Coherence of a squeezed sodium atom laser generated from Raman output coupling

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The coherence of a squeezed sodium atom laser generated from a Raman output coupler, in which the sodium atoms in Bose-Einstein condensate (BEC) interact with two light beams consisting of a weaker squeezed coherent probe light and a stronger classical coupling light, is investigated. The results show that in the case of a large mean number of BEC atoms and a weaker probe light field, the atom laser is antibunching, and this atom laser is second-order coherent if the number of BEC atoms in traps is large enough.

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The achievements of atomic Bose-Einstein condensate (BEC) and atom laser have pioneered a new research area of atomic physics and optics[1−3]. As a laser is made through the coupling output from a laser cavity of a coherent photon beam by means of a partial reflection mirror, a coherent beam, i.e., "atom laser", can be formed similarly when the ultra-cooling atoms are drawn out of the BEC in the trap. In 1997, Mewes *et al.* created an atom laser with the atom output from BEC by use of the radio frequency[4]. In 1999, Hagley *et al.* successfully developed the atom laser apparatus with controllable, adjustable, highly collimated, and "quasi continuous" characteristics, and made an important breakthrough in the development of a laser apparatus of the matter wave[5].

In recent years, scientists have put huge efforts into the investigation of the atomic BEC and the atom laser. They have already made a series of important successes[6−20]. In this letter, we present a scheme to generate a squeezed atom laser via stimulated Raman transition of the atoms in BEC interacting with two light beams, including a weaker squeezed coherent probe light and a stronger classical coupling light. The coherence of this atom laser is also analyzed.

We consider a Raman coupling system as follows. A large number of BEC sodium atoms in a trap are in the trapped state $|1\rangle(3S_{1/2},\ F=1,\ m_F=-1)$, which is coupled to the state $|3\rangle(3P_{3/2},\ F=2)$ via a weaker squeezed coherent probe light field with frequency ω_1, and the state $|3\rangle$ is coupled to the untrapped state $|2\rangle(3S_{1/2},\ F=1,\ m_F=0)$ via a stronger classical coupling light field with frequency ω_2. We assume that each light beam is identically largely detuned from $|1\rangle\rightarrow|3\rangle$ and $|2\rangle\rightarrow|3\rangle$ transitions. The interaction scheme is shown in Fig. 1. The coupling output of the atom beam in the untrapped state $|2\rangle$ should form the sodium atom laser. It is necessary to point out that our scheme is in a departure from that of Ref. [5], that is, we replace the general laser by a squeezed coherent probe light.

The second quantized Hamiltonian describing the above system takes the following form ($\hbar=1$) :

$$H=H_{\rm p}+H_{\rm a}+H_{\rm af}+H_{\rm aa}, \qquad (1)$$

where $H_{\rm p}$ and $H_{\rm a}$ give the free evolution of the probe-light field and the atomic fields respectively, $H_{\rm af}$ and $H_{\rm aa}$ describe the interaction between the atomic fields and the two light fields and the inter-atom interaction, respectively. They are respectively given by

$$H_{\rm p}=\omega_1 a_1^\dagger a_1, \qquad (2)$$

where $a_1^\dagger$ and a_1 denote the photon creation and annihilation operators of the probe-light field;

$$H_{\rm a}=\sum_{i=1}^{3}\nu_i b_i^\dagger b_i, \qquad (3)$$

where $b_i^\dagger$ and b_i represent the atom creation and annihilation operators in the state $|i\rangle(i=1,2,3)$, and ν_i gives the energy of i-mode with $\hbar=1$;

$$\begin{aligned}H_{\rm af}=&\ (g_1 b_3^\dagger a_1 b_1 {\rm e}^{-{\rm i}\omega_1 t}+g_1^* a_1^\dagger b_1^\dagger b_3 {\rm e}^{{\rm i}\omega_1 t})\\&+(g_2 b_3^\dagger b_2 {\rm e}^{-{\rm i}\omega_2 t}+g_2^* b_2^\dagger b_3 {\rm e}^{{\rm i}\omega_2 t}),\end{aligned} \qquad (4)$$

where g_1 and g_2 stand for the light-atom dipole interaction constants; and

$$H_{\rm aa}=\sum_{i=1}^{3}\lambda_i b_i^{\dagger 2} b_i^2+\sum_{\substack{i,j=1\\(i\neq j)}}^{3}\lambda_{ij} b_i^\dagger b_j^\dagger b_i b_j, \qquad (5)$$

