

Dispersion measurements of water with spectral interferometry

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The spectral relative phase is directly derived from the spectrally resolved interferogram by the Fourier transform method. Furthermore, the spectral absolute phase can also be found. The spectral absolute phase is used to measure the refractive index. In addition, the group index and the derivatives of the refractive index with respect to the wavelength are given through the polynomial fitting process. The measured results are compared with the published data and the new measurement results of the water are given for the wavelength larger than 1.3 μm.

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The optical dispersion of media is an important parameter in understanding and controlling the femtosecond pulse propagation^[1]. Water is an inevitable propagation medium in atmosphere and ocean, as well as in biological tissues. Femtosecond pulses can be broadened and split by dispersion with nonlinear self-focusing and diffraction^[2]. The broadened pulses have much to do with the peak power density in laser eye surgery. Interferometry has been applied to measure the refractive index and dispersion in spectral domain and time domain^[3–13]. In time domain, the optical path in one arm of the interferometer is varied continuously with the optical delay line or the movement of the reflective mirror when recording the interference signal. The group delay can be measured by evaluating the shift of cross-correlation peak of the interferogram with the variation of wavelength, or by a Fourier transform of the cross-correlation interferogram, the spectral phase delay can be obtained. However, it is difficult for us to determine the zero-delay point that serves as a time reference, and the derived phase is a relative value and needs to be unwrapped^[14]. In spectral domain, the optical path difference (OPD) between two arms of the interferometer is fixed while the interference signal is spectrally resolved by a spectrometer. The group index can be obtained directly from the local fringes' periodicity at the stationary phase point, the dispersion of the group refractive index can be acquired through the equalization wavelength as well. Alternatively, the spectral phase can be derived directly from the spectrally resolved interferogram by the rule that the phase at a fringe peak is equal to an integer multiple of π^[9,11]. However, the interferogram cannot be spectrally resolved when the OPD far from the stationary phase point becomes too large^[10].

In this letter, the Fourier transform method^[15] is used to analyze the spectrally resolved interferogram, and the spectral relative phase is directly derived from the first order spectrum of the Fourier transformed interferogram. Compared with the phase retrieval methods in Refs. [9] and [11], the Fourier transform method is simple and

easy. In addition, the spectral absolute phase can be acquired by the sampling theory in combination with the spectral relative phase, which is different from the way the absolute phase is obtained^[9]. The spectral absolute phase can be used to measure the refractive index. Furthermore, the group index and the derivatives of the refractive index with respect to the wavelength λ are given through the polynomial fitting process.

The experimental schematic illustration is shown in Fig. 1, which is a Michelson interferometer with a superluminescent light emitting diode (SLED) broadband light source, whose nominal central wavelength λ_c = 1316 nm and bandwidth Δλ = 65 nm. After being collimated through a lens L1, the broadband light is split into two beams by a beam splitter (BS). The two beams pass through a pair of quartz glass cells and are reflected back from mirrors M1 and M2, the interference signal is coupled into a spectrometer by a fiber. In order to cancel out the effect of the wall of the glass cell, one of the pair of the same quartz glass cells is empty while another has a sample inside.

The recorded interferogram can be written as

$$I(\nu) = I_0(\nu)\{1 + r(\nu)^2 + 2r(\nu)\cos[\phi(\nu)]\}, \quad (1)$$

where ν is defined as 1/λ, λ is the wavelength, I₀(ν) is the light source spectral distribution, r(ν) is the ratio of

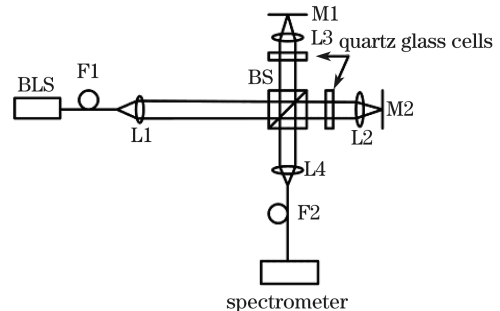


Fig. 1. Experimental schematic illustration. BLS: broadband light source; F1, F2: fibers; L1, L2, L3, L4: lenses; M1, M2: mirrors.

