Influence of laser mode on splitting beam illumination effect of Dammann grating

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The influences of various laser modes on the splitting beam effect of Dammann grating are studied in theory and by numerical simulation. The results show that fundamental mode laser resembles plane wave while high order mode laser differs from plane wave in the splitting beam effect by Dammann grating. Therefore, the fundamental mode laser is more suitable to be the light source to improve the energy efficiency in far-distance image detecting systems, such as laser image ladar, which use Dammann grating in the illumination system.

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Dammann grating[1,2] is a kind of phase grating which could transform monochromatic plane wave to light spot array in far field of Fourier transform with high efficiency. Since the 1980s, Dammann grating has been developed rapidly with the improvement of optical computing and optical interconnect[3−7]. Now it is likely to design and fabricate Dammann gratings with a larger light spot array[8−10], which could develop the application of Dammann grating to far-distance image detecting fields. For example, in the transmitting system of a laser image ladar without scanner, Dammann grating could be adopted to illuminate the target in the form of spot array to improve the energy efficiency. This application would effectively increase the detecting distance and detecting precision of a far-distance system, which is important to realize the detecting system miniaturization.

Dammann grating is generally designed for the condition that the incident wave is a plane wave. This makes it possible to complete a large amount of calculation in optimizing the design of Dammann grating, which is important especially in optimizing a larger spot array Dammann grating[11,12]. However, for the use of far-distance image detection, laser is the best light source because it could achieve high power and small scattering angle. One of the keys to apply Dammann grating to a far-distance image detecting system is to study the splitting beam effect of Dammann grating when the incident wave is laser instead of a plane wave. In this letter, the splitting beam effect of Dammann grating is studied in theory and by numerical simulation when the incident wave is laser with various modes.

Take one-dimensional (1D) mth order Hermite-Gaussian laser for example. The complex amplitude distribution of the optical field can be written as[13]

$$E_m = H_m(\frac{\sqrt{2}x}{\omega_0})\exp(-\frac{x^2}{\omega_0^2}), \qquad (1)$$

where ω_0 is the radius of the beam waist, $H_m\left(\dfrac{\sqrt{2}x}{\omega_0}\right)$ is the Hermite function in the form of $c_m x^m +$ $c_{m-1}x^{m-1}+\ldots+c_k x^k+\ldots+c_1 x + c_0$ with c_k being constant.

The frequency spectrum of this optical field is the Fourier transform of Eq. (1), which can be written as

$$
\begin{aligned}
F(E_m) &= \int\limits_{-\infty}^{+\infty} H_m\left(\frac{\sqrt{2}x}{\omega_0}\right)\exp\left(-\frac{x^2}{\omega_0^2}\right)\exp(-\mathrm{i}2\pi px)\mathrm{d}x \\
&= \exp(-\pi^2 p^2 \omega_0^2)\int\limits_{-\infty}^{+\infty} H_m\left(\frac{\sqrt{2}x}{\omega_0}\right) \\
&\quad \times \exp\left[-\left(\frac{x}{\omega_0}+\mathrm{i}\pi\omega_0 p\right)^2\right]\mathrm{d}x,
\end{aligned}
\qquad (2)
$$

where p is the space frequency.

Let $\mathrm{i}\pi\omega_0 p = b$. For simplifying the analysis while keeping the result unaffected, suppose $\omega_0 = 1$. Then $\int\limits_{-\infty}^{+\infty}\left(\dfrac{x}{\omega_0}\right)^m \exp\left[-\left(\dfrac{x}{\omega_0}+\mathrm{i}\pi\omega_0 p\right)^2\right]\mathrm{d}x$ could be expressed as

$$\int\limits_{-\infty}^{+\infty}\{(x+b)^m + [x^m-(x+b)^m]\}\exp[-(x+b)^2]\mathrm{d}x. \quad (3)$$

For the item $\int\limits_{-\infty}^{+\infty}(x+b)^m\exp[-(x+b)^2]\mathrm{d}x$ in the expression (3), when m is an odd number,

$$\int\limits_{-\infty}^{+\infty}(x+b)^m\exp[-(x+b)^2]\mathrm{d}x = 0. \qquad (4)$$

When m is an even number,

$$\int\limits_{-\infty}^{+\infty}(x+b)^m\exp[-(x+b)^2]\mathrm{d}x = \Gamma\left(\frac{m+1}{2}\right). \qquad (5)$$

The factor $x^m - (x+b)^m$ can be expressed as

$$(x+b)^m - x^m = A_{m-1}(x+b)^{m-1} + A_{m-2}(x+b)^{m-2} + \ldots + A_k(x+b)^k + \ldots + A_1(x+b) + A_0, \tag{6}$$

where A_k would be educed thereinafter. As a result, when m is an odd number,

$$\int\limits_{-\infty}^{+\infty} \{(x+b)^m + [x^m - (x+b)^m]\}\exp[-(x+b)^2]\mathrm{d}x = A_{m-1}\Gamma\left(\frac{m}{2}\right) + A_{m-3}\Gamma\left(\frac{m-2}{2}\right) + \ldots$$

$$+ A_k\Gamma\left(\frac{k+1}{2}\right) + \ldots + A_2\Gamma\left(\frac{3}{2}\right) + A_0\sqrt{\pi}. \tag{7}$$

When m is an even number,

$$\int\limits_{-\infty}^{+\infty} \{(x+b)^m + [x^m - (x+b)^m]\}\exp[-(x+b)^2]\mathrm{d}x = \Gamma\left(\frac{m+1}{2}\right) + A_{m-2}\Gamma\left(\frac{m-1}{2}\right) + \ldots$$

$$+ A_k\Gamma\left(\frac{k+1}{2}\right) + \ldots + A_2\Gamma\left(\frac{3}{2}\right) + A_0\sqrt{\pi}. \tag{8}$$

By expanding Eq. (6) on each side, A_k could be educed as follows:

$$C_m^1 b = A_{m-1} = mb,$$
$$C_m^2 b^2 = A_{m-1}C_{m-1}^1 b + A_{m-2},$$
$$C_m^3 b^3 = A_{m-1}C_{m-1}^2 b^2 + A_{m-2}C_{m-1}^1 b + A_{m-3},$$
$$C_m^k b^k = A_{m-1}C_{m-1}^{k-1}b^{k-1} + A_{m-2}C_{m-2}^{k-2}b^{k-2} + \ldots$$
$$+ A_{m-k},$$
$$C_m^m b^m = A_{m-1}b^{m-1} + A_{m-2}b^{m-2} + \ldots + A_0. \tag{9}$$

Writing the above equations in matrix form, we can get

$$\begin{pmatrix} A_{m-1} \\ A_{m-2} \\ \ldots \\ A_{m-k} \\ \ldots \\ A_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -C_{m-1}^1 b & 1 & 0 & 0 & 0 & 0 \\ -C_{m-1}^2 b^2 + C_{m-1}^1 C_{m-2}^1 b^2 & -C_{m-2}^1 b & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & 1 \end{pmatrix} \begin{pmatrix} C_m^1 b \\ C_m^2 b^2 \\ \ldots \\ C_m^k b^k \\ \ldots \\ C_m^m b^m \end{pmatrix}. \tag{10}$$

From Eq. (10), it is certain that any A_{m-k} contains the factor b^n, in other words, any A_{m-k} contains the factor p^n.

To sum up, for any mth order Hermite-Gaussian laser, when m is an even number

$$F(E_m) = \Gamma\left(\frac{m+1}{2}\right)\exp\left(-\pi^2 p^2\right) + \left(A_{m-2}\Gamma\left(\frac{m-1}{2}\right) + \ldots + A_k\Gamma\left(\frac{k+1}{2}\right)\right.$$

$$+ \ldots + A_2\Gamma\left(\frac{3}{2}\right) + A_0\sqrt{\pi})\exp\left(-\pi^2 p^2\right); \tag{11}$$

when m is an odd number,

$$F(E_m) = \left(A_{m-1}\Gamma\left(\frac{m}{2}\right) + A_{m-3}\Gamma\left(\frac{m-2}{2}\right) + \ldots + A_k\Gamma\left(\frac{k+1}{2}\right) + \ldots + A_2\Gamma\left(\frac{3}{2}\right) + A_0\sqrt{\pi})\exp\left(-\pi^2 p^2\right). \tag{12}$$

When the incident wave is a fundamental mode Gaussian laser, which means $m=0$ in Eq. (11), then

$$F(E_0) = \Gamma\left(\frac{1}{2}\right)\exp(-\pi p^2). \tag{13}$$

Obviously, the frequency spectrum of a fundamental mode Gaussian laser is a Gaussian function. However, if the width of the fundamental Gaussian laser is sufficient in space range, this function are similar to δ function that is the Fourier transform of plane wave in spatial frequency field. It could be inferred that the splitting beam effects by Dammann grating are similar for fundamental Gaussian laser and for plane wave if only the width of the fundamental Gaussian laser is sufficient in space range.

When m is an even number, but $m \neq 0$, the incident wave could be divided into two parts for analysis. The first part is the first item in Eq. (11) that is identical with a fundamental Gaussian laser. The second part contains the other items except the first item in Eq. (11). Every item of the second part contains the factor p and p changes in a certain region, which means that some incident wave is not vertical to the surface of the Dammann grating. Therefore, in the receiving plane, some intensity will depart from the position corresponding to the case that the incident wave is vertical to the surface of the Dammann grating. Thus, the intensity uniformity of the spot array and the energy received by the spot array will reduce in the position that corresponds to the condition that the incident wave is vertical to the surface of the Dammann grating.

When m is an odd number, by Eq. (12), in the receiving plane there is not the intensity whose spatial frequency is zero. There would be spot arrays which stagger or superpose in the receiving plane because the incident wave is not vertical to the surface of the Dammann grating and the spatial frequency p changes in certain region. This would impact both the distribution of the energy in the receiving plane and the energy efficiency of the light source.

Adopting the data of a 5×5 spot array Dammann grating[14], the splitting beam effect was numerically simulated when the incident waves were fundamental Gaussian laser and plane wave. The period of the grating was T=0.25 mm, the grating contained 20×20 periods, and the aperture of the grating was 5×5 (mm).

Let P express the non-uniformity of the spot array intensity defined as the ratio of the standard deviation of spot array intensity to the average intensity. According to the data above, for a plane wave, P=13.03%. For a fundamental Gaussian laser, when the beam waist is in the grating, the incident wave field is expressed by

$$E(x,y) = A\exp\left[-\left(\frac{x^2+y^2}{\omega^2}\right)\right], \qquad (14)$$

where $E(x,y)$ is the intensity of the electric field, A is the amplitude, and ω is the radius of the beam waist.

Figure 1 shows the curve that P changes with the ratio of ω to T when the beam waist of the fundamental Gaussian laser is in the Dammann grating. P is comparatively high when ω is smaller than $0.6T$. However, when $\omega \geq 0.6T$, P keeps about 13%, which is the same to that for a plane wave. This validates the result of the theoretical study.

In addition, the simulation was accomplished when the beam waist of the fundamental Gaussian laser was not in the grating. The result shows that P is still as the same as that of a plane wave if $\omega \geq 0.6T$. So, the splitting beam effect of Dammann grating is sensitive only to the size of the laser beam waist whereas immune to the position of the laser beam waist.