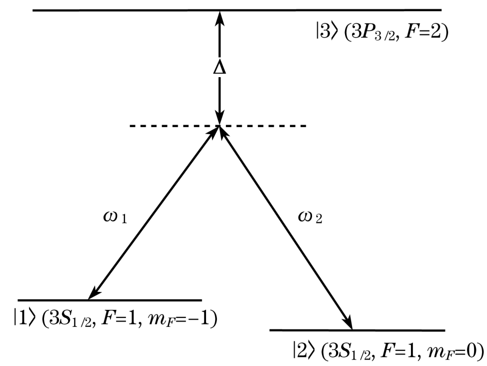

Fig. 1. Three-level Λ-shaped atoms coupled to two light beams[5].

where λ_i and λ_{ij} $(i,j=1,2,3)$ describe the inter-atom interactions.

Taking an interaction picture, we assume the Hamiltonian to be the sum of the free evolution Hamiltonian H_0 and the interaction Hamiltonian V:

$$H = H_0 + V, \tag{6}$$

where

$$\begin{aligned} H_0 = \ & \omega_1 a_1^\dagger a_1 + \nu_1 \sum_{i=1}^{3} b_i^\dagger b_i \\ & +(\omega_1-\omega_2) b_2^\dagger b_2 + \omega_1 b_3^\dagger b_3, \end{aligned} \tag{7}$$

$$\begin{aligned} V = \ & (\nu_3-\nu_1-\omega_1) b_3^\dagger b_3 + (g_1 b_3^\dagger a_1 b_1 + g_1^* a_1^\dagger b_1^\dagger b_3) \\ & +(g_2 b_3^\dagger b_2 + g_2^* b_2^\dagger b_3) + \sum_{i=1}^{3} \lambda_i b_i^{\dagger 2} b_i^2 \\ & + \sum_{\substack{i,j=1 \\ (i\neq j)}}^{3} \lambda_{ij} b_i^\dagger b_j^\dagger b_i b_j. \end{aligned} \tag{8}$$

We suppose that a large enough number of BEC atoms in the trap are in the trapped state $|1\rangle$ at the initial moment. Neglecting the thermal excitation and the quantum depletion, the states $|2\rangle$ and $|3\rangle$ can be treated as vacuum states. In the case of the large detuning for a weaker probe light field and a stronger coupling light field, the atom numbers in the states $|2\rangle$ and $|3\rangle$ are substantially less than that in state $|1\rangle$ and do not have a noticeable change. On the one hand, the inter-atom interaction of the atoms in the states $|2\rangle$ and $|3\rangle$ can be neglected. Therefore, Eq. (8) can be simplified as

$$\begin{aligned} V = \ & (\nu_3-\nu_1-\omega_1) b_3^\dagger b_3 + (g_1 b_3^\dagger a_1 b_1 + g_1^* a_1^\dagger b_1^\dagger b_3) \\ & +(g_2 b_3^\dagger b_2 + g_2^* b_2^\dagger b_3) + \lambda_1 b_1^\dagger b_1^\dagger b_1 b_1. \end{aligned} \tag{9}$$

On the other hand, the atomic operators b_3 and $b_3^\dagger$ can be adiabatically eliminated from Eq. (9) by the equations $b_3 = -\dfrac{g_1 a_1 b_1 + g_2 b_2}{\nu_3-\nu_1-\omega_1}$ and $b_3^\dagger = -\dfrac{g_1^* a_1^+ b_1^+ + g_2^* b_2^+}{\nu_3-\nu_1-\omega_1}$, since $\mathrm{i}\dot{b}_3 = [b_3, V] \approx 0$ and $\mathrm{i}\dot{b}_3^\dagger = [b_3^\dagger, V] \approx 0$ from the Heisenberg equations. Hence, Eq. (9) can be reduced to