the amplitudes of the two superposed interference beams, and the phase $\phi(\nu)$ can be given as

$$\phi(\nu) = 2\pi\nu\{2d[n(\nu) - 1] + 2L\}, \quad (2)$$

here d is the thickness of the quartz glass cell, L is the extra OPD between the two arms of the interferometer by moving the reflective mirror in one arm away from the zero OPD position without the sample, $n(\nu)$ is the phase refractive index of the sample. By Fourier transforming two sides, Eq. (1) can be further written as

$$F[I(\nu)] = F[I_0(\nu)] \otimes \{[1 + r(\nu)^2]\delta(s) + C[s - 2(L - d)] + C^*[s + 2(L - d)]\}, \quad (3)$$

$$\begin{aligned} \phi(\nu) = & 4\pi\nu_0 dn(\nu_0) + 4\pi\nu_0(L - d) + \left[4\pi d \left[n(\nu) + \nu \frac{dn(\nu)}{d\nu} \right] + 4\pi(L - d) \right] \Big|_{\nu=\nu_0} (\nu - \nu_0) \\ & + 4\pi d \left[2 \frac{d^2n(\nu)}{d\nu^2} + \nu \frac{d^3n(\nu)}{d\nu^3} \right] \Big|_{\nu=\nu_0} \frac{(\nu - \nu_0)^2}{2} + 4\pi d \left[3 \frac{d^3n(\nu)}{d\nu^3} + \nu \frac{d^4n(\nu)}{d\nu^4} \right] \Big|_{\nu=\nu_0} \frac{(\nu - \nu_0)^3}{6} + \dots \end{aligned} \quad (5)$$

And the obtained phase value $\phi(\nu)$ from Eq. (4), by the polynomial fitting process, could be written as

$$\begin{aligned} \phi(\nu) = & \phi_0 + \phi_1(\nu - \nu_0) + \phi_2(\nu - \nu_0)^2 \\ & + \phi_3(\nu - \nu_0)^3 + \dots \end{aligned} \quad (6)$$

The fitting coefficients at different wavelengths can be obtained with the variation of ν_0 . We can get the refractive index and the group index of the sample by comparing the coefficients in Eq. (5) and (6) if the values L and d are known. However, the phase $\phi(\nu)$ in Eq. (6) is relative but not absolute because of a multiple of 2π , while the phase $\phi(\nu)$ in Eq. (5) is absolute. The absolute phase at certain wavelength is firstly obtained and then the absolute phase for the full wavelength range can be derived from the relative phase values. The distance between the zeroth and the first order spectra in Fourier domain can be expressed as

$$S = dn_0 + L - d, \quad (7)$$

where n_0 represents the refractive index at the central frequency ν_c in correspondence with the central sampling point of Fourier transform. According to the sampling theory, the sampling space is given as

$$ds = \frac{1}{2\Delta\nu}, \quad (8)$$

where $\Delta\nu = 1/\lambda_{\min} - 1/\lambda_{\max}$, λ_{\min} and λ_{\max} stand for the minimum and maximum detected wavelengths, respectively. From Eq. (2), the absolute phase at ν_c can be got, so the absolute phase for the full wavelength range could be written as

$$\phi_a(\nu) = \phi(\nu) - \phi(\nu_c) + 4\pi\nu_c S. \quad (9)$$

If any two variables of L , d , and n_0 are known, the third one can be solved from Eq. (7). Then the refractive index and the group index can be acquired from Eqs. (5) and (9).

where C expresses the Fourier transform of $r(\nu)\exp[j4\pi\nu dn(\nu)]$, F expresses the Fourier transform, \otimes stands for convolution.

The positive first-order Fourier spectrum of Eq. (3) is inversely Fourier transformed to give the phase and amplitude as

$$c_1 = I_0(\nu)r(\nu)\exp[j\phi(\nu)], \quad (4)$$

where c_1 represents the result by inversely Fourier transforming the positive first-order Fourier spectrum.

The phase value $\phi(\nu)$ in Eq. (2) can be expanded in a Taylor series about $\nu = \nu_0$ as

In our experiment, there are not enough known variables of L , d , and n_0 , but we still have

$$\begin{aligned} \frac{d^2\phi(\nu)}{d\nu^2} = & 4\pi d \lambda^3 \frac{d^2n}{d\lambda^2}, \\ \frac{d^3\phi(\nu)}{d\nu^3} = & -4\pi d \left(3\lambda^4 \frac{d^2n}{d\lambda^2} + \lambda^5 \frac{d^3n}{d\lambda^3} \right). \end{aligned} \quad (10)$$

Substituting Eq. (6) into Eq. (10), we can solve $d^2n/d\lambda^2$ and $d^3n/d\lambda^3$. Besides, $d^2k/d\omega^2$ and $d^3k/d\omega^3$ can be derived from $d^2n/d\lambda^2$ and $d^3n/d\lambda^3$ in the form of Eqs. (8) and (9) in Ref. [8].