Adopting the data of an 8×8 array Dammann grating[15], we simulated the light intensity arrays after split by the Dammann grating when the incident waves are TEM_{10}, TEM_{11}, and TEM_{20} mode laser, respectively. The results are shown in Figs. 2−4.
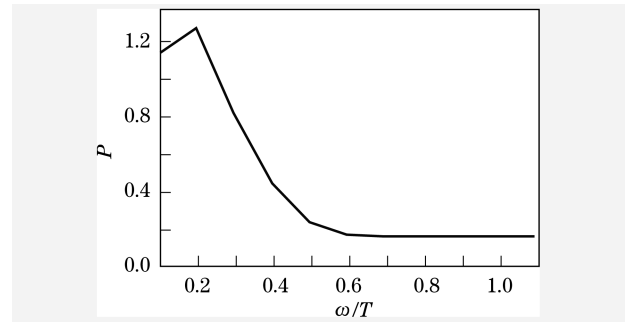

Fig. 1. Non-uniformity of the spot array intensity P changes with the radius of the beam waist ω.
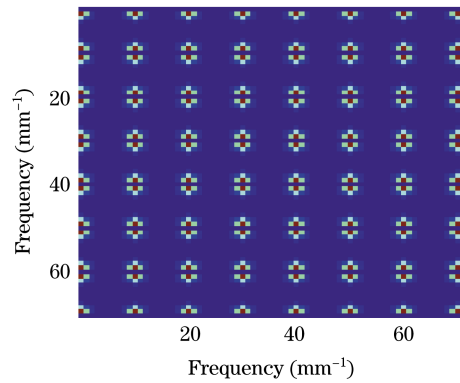

Fig. 2. Intensity array after split by the 8×8 Dammann grating when the incident wave is a TEM_{10} mode laser.
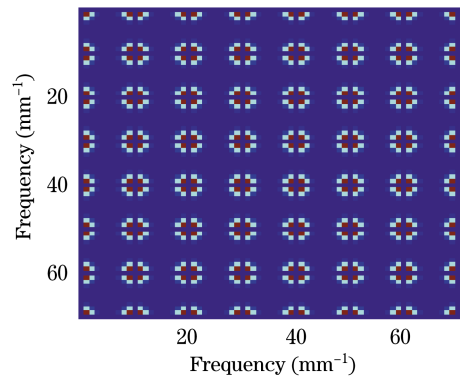

Fig. 3. Intensity array after split beam by the 8×8 Dammann grating when the incident wave is a TEM_{11} mode laser.
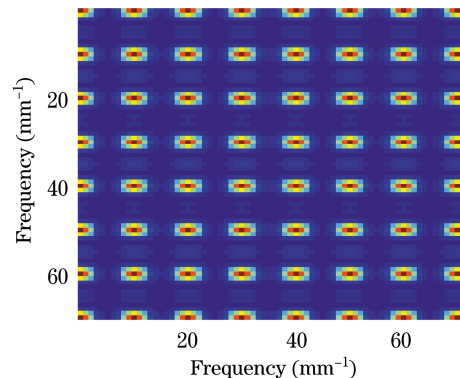

Fig. 4. Intensity array after split by the 8×8 Dammann grating when the incident wave is a TEM_{20} mode laser.

In Fig. 2, every light spot of the 8×8 array is split into two spots in y direction, which is corresponding to the two maxima of the mode TEM_{10} in frequency field. Figure 3 shows that every light spot of the 8×8 array is split into four spots, two in x direction and two in y direction, corresponding to the four maxima of the mode TEM_{11} in frequency field. Figure 4 shows that the light spot of the 8×8 array is not split any more, but the symmetry of every light spot is reduced. This result indicates that the first item of Eq. (11) is primary in intensity, so the position of the light spot intensity center is the same to the case of an incident plane wave. The effect of the other items in Eq. (11) except the first item is to debase the symmetry of every light spot. The simulation results have validated the result of the theoretical study.

In conclusion, the theoretical study and simulation are accomplished on the splitting beam effect by Dammann grating when the incident wave is of various laser modes. The effect of splitting beam by Dammann grating when the incident wave is a fundamental mode Gaussian laser is the same to the case of an incident plane wave. The splitting beam effect for a high order mode laser differs from that for a plane wave, which would lead to the change in array mode, as well as the fall in energy efficiency. The fundamental mode Gaussian laser is more suitable to be the light source in far-distance image detecting system that adopts Dammann grating in the illumination system. The conclusion is important for space applications.

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