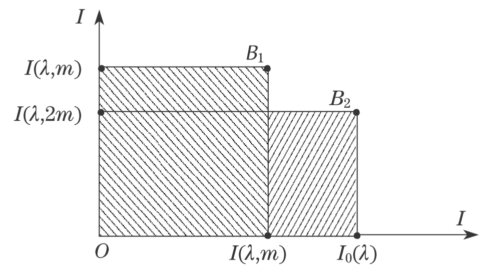

Fig. 2. Geometric interpretation of suggested method of equal areas.

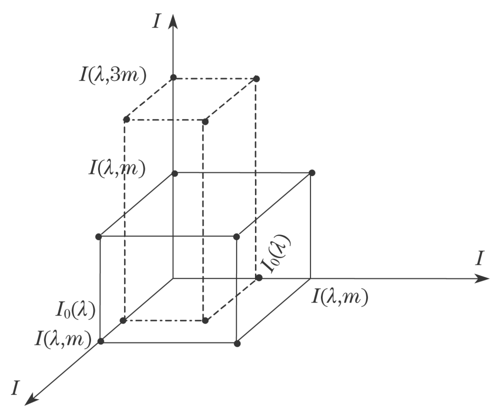

Fig. 3. Geometric interpretation of suggested method of equal volumes.

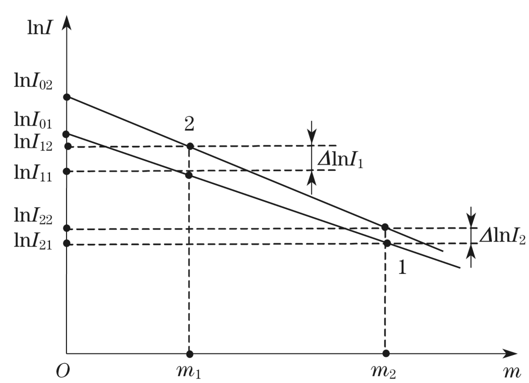

Fig. 4. Graphical explanation of error analysis.

$$\ln I_{01} = \ln I_{21} + k_1 m_2, \tag{15}$$

where

$$k_1 = \frac{\ln I_{11} - \ln I_{21}}{m_2 - m_1}. \tag{16}$$

Suppose that we have the measurement errors of any nature in points m_1 and m_2 which are equal to $\Delta \ln I_1$ and $\Delta \ln I_2$ accordingly, the function of experimental line of Langley plot diagram 2 may be written as

$$\ln I_{02} = \ln I_{22} + k_2 m_2. \tag{17}$$

where

$$k_2 = \frac{I_{12} - I_{22}}{m_2 - m_1}. \tag{18}$$

The absolute error may be given as

$$\Delta \ln I_{0L} = \ln I_{02} - \ln I_{01}. \tag{19}$$

To calculate the absolute value of error, we should take logarithm with Eq. (11). As a result, we get

$$\ln I_0(\lambda) = \frac{k}{k-1}\ln I(\lambda, m_1) - \frac{1}{k-1}\ln(\lambda, km_1). \tag{20}$$

If increments $\Delta I(\lambda, m_1)$ and $\Delta I(\lambda, km_1)$ occur, Eq. (20) may be written as

$$\ln\ [I_0(\lambda) + \Delta I_0(\lambda)] = \frac{k}{k-1}\ln[I(\lambda, m_1) + \Delta I(\lambda, m_1)] - \frac{1}{k-1}\ln[I(\lambda, km_1) + \Delta I(\lambda, km_1)]. \tag{21}$$

From Eqs. (20) and (21), we have

$$\Delta \ln I_0(\lambda) = \frac{k}{k-1}\ln\left[1 + \frac{\Delta I(\lambda, m_1)}{I(\lambda, m_1)}\right] - \frac{1}{k-1}\ln\left[1 + \frac{\Delta I(\lambda, km_1)}{I(\lambda, km_1)}\right]. \tag{22}$$

Taking into consideration $k = \frac{m_2}{m_1}$, Eq. (22) may be written as

$$\Delta \ln I_0(\lambda) = \frac{m_2}{m_2 - m_1}\ln\left[1 + \frac{\Delta I(\lambda, m_1)}{I(\lambda, m_1)}\right] - \frac{m_1}{m_2 - m_1}\ln\left[1 + \frac{\Delta I(\lambda, km_1)}{I(\lambda, km_1)}\right]. \tag{23}$$

It is obvious that the advantage in accuracy of calibration may be reached upon the following condition:

$$\Delta \ln I_{0L} > \Delta \ln I_0(\lambda). \tag{24}$$

In view of Eqs. (19), (23), and (24), we have

$$\ln I_{02} + \frac{m}{m_2 - m_1}\ln\left[1 + \frac{\Delta I(\lambda, m_2)}{I(\lambda, m_2)}\right] > \ln I_{01} + \frac{m_2}{m_2 - m_1}\ln\left[1 + \frac{\Delta I(\lambda, m_1)}{I(\lambda, m_1)}\right]. \tag{25}$$

From the inequality (25), we can finally get the following condition for the presence of advantage in accuracy of calibration:

$$\frac{I_{02}}{I_{01}} > \frac{\left[1 + \frac{\Delta\, I\,(\lambda,\, m_1)}{I(\lambda,\, m_1)}\right]^{m_2/m_2\, -\, m_1}}{\left[1 + \frac{\Delta\, I\,(\lambda,\, m_2)}{I(\lambda,\, m_2)}\right]^{m_1/m_2\, -\, m_1}}. \tag{26}$$

Thus, the formula (26) defines the condition for presence of advantage in accuracy of calibration in the suggested method in comparison with Langley diagram method. It is obvious that

$$\frac{m_2}{m_2\ -\ m_1} > \frac{m_1}{m_2\ -\ m_1}.$$

Therefore, the absolute condition for the presence of aforesaid advantage is

$$\frac{\Delta\, I\ (\lambda,\, m_1)}{I\ (\lambda,\, m_1)} \geq \frac{\Delta\, I\,(\lambda,\, m_2)}{I\ (\lambda,\, m_2)}. \tag{27}$$

It is clear that condition (26) may also be met upon some violation of condition (27).

Another advantage of the suggested method in comparison with classic Langley method is that we have an analytical formula making it possible to calculate the value of $I_0(\lambda)$ using various derivative formulas, while in the Langley method the value of $I_0(\lambda)$ may be found using only geometrical method.

Experimental researches were performed by comparing two photometers of the same construction. The first photometer was calibrated using the Langley method and the second one using the suggested method of equal areas. The "input-output" characteristics of both photometers were linearized with accuracy equal to 3%, which allows to compare the output signals of photometers with sufficient accuracy. The photometric measurements were carried out at air masses equivalent to elevation angles of the Sun $\alpha_1 = 20°$, $25°$, $30°$, and $35°$ (correspondingly, $2\alpha_1 = 40°$, $50°$, $60°$, and $70°$). The relative error of determination of solar constant values is calculated by

$$\gamma = \frac{U_{01}\,(\lambda)\ -\ U_{02}\,(\lambda)}{U_{02}\,(\lambda)}, \tag{28}$$

where $U_{01}(\lambda)$ is the value of the output signal of the photometer calibrated using suggested method of equal areas; $U_{02}(\lambda)$ is that using the Langley method; $\lambda = 557 \pm 3$ nm. The calculated values of γ are shown in Table 1.

Table 1. Comparisons of Two Photometers with Relative Error Calculated by Eq. (28)

Measurements	1	2	3	4
α_1	20°	25°	30°	35°
γ (%)	+5.42	−2.33	−4.97	+3.06

As can be seen from Table 1, the error shows a dual-polarity characteristic, which presumably can be explained by effects of many factors, such as error of linearization of photometric channels, error of calculation on the Langley diagrams, error due to aerosol variability, and so on.

Concerning the scientific value of the Langley method and the suggested one, it should be noted that both methods are valid for homogenized atmosphere. Limitations concerning the values of optical air mass in Eq. (7) are also valid for both methods. But the factual advantage in calculation of solar constant shown above in error analysis may be considered as a significant positive feature of the suggested method.

In conclusion, the main shortages of the classic method of Langley diagrams include the effect of variations of atmospheric optical depth to the accuracy of calculations and the absence of analytical formula allowing to calculate the calibration parameter values without geometric procedures. The suggested calibration methods make it possible to determine the calibration parameter values using photometric measurements on air masses m, $2m$, and $3m$ and to further calculate the wanted values using analytical or geometric calculation procedures.

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