$$\begin{aligned} V = \ & -\omega_2' b_2^\dagger b_2 - (g' b_2^\dagger a_1 b_1 + g'^* a_1^\dagger b_1^\dagger b_2) \\ & -\omega_1' a_1^\dagger b_1^\dagger a_1 b_1 + \lambda_1 b_1^{\dagger 2} b_1^2, \end{aligned} \tag{10}$$

where $\omega_1' = \frac{|g_1|^2}{\Delta}$, $\omega_2' = \frac{|g_2|^2}{\Delta}$, and $g' = \frac{g_1 g_2^*}{\Delta}$ with the detuning $\Delta = \nu_3-\nu_1-\omega_1$. Similarly, since $\mathrm{i}\dot{b}_2 = [b_2, V] \approx 0$ and $\mathrm{i}\dot{b}_2^\dagger = [b_2^\dagger, V] \approx 0$ from the Heisenberg equations, the atomic operators of state $|2\rangle$ can be deleted in Eq. (10) by relations $b_2 = -\frac{g' a_1 b_1}{\omega_2'}$ and $b_2^\dagger = -\frac{g'^* a_1^+ b_1^+}{\omega_2'}$. Therefore, the final effective interaction Hamiltonian is given by

$$V_{\text{eff}} = 2\omega_1' a_1^\dagger a_1 b_1^\dagger b_1 + \lambda_1 b_1^\dagger b_1^\dagger b_1 b_1. \tag{11}$$

Assuming that at the initial moment all the BEC atoms stay in the coherent state $|\beta\rangle$ and the probe light field is in the squeezed coherent state $|\alpha,\xi\rangle$, we can write the initial state vector as

$$|\Psi(0)\rangle = |\alpha,\xi\rangle \otimes |\beta\rangle, \tag{12}$$

where

$$\begin{aligned} |\alpha,\xi\rangle = \ & \mathrm{e}^{-\frac{1}{2}|\alpha|^2 + \frac{\tanh r}{2}\alpha^2} \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} \frac{1}{\sqrt{\cosh r}} \left(\frac{\tanh r}{2}\right)^{n/2} \\ & \times H_n\left(\frac{\alpha}{\sqrt{2\sinh r \cosh r}}\right)|n\rangle, \end{aligned} \tag{13}$$

$$|\beta\rangle = \mathrm{e}^{-\frac{|\beta|^2}{2}} \sum_{m=0}^{\infty} \frac{\beta^m}{\sqrt{m!}} |m\rangle, \tag{14}$$

where $\xi = r\mathrm{e}^{\mathrm{i}\theta}$, $\alpha = \sqrt{\bar{n}}\mathrm{e}^{\mathrm{i}\varphi}$, $\bar{n}$ denotes the mean number of photons of the probe light field and r is the squeezing factor, $H_n(x)$ is the n-order Hermite polynomials in Eq. (13), $\beta = \sqrt{\bar{N}_1}\mathrm{e}^{\mathrm{i}\eta}$ in Eq. (14). For simplicity, we take $\theta = 0$, $\varphi = 0$, $\eta = 0$.

To solve the Schröinger equation, we obtain the state vector of the system at any moment $t > 0$ as

$$\begin{aligned} |\Psi(t)\rangle = \ & \sum_{n,m=0}^{\infty} \exp\left\{ -\frac{1}{2}|\alpha|^2 + \frac{\tanh r}{2}\alpha^2 - \frac{|\beta|^2}{2} \right. \\ & \left. -\mathrm{i}[2\omega_1' nm + \lambda_1 m(m-1)]t \right\} \\ & \times \frac{1}{\sqrt{n!m!}} \frac{\beta^m}{\sqrt{\cosh r}} \left(\frac{\tanh r}{2}\right)^{n/2} \\ & \times H_n\left(\frac{\alpha}{\sqrt{2\sinh r \cosh r}}\right)|n,m\rangle. \end{aligned} \tag{15}$$

Next, we will investigate the coherent properties of the atom laser by calculating the second-order normalized correlation functions of this atom laser:

$$g^{(2)} = \frac{\langle b_2^\dagger b_2^\dagger b_2 b_2\rangle}{\langle b_2^\dagger b_2\rangle^2} = \frac{\langle (b_2^\dagger b_2)^2\rangle - \langle b_2^\dagger b_2\rangle}{\langle b_2^\dagger b_2\rangle^2}, \tag{16}$$

where $b_2 = -\dfrac{g' a_1 b_1}{\omega_2'}$ and $b_2^\dagger = -\frac{g'^* a_1^\dagger b_1^\dagger}{\omega_2'}$.

In the case of a weaker probe light field and the mean number $\bar{N}_1$ of BEC atoms at the initial moment $t = 0$ is so large that the influence of transition of the mean number of BEC atoms can be neglected in the evolution of the system, the state vector of the system at any moment $t > 0$ can be simplified as follows. Firstly, since the mean number of BEC atoms is large enough, the population function P_n of BEC atoms can be approximately replaced by a δ-function as shown in Fig. 2, and the sum can be transformed into the integral in Eq. (15) as

$$\sum_{m=0}^{\infty} \frac{\mathrm{e}^{-\bar{N}_1} \bar{N}_1^m}{m!} \quad \rightarrow \quad \int_0^\infty \mathrm{d}m\, \delta\left(m - \bar{N}_1\right). \tag{17}$$

Secondly, since the number of the photons is much less than that of the BEC atoms for the weaker probe light field, the state vector in Eq. (15) is finally reduced to

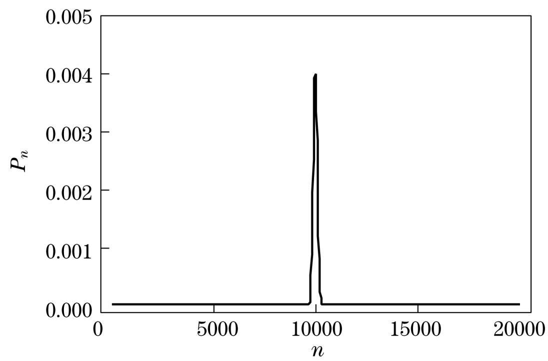

Fig. 2. Population function P_n of BEC atoms with $\bar{N}_1 = 10000$.

$$
\begin{aligned}
|\psi(t)\rangle = {} & \mathrm{e}^{-\mathrm{i}\lambda_1 \bar{N}_1(\bar{N}_1-1)t}\mathrm{e}^{-\frac{1}{2}|\alpha|^2+\frac{\tanh\ r}{2}\alpha^2} \\
& \sum_{n=0}^{\infty}\mathrm{e}^{-\mathrm{i}\,n\,\omega\,t}\frac{1}{\sqrt{n!}}\frac{1}{\sqrt{\cosh\ r}}\left(\frac{\tanh\ r}{2}\right)^{n/2} \\
& \times H_n\left(\frac{\alpha}{\sqrt{2\sinh\ r\ \cosh\ r}}\right)|n\rangle|\bar{N}_1\rangle, \qquad (18)
\end{aligned}
$$

where $\omega = 2\omega_1'\,\bar{N}_1 = 2\frac{|g_1|^2}{\Delta}\,\bar{N}_1$.

Using $b_2 = -\frac{g' a_1 b_1}{\omega_2'}$, $b_2^{\dagger} = -\frac{g'^{*} a_1^{\dagger} b_1^{\dagger}}{\omega_2'}$, and Eq. (18), we find that in the state $|\psi(t)\rangle$, the second-order normalized correlation functions of the atom laser can be expressed as

$$g^{(2)} = 1 - \frac{1}{\bar{N}_1}. \qquad (19)$$

For a huge $\bar{N}_1$, $g^{(2)} \rightarrow 1$, the atom laser is second-order coherent. Since in the experiment made by Hagley *et al.*[5], the number of atoms reaches the order of 10^6, their atom laser should has a very good coherence.

In summary, we have proposed a theoretical model and constructed its Hamiltonian regarding the system of Λ-type three-level atomic BEC interacting with two light beams, via stimulated Raman transition. We have also investigated the second-order coherent properties of the atom laser extracted from this system. Our results demonstrate that in the case of a large mean number of BEC atoms and a weaker probe light field, the atom laser is antibunching, and this atom laser is second-order coherent if the number of BEC atoms in traps is large enough.

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