The recorded spectrally resolved interferogram of the distilled water is shown in Fig. 2(a), whose Fourier transform is shown in Fig. 2(b). In our experiment, the nominal thickness of the quartz glass cell $d = 1$ mm, the central frequency ν_c is located at $\lambda_c = 1.3288 \mu\text{m}$, the minimum wavelength $\lambda_{\min} = 1.2288 \mu\text{m}$, the maximum wavelength $\lambda_{\max} = 1.4288 \mu\text{m}$, the actual resolution of the spectrometer is 0.94 nm, and the sampling

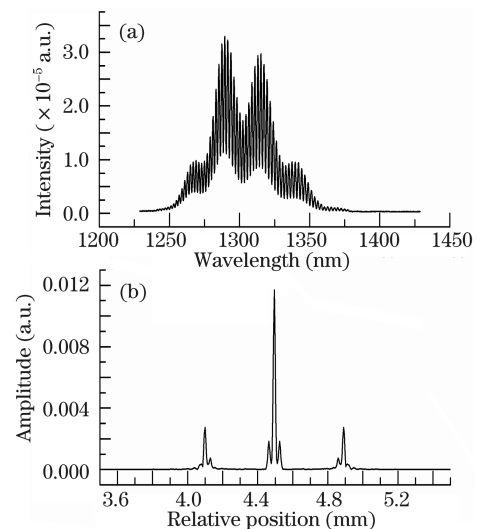


Fig. 2. (a) Spectrally resolved interferogram and (b) its Fourier transform.

space $ds = 4.3893 \mu\text{m}$ from Eq. (8). The mean distance from five interferograms $S = -0.395 \pm 0.002 \text{ mm}$, whose precision is determined by the sampling space. Then the absolute phase can be obtained from Eq. (9) with a precision of $\pm 6.6\pi$, the measured refractive index should have the error of ± 0.002 from Eqs. (7) and (5). According to Eq. (12) in Ref. [8], the refractive index at the central wavelength $n_0 = 1.32007$, the value of L could be found from Eq. (7). Then the refractive index is derived from the absolute phase, as shown in Fig. 3(d). The measured refractive index agrees with that in Ref. [8] within the error range of ± 0.002 . Because the calculated values are extrapolated from the valid wavelengths in Eq. (12) of Ref. [8], the variation trend of refractive index with wavelength cannot be certainly right. The measured group index in Fig. 3(b) shows a systematic error in comparison with the result from Eq. (12) of Ref. [8]. The values of $d^2n/d\lambda^2$ and $d^3n/d\lambda^3$ can be measured from the spectral relative phase from Eq. (10). The measured values of $d^2n/d\lambda^2$ of the distilled water are given in Fig. 3(c). Our measured values of $d^2n/d\lambda^2$ at 1.3 and 1.25 μm are respectively $-0.028 \pm 0.004 \mu\text{m}^{-2}$ and $-0.02 \pm 0.03 \mu\text{m}^{-2}$, both agree well with the published data^[8]. The measured values of $d^3n/d\lambda^3$ are given with large standard errors as shown in Fig. 3(a). The new measured results of $d^2n/d\lambda^2$ and $d^3n/d\lambda^3$ are given for the wavelength larger than 1.3 μm .

The main errors come from the thickness d , the sampling space ds , the fitting process, the spectral relative phase, and Eq. (12) in Ref. [8]. We can also see from Figs. 3(b) and (c) that the larger the standard error of the mean, the farther away from the zero OPD position because of the denser interference fringe and the lower fringe visibility. Another factor which maybe influences the measurement is the parasitic interference,

which can be seen in Figs. 2(a) and (b), for the spectral distribution of the light source is nearly Gaussian. In addition, the refractive index and the group index from Eq. (12) of Ref. [8] are extrapolated to the wavelength range of the experiment, which should be noticed. For example, there is a dramatic difference between the measured values of $d^2n/d\lambda^2$ and the calculated values from Eq. (12) in Ref. [8].

In conclusion, the spectral relative phase can be derived directly from the first-order spectrum of the Fourier transform of the spectrally resolved interferogram. The distance between the zeroth and first order spectra of the Fourier transform could be found from the sampling theory. Furthermore, the spectral absolute phase can be acquired from the distance and the spectral relative phase. The absolute phase can be used to solve the refractive index and the group index if we know any two variables of the sample thickness d , the extra OPD, and the refractive index n_0 at the central frequency of ν_c . The measured values of $d^2n/d\lambda^2$ agree well with the published data and the new measured values of $d^2n/d\lambda^2$ and $d^3n/d\lambda^3$ are given for the wavelength larger than 1.3 μm .

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References

1. I. Walmsley, L. Waxer, and C. Dorrer, *Rev. Sci. Instrum.* **72**, 1 (2001).
2. S. A. Diddams, H. K. Eaton, A. A. Zozulya, and T. S. Clement, *IEEE J. Sel. Top. Quantum Electron.* **4**, 306 (1998).
3. W. H. Knox, N. M. Pearson, K. D. Li, and C. A. Hirli- mann, *Opt. Lett.* **13**, 574 (1988).
4. M. Beck and I. A. Walmsley, *Opt. Lett.* **15**, 492 (1990).
5. K. Naganuma, K. Mogi, and H. Yamada, *Opt. Lett.* **15**, 393 (1990).
6. V. N. Kumar and D. N. Rao, *J. Opt. Soc. Am. B* **12**, 1559 (1995).
7. S. Diddams and J.-C. Diels, *J. Opt. Soc. Am. B* **13**, 1120 (1996).
8. A. G. Van Engen, S. A. Diddams, and T. S. Clement, *Appl. Opt.* **37**, 5679 (1998).
9. D. X. Hammer, A. J. Welch, G. D. Noojin, R. J. Thomas, D. J. Stolarski, and B. A. Rockwell, *J. Opt. Soc. Am. A* **16**, 2092 (1999).
10. H. Delbarre, C. Przygodzki, M. Tassou, and D. Boucher, *Appl. Phys. B* **70**, 45 (2000).
11. J. Y. Lee and D. Y. Kim, *Opt. Express* **14**, 11608 (2006).
12. P. Hlubina, D. Ciprian, and L. Knyblová, *Opt. Commun.* **269**, 8 (2007).
13. Y. Jiang, B. Zhou, Y. Leng, X. Chen, R. Li, and Z. Xu, *Chin. Opt. Lett.* **4**, 614 (2006).
14. M. Galli, F. Marabelli, and G. Guizzetti, *Appl. Opt.* **42**, 3910 (2003).
15. M. Takeda, H. Ina, and S. Kobayashi, *J. Opt. Soc. Am.* **72**, 156 (1982).

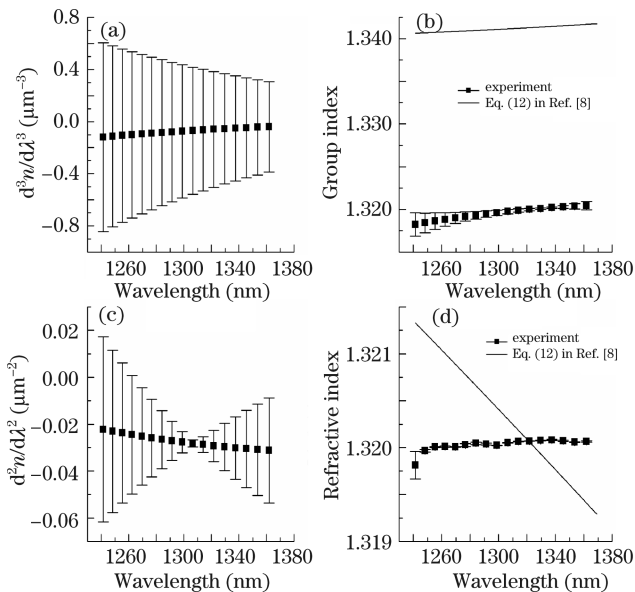


Fig. 3. (a) Measured third derivative of the refractive index $d^3n/d\lambda^3$; (b) comparison between the measured group index and the calculated values from Eq. (12) in Ref. [8]; (c) measured second derivative of refractive index $d^2n/d\lambda^2$; (d) comparison between the measured refractive index and the calculated values from Eq. (12) in Ref. [8].