Sudden birth of entanglement between two atoms in a double JC model

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Sudden birth of entanglement between two initially separate atoms interacting with two entangled photons in a double JC model is investigated, and the influences of different atomic initial states on entanglement among atoms are discussed. The results show that sudden birth of entanglement can occur when the two atoms are initially in excited states.

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Entanglement is one of the most striking features of quantum mechanics. It is a key problem in Einstein-Podolsky-Rosen (EPR) paradox[1], quantum cryptography[2], quantum teleportation[3], quantum computation[4], and so on. The simplest situation, studying entanglement dynamics of qubit pairs in different scenarios, is of great importance and has become the focus of much theoretical and experimental work[5−9].

Recently, it has been shown that entanglement may terminate abruptly in a finite time. This phenomenon is usually called sudden death of entanglement (ESD). ESD is an intriguing and potentially very important discovery. Since the first theory of ESD is demonstrated, further investigations have been made by different groups. Yu and Eberly[10,11] studied the ESD between the two particles coupled with two independent environments. And Zhang[12,13], Yang[14], and Metwally[15] explored that ESD between two atoms occured in a T-C model. In addition, the experimental researchers[16,17] carried out ESD through using engineered interaction between systems and environments. Interestingly, as the entanglement can suddenly disappear, it can be suddenly generated, and the process is called entanglement sudden birth[18−20] (ESB). In this letter, we show that there is ESB between two initially separate atoms under the certain condition in a double JC model.

A double JC model consists of two two-level atoms labeled A and B and two single-mode fields labeled a and b (A interacting only with a and similarly for B and b). The Hamiltonian of the system (in the rotating wave approximation and setting $\hbar = 1$) is given by

$$\hat{H}_{\mathrm{tot}} = \hat{H}_{\mathrm{A}} + \hat{H}_{\mathrm{B}}, \quad (1)$$

$$\hat{H}_n = \frac{\omega_n \hat{\sigma}_{\mathrm{z}}^n}{2} + \nu_n(\hat{a}_n^\dagger \hat{a}_n + \frac{1}{2}) + g_n(\hat{a}_n^\dagger \sigma_-^n + \hat{a}_n \hat{\sigma}_+^n). \quad (2)$$

Where n=A, B denote n-th atoms; $\hat{a}_n^\dagger$ and $\hat{a}_n$ are the creation and annihilation operators of the field mode of frequency ν_n; ω_n is the atomic transition frequency; $\hat{\sigma}_{\mathrm{z}}^n$, $\hat{\sigma}_+^n$, and $\hat{\sigma}_-^n$ are the usual pseudo-spin operators of the two-level atom; and g_n is the atom-field coupling constant.

The usual way to identify entanglement between two qubits in a mixed state is to examine Wootters' concurrence and the concurrence C is defined as

$$C(t) = \max\left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\}, \quad (3)$$

where the quantities λ_i are the eigenvalues in a decreasing order of the matrix

$$R = \rho(\hat{\sigma}_{\mathrm{y}} \otimes \hat{\sigma}_{\mathrm{y}})\rho^*(\hat{\sigma}_{\mathrm{y}} \otimes \hat{\sigma}_{\mathrm{y}}), \quad (4)$$

where ρ^* denotes the complex conjugation of ρ in the standard basis and $\hat{\sigma}_{\mathrm{y}}$ is the Pauli matrix. The range of concurrence is from 0 to 1, when $C = 0$ indicates zero entanglement (two separated atoms) and $C = 1$ means maximally entangled atoms.

We consider that at $t = 0$ the fields are in the photon entangled states

$$|\Psi_{(0)}\rangle_f = \cos\alpha|01\rangle + \mathrm{e}^{\mathrm{i}\varphi}\sin\alpha|10\rangle, \quad (5)$$

and the two atoms are specified below. Here, we assume that the two atoms are initially in excited states $|ee\rangle$, so the total system is

$$|\Psi_{(0)}\rangle = \cos\alpha|ee01\rangle + \sin\alpha|ee10\rangle. \quad (6)$$

At time t, the state vector of the total system can be expressed in the standard basis

$$\begin{aligned}|\Psi(t)\rangle = {} & x_1|ee01\rangle + x_2|ge11\rangle + x_3|eg02\rangle + x_4|gg12\rangle \\ & + x_5|ee10\rangle + x_6|ge20\rangle \\ & + x_7|eg11\rangle + x_8|gg21\rangle,\end{aligned} \quad (7)$$

where the coefficients are given by

$$\begin{aligned} x_1 &= r_{\mathrm{A}}(t)u_{\mathrm{B}}(t)\cos\alpha, \\ x_2 &= s_{\mathrm{A}}(t)u_{\mathrm{B}}(t)\cos\alpha, \\ x_3 &= r_{\mathrm{A}}(t)v_{\mathrm{B}}(t)\cos\alpha, \\ x_4 &= s_{\mathrm{A}}(t)v_{\mathrm{B}}(t)\cos\alpha, \\ x_5 &= u_{\mathrm{A}}(t)r_{\mathrm{B}}(t)\sin\alpha, \\ x_6 &= v_{\mathrm{A}}(t)r_{\mathrm{B}}(t)\sin\alpha, \\ x_7 &= u_{\mathrm{A}}(t)s_{\mathrm{B}}(t)\sin\alpha, \\ x_8 &= v_{\mathrm{A}}(t)s_{\mathrm{B}}(t)\sin\alpha, \end{aligned} \quad (8)$$

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with

$$
\begin{aligned}
r_n(t) &= \mathrm{e}^{-\mathrm{i}\nu_n t}\left[\cos(\Omega_n t) - \frac{\mathrm{i}\Delta_n \sin(\Omega_n t)}{2\Omega_n}\right],\\
s_n(t) &= \frac{-\mathrm{i}g_n \mathrm{e}^{-\mathrm{i}\nu_n t}\sin(\Omega_n t)}{\Omega_n},\\
u_n(t) &= \mathrm{e}^{-2\mathrm{i}\nu_n t}\left[\cos(\Omega_n^{'} t) - \frac{\mathrm{i}\Delta_n \sin(\Omega_n^{'} t)}{2\Omega_n^{'}}\right],\\
v_n(t) &= \frac{-\sqrt{2}\mathrm{i}g_n \mathrm{e}^{-2\mathrm{i}\nu_n t}\sin(\Omega_n^{'} t)}{\Omega_n^{'}},
\end{aligned} \tag{9}
$$

where the Rabi frequencies are $\Omega_n = \left(g_n^2 + \frac{\Delta_n^2}{4}\right)^{\frac{1}{2}}$, $\Omega_n^{'} = \left(2g_n^2 + \frac{\Delta_n^2}{4}\right)^{\frac{1}{2}}$, $\Delta_n = \omega_n - \nu_n$. For simplicity, we consider only the resonant case ($\Delta_{\mathrm{B}} = \Delta_{\mathrm{A}} = 0$).

The reduced density matrix ρ^{AB} for the two atoms can be obtained by tracing out the photonic part of the total pure state. ρ^{AB} can be given by in the basis $|ee\rangle$, $|ge\rangle$, $|eg\rangle$, and $|gg\rangle$:

$$
\rho^{\mathrm{AB}} = \begin{pmatrix}
|x_1|^2 + |x_5|^2 & 0 & 0 & 0\\
0 & |x_2|^2 + |x_6|^2 & x_2 x_7^* & 0\\
0 & x_2^* x_7 & |x_3|^2 + |x_7|^2 & 0\\
0 & 0 & 0 & |x_4|^2 + |x_8|^2
\end{pmatrix}, \tag{10}
$$

and the concurrence for the matrix is given by

$$
\begin{aligned}
C^{\mathrm{AB}}(t) = \; & 2\max\{0, |x_2 x_7^*| \\
& - \sqrt{(|x_1|^2 + |x_5|^2)(|x_4|^2 + |x_8|^2)}\}.
\end{aligned} \tag{11}
$$

We present numerical results of the concurrence C given by Eq. (11) for different initial state parameters of two fields and the ratio of two atom-field coupling constants.

In Fig. 1(a), we take the ratio of two atom-field coupling constants $g_{\mathrm{B}}/g_{\mathrm{A}} = 1$ and the fields parameter $\alpha = \pi/4$ (the two fields are initially in maximal entangled state). It is obvious that the concurrence C between two atoms is zero at earlier time, corresponding to the two atoms are in disentangled states. After a finite time, C abruptly increases to peak value, and the ESB occurs. Coming through a short time, C abruptly decreases to zero value, and the ESD happens. As the time goes on, the ESB and ESD appear repeatedly. When $\alpha = \pi/8$, corresponding to the fields are initially in partial entangled states, the peak values of C reduce and the ESB and ESD still occur, as shown in Fig. 1(b). When two coupling constants g_{B} and g_{A} are unequal, the peak value width of C decreases and the times of ESB and ