Study of weighted space deconvolution algorithm in computer controlled optical surfacing formation

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Theoretical and experimental research on the deconvolution algorithm of dwell time in the technology of computer controlled optical surfacing (CCOS) formation is made to get an ultra-smooth surface of space optical element. Based on the Preston equation, the convolution model of CCOS is deduced. Considering the morbidity problem of deconvolution algorithm and the actual situation of CCOS technology, the weighting spatial deconvolution algorithm is presented based on the non-periodic matrix model, which avoids solving morbidity resulting from the noise induced by measurement error. The discrete convolution equation is solved using conjugate gradient iterative method and the workload of iterative calculation in spatial domain is reduced effectively. Considering the edge effect of convolution algorithm, the method adopts a marginal factor to control the edge precision and attains a good effect. The simulated processing test shows that the convergence ratio of processed surface shape error reaches 80%. This algorithm is further verified through an experiment on a numerical control bonnet polishing machine, and an ultra-smooth glass surface with the root-mean-square (RMS) error of 0.0088 μm is achieved. The simulation and experimental results indicate that this algorithm is steady, convergent, and precise, and it can satisfy the solving requirement of actual dwell time.

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More and more aspheric surfaces are used in optical systems. Due to the unique geometric shape of aspheric surfaces, the manufacture difficulty and cost of aspheric surfaces are greatly higher than that of spherical surface optical elements[1,2]. A breakthrough of modern space optical element processing technique is computer controlling "small grinding tool" to polish the element. The processing technology is often referred to as computer controlled optical surfacing (CCOS) technology, namely computer-controlled optical surface shaping technology. CCOS technology is limited to the computer controll of small grinding tool for polishing, the processing principle is based on the Preston hypothesis, and the polishing process can be modeled. So this method is also known as the deterministic polishing technique[3]. Currently, except computer-controlled polishing (CCP), the developing CCOS technology includes many representative technologies, such as the dynamic stressed lap polishing technology, magnetorheological finishing technology[4], and ion beam polishing technology[5] . The common characteristics of these technologies are a computer-controlled "small grinding tool" polishing on the surface of optical components. In the processing of large-scale optical components, particularly in non-spherical optical parts processing, these CCOS technologies are increasingly being applied. Bonnet polishing with novel polishing tool and special motion trait is a high precision and efficient optical component machining method, especially for aspheric surface machining[6−8]. It is a new polishing method presented by London Optical Science Laboratory and has an extensive application prospect.

In CCOS technology, the solving algorithm of dwell function is the key factor affecting the processing efficiency and error convergence. Based on the Preston equation, the material removal amount can be derived from the convolution between dwell time and removal function. Thus solving the dwell function is a deconvolution essentially. But the deconvolution solution is always a pathological problem. The popular algorithms of solving dwell time function are iterative algorithm and Fourier transform[9]. Iterative algorithm adopts approaching method in numerical calculation, but the disadvantage is that the iterative convergence rate is slow. Furthermore, this approach will oscillate without convergence under certain circumstances. Fourier transform method changes the convolution to product operation by Fourier transform, and then the inverse Fourier transform can be done to obtain the final result. The computation amount is smaller than that of the iterative method, but it is difficult for Fourier transform when the removal function is close to zero, and the results need special treatment, especially it cannot guarantee convergence. The algorithm to solve dwell time function based on different optical processing methods has been researched in recent years. For example, the distribution of dwell time was obtained according to linear algebra and Tikhonov regularization by Deng *et al.*[10], and a discrete linear model was established by Zhou *et al.* to analyze the relationship between the dwell time and removal amount by L-curve, further to calculate the dwell time[11]. These two methods changed the fabrication process from convolution to matrix product based on linear algebra to solve the illposed problem caused by noise. In theory, the methods obtained satisfying numerical results, but the calculation quantity was large and the edge effect problem in actual process was not considered.

Weighted spatial deconvolution algorithm, based on the non-periodic matrix model, can avoid the morbidity problem of solving deconvolution, allowing the sequence of convolution and convolution kernel alternate iteration estimation, and in the space domain iterative algorithm can reduce the workload of calculation. Aiming at the edge effect of convolution, a marginal modified factor is introduced to control the edge precision. We have gained a good result by using this approach. An ultra-smooth optical surface is obtained through verification of a process experiment.

Optical surface grinding and polishing are constrained by many factors and the quantitative control is very difficult. The mathematical model commonly used to describe the optical surface for processing is the Preston equation[1]:

$$\Delta h(x,y) = k \cdot \nu(x,y) \cdot p(x,y), \tag{1}$$

where $\Delta h(x,\mathrm{y})$ is the Removal rate in unit time of point(x,y); k is the processing factor related to the workpiece material, polishing type, polishing liquid, and temperature of work area; $\nu(x,y)$ is the instantaneous relative velocity of polishing pad at point(x,y); $p(x,y)$ is the instantaneous pressure of polishing pad at point(x,y).

An assumed condition is that the value of removal function is invariable with time and space. We define the average removal value of surface materials $r(x,y)$ in unit time T as the polish pad removal function, namely:

$$\begin{aligned} r(x,y) &= \frac{1}{T}\int_0^T \Delta h(x,y)\cdot \mathrm{d}t \\ &= \frac{1}{T}\int_0^T k\cdot\nu(x,y)\cdot p(x,y)\cdot \mathrm{d}t. \end{aligned} \tag{2}$$

As shown in Fig. 1, during the process, the tool dwells for a certain time $d(x,y)$ at each point. When the removal function is polishing at point $o(\alpha,\beta)$, the function has different impacts on the circle domain centered at $o(\alpha,\beta)$, and the radius is r_0. When the polishing pad moves to the point $p(x,y)$ in accordance with the scheduled track, the removed material in each region will be superimposed. So we can get the distribution function $h(x,\ y)$ of the entire processing surface material removal amount:

$$h(x,y) = \sum_\alpha \sum_\beta [d(\alpha,\beta)\cdot r(x-\alpha,y-\beta)]\delta\alpha\delta\beta. \tag{3}$$

When $\delta\alpha$ and $\delta\beta$ reach zero, Eq. (3) can be written as

$$h(x,y) = \int_\alpha\int_\beta d(\alpha,\beta)\cdot r(x-\alpha,y-\beta)\mathrm{d}\alpha\mathrm{d}\beta. \tag{4}$$

The integral formula (4) shows that the surface material removal amount $h(x,y)$ is equal to the two-dimensional (2D) convolution along with motion track between the polishing removal function $r(x,y)$ and the dwell time $d(x,y)$:

$$h(x,y) = r(x,y) ** d(x,y). \tag{5}$$

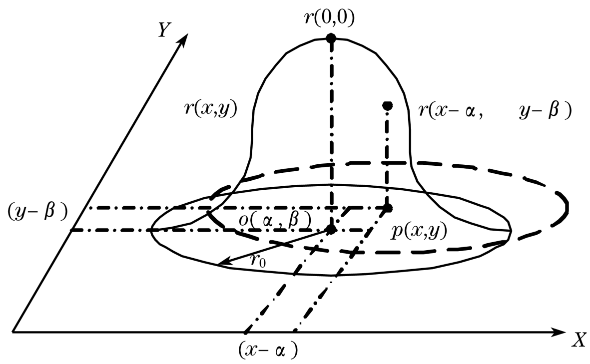

Fig. 1. Superposition of material removal.

$r(x,y)$ is related to the size of polishing pad and material properties, work pressure, relative velocity, and other factors, which can be obtained through computer simulation and test technology. $h(x,y)$ is the surface shape error distribution, which is determined by measurement. When we have known $r(x,y)$ and $h(x,y)$, the dwell time function $d(x,y)$ can be obtained through iterative operation. Choosing reasonable parameters based on the above analysis, computer numerical control (CNC) documents are generated under the control of computer. The grinding tool moves on the optical surface, abiding specific path, speed, and pressure. By controlling the dwell time of each region, the goal is achieved, which amends the error and enhances the accuracy. The residual error after process is

$$e(x,y) = h(x,y) - r(x,y) ** d(x,y). \tag{6}$$

For obtaining the removal amount in the next step, the polishing process is needed once again, which can gradually increase the surface accuracy. These steps are a repeatedly iterative process, and will gradually converge to the ideal shape.

According to the above analysis, firstly, the dwell time of each manufacturing point is solved by deconvolution in the procedure of CCOS. Then the removal amount of every point is controlled by dwell time of this point, and the control of machining accuracy and surface convergence rate depends on the accuracy of solving deconvolution. Deconvolution, whose solution is a pathological problem, belongs to the first Fredholm integral equation category in mathematics. The quantity of awaiting process is a limited discrete convolution in the CCOS procedure. In non-periodic matrix model, it is assumed that the size of dwell time matrix is smaller than the matrix of surface error, as a guarantee for the unknown variant number less than the number of equations. Thus solving an underdetermined problem is transformed to solving an over determined problem[9]. Weighting the spatial deconvolution algorithm based on the non-periodic matrix model can avoid the morbidity problem of solving deconvolution, allowing the sequence of convolution and convolution kernel alternate iteration estimation, and in the space domain the iterative algorithm can reduce the workload of calculation.

If the size of the unknown sequence $x(m_1,n_1)$ is assumed to be smaller than the known one $y(m_2,n_2)$, it is feasible to estimate the unknown sequence with a smaller size according to the known one. In the one-dimensiond

(1D) case, the length of the convolution kernel sequence is assumed to be M. The length of the sequence $\{x(n)\}$ is N. Then the length of the convolution sequence L is $L = M + N - 1$. The form of the equations is

$$\begin{bmatrix} h_0 & & & \\ h_1 & h_0 & & \\ \vdots & h_1 & \ddots & \\ h_{M-1} & \vdots & \ddots & \\ & h_{M-1} & & h_0 \\ & & \ddots & h_1 \\ & & \ddots & \vdots \\ & & & h_{M-1} \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ \vdots \\ x_{N-1} \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ \vdots \\ y_{L-1} \end{bmatrix}. \quad (7)$$

The concise form of Eq. (7) is $F_h x = y$, where F_h is named convolution kernel matrix, whose size is $L \times N$.

In the 2D case, when the sizes of matrices $h(m_1, m_2)$, $x(n_1, n_2)$, and $y(l_1, l_2)$ are assumed to be $M_1 \times M_2$, $N_1 \times N_2$, and $L_1 \times L_2$, $L_1 = M_1 + N_1 - 1$, $L_2 = M_2 + N_2 - 1$, the convolution equation is deduced to be $F_{(h,N_1,N_2)} x = y$. The convolution kernel matrix $F_{(h,N_1,N_2)}$ is evolved from h. There are N_1 matrix columns in $F_{(h,N_1,N_2)}$ and N_2 columns in submatrices in every matrix column. The expansion equation is

$$F_{(h,N_1,N_2)} = \begin{bmatrix} F_{(h_1,N_2)} & & & \\ F_{(h_2,N_2)} & F_{(h_1,N_2)} & & \\ \vdots & F_{(h_2,N_2)} & \ddots & \\ F_{(h_{M1},N_2)} & \vdots & & F_{(h_1,N_2)} \\ & F_{(h_{M1},N_2)} & & F_{(h_2,N_2)} \\ & & \ddots & \vdots \\ & & & F_{(h_{M1},N_2)} \end{bmatrix}, \quad (8)$$

where $\mathbf{h}_i$ is a vector, which is formed with the ith row of $h(m_1, m_2)$, $F_{(\mathbf{h}_i,N_2)}$ is a 1D convolution kernel matrix. Then the size of $F_{(h,N_1,N_2)}$ is $L_1 L_2 \times N_1 N_2$. In this way, $F_{(x,M_1,M_2)}$ is evolved from $\boldsymbol{x}$ and the size is $L_1 L_2 \times M_1 M_2$.

The deconvolution model of the last non-periodic matrix is in accordance with the solution of the dwell time in CCOS essentially. In the process of CCOS, it is assumed that the edge of polished die is within the workpiece boundary, then the spatial distribution of the dwell time is smaller in size than the surface error, and the measurement of the surface error is equivalent to the sum of the sizes of the dwell time and removal function. It ensures that the quantity of the unknown variables is less than the one of the equations and will improve the ill-conditioned characteristic of the problem. Furthermore the surface error itself is a finite discrete convolution and the non-periodic convolution matrix is always non-singular, which has a much smaller condition number under any circumstances, and is not sensitive to the length of convolution and the changes of the Rernel function. For these reasons, it will obtain an accurate result using non-periodic matrix model.

Giving locally adaptive control to deconvolution and smooth is the main idea of weighted space deconvolution algorithm. This approach can strengthen the smoothness and weaken the deconvolution in the flat region of surface error. Contrarily, near the edge the deconvolution is strengthened and the smoothness is weakened. Weakening the deconvolution in the flat region will weaken the noise amplification, and thus the total effect of the solution can be improved. The residual error of deconvolution is bounded, and the remaining errors are associated with noise. In order to overcome the morbidity problems of deconvolution, the equation should be regularized to guarantee the second order derivative's norm square $\|x''\|^2$ to the smallest. In discrete circumstances, through replacing the second order derivatives by the second order difference, $x(m, n) * c(m, n)$ can be used to calculate the second order difference of $x(m, n)$. The second order differential operator is

$$c(m, n) = \frac{1}{8} \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad (9)$$

which is also called the Laplace operator. Define two restrictive conditions:

$$\|y - Bx\| \leq \varepsilon \text{ and } \|Cx\| \leq E, \quad (10)$$

where ε and E are the specified borders. Choosing the value of ε is decided by the noise energy from the error of surface measurement. Similarly, E relies on the high frequency energy from allowable surface error. For controlling the local adaptability of deconvolution, two weighted matrices are introduced, so the result is

$$\|y - Bx\|_R = \left[(y - Bx)^{\mathrm{T}} R (y - Bx)\right]^{\frac{1}{2}} \leq \varepsilon, \quad (11)$$

$$\|Cx\|_S = [(Cx)^{\mathrm{T}} S (Cx)]^{\frac{1}{2}} \leq E, \quad (12)$$

where R and S are two diagonal matrices, which include the weight coefficients r_{ij} and s_{ij} influencing each sampling point. The role of r_{ij} is ensuring the edge precision of surface error, controlling the nonstationarity with the variation of noise, and enhancing the recovery of the missing data. Meanwhile, the value of s_{ij} can control the smoothness locally to eliminate the ring shape parasitic ripple. Combining Eqs. (11) and (12), we can get

$$\begin{aligned} J_w(x) = & \left[(y - Bx)^{\mathrm{T}} R (y - Bx)\right] \\ & + \alpha[(Cx)^{\mathrm{T}} S (Cx)] \leq 2\varepsilon^2, \end{aligned} \quad (13)$$

where $\alpha = (\frac{\varepsilon}{E})^2$. Minimizing $J_w(x)$, we can obtain

$$(B^{\mathrm{T}} R B + \alpha C^{\mathrm{T}} S C) x \leq B^{\mathrm{T}} R y. \quad (14)$$

Matrices B and C are non-periodic convolution kernel matrices, especially, denoted as F_h and F_c. Then we resolve

$$(F_h^{\mathrm{T}} R F_h + \alpha F_c^{\mathrm{T}} S F_c) x \leq F_h^{\mathrm{T}} R y. \quad (15)$$

For a faster convergence, conjugate gradient approach is adopted in the application of this algorithm. The equation after regularization is in well-condition. So the key steps of resolving the inequality (15) by conjugate gradient approach is

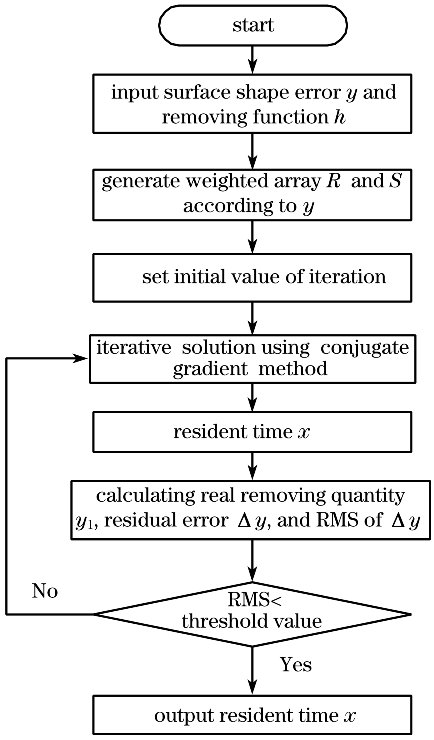

Fig. 2. Flowchart of deconvolution algorithm.

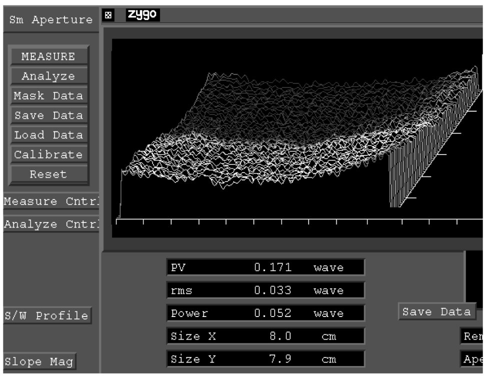

Fig. 3. Original measurement results.

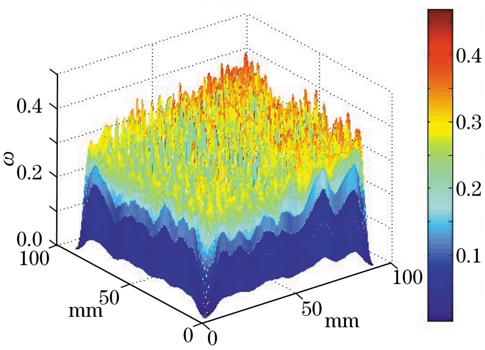

Fig. 4. Dwell time.

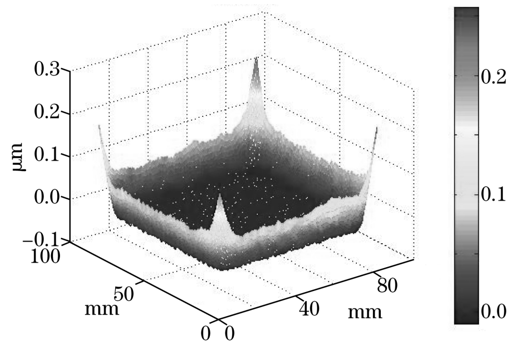

Fig. 5. Residual error.

$$\mathbf{q} = (F_h^{\mathrm{T}} R F_h + \alpha F_c^{\mathrm{T}} S F_c) p. \quad (16)$$

Replacing $F_h p$ by $h * p$ in iteration can reduce the computational complexity. p is the iterative initial and $\mathbf{q}$ is the iterative vector of the equation. The calculation flowchart is shown in Fig. 2.

In order to verify the effectiveness of the approach, we simulate the algorithm with the actual measurement results. Without loss of generality, an 80 × 80 (mm) square plane is chosen as the workpiece to be processed, and the measurement results are shown in Fig. 3. The root-mean-square (RMS) value is 0.0209 μm (0.033λ, λ= 0.6328 μm). In order to get the dwell time of every point within the whole processing region, the original measurement data are dealt with edge extension, and the extension point number relies on the removal function value. The removal function can be measured by means of experiments. Figure 4 shows the simulated dwell time. Figure 5 shows the simulated residual error. The final surface RMS value of the workpiece is 0.0146 μm.

The edge effect is inevitable in convolution[12]. In order to control the edge accuracy, an edge correction factor is proposed in the algorithm, that is, the original surface data on the edge are amended. Supposing that $\Delta z(x, y)$ is the specified edge surface shape error, $\Delta z_1(x, y)$ is the modified edge residual error, $R(x, y)$ is the removal function, $\varepsilon(x, y)$ is the optimized edge processing residual error, and α is a modifying factor, we have

$$\Delta z_1(x, y) = \Delta z(x, y) + \alpha \cdot \varepsilon(x, y). \quad (17)$$

The edge modifying factor α can be obtained when the RMS value of $\Delta z_1(x, y)$ is the minimum.

Figure 6 is the amendment process flowchart. Figure 7 shows the residual error after the edge compensation. The RMS value reaches 0.0071 μm and the convergence ratio of RMS value is more than 80%. The simulation results show that the algorithm is steady, convergent, and precise, which can meet the request of solving actual dwell time.

The processing experiment was achieved in a bonnet CNC polishing machine. The experimental piece was just the piece used in the simulation, a nucleated glass material. After 70 min of polishing processing, the RMS value of the surface accuracy reaches 0.0088 μm (0.013λ). The experimental results shown in Fig. 8 prove the effectivity of the algorithm. At present, the software to

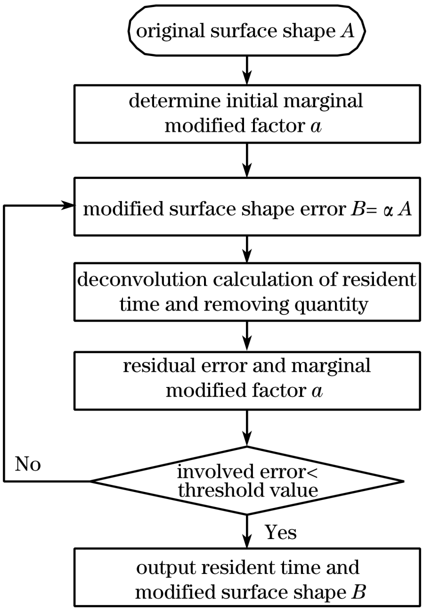

Fig. 6. Flowchart of marginal compensation algorithm.

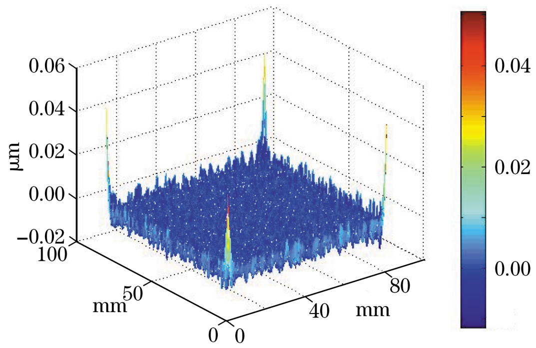

Fig. 7. Residual error after marginal compensation.

perform the algorithm has been completed.

In summary, compared with the traditional manual polishing process, the CCOS process, which has high certainty and fast surface convergence, basically does not rely on the skill level and experience of the technologist. The solving algorithm of dwell function is the key factor which is effective on processing efficiency and error convergence. We present the weighting spatial deconvolution algorithm based on the non-periodic matrix model, solve the discrete convolution function by the conjugate gradient iterative method, and test the algorithm by simulation and experiments. The results show that this algorithm is stably convergent, highly accurate, and can avoid the morbidity problem. Furthermore, the algorithm introduces the edge correction factor, which effectively controls the edge precision.

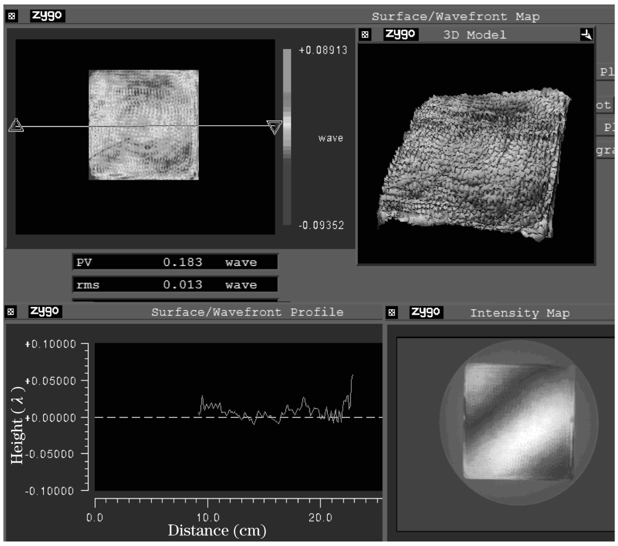

Fig. 8. Processing results.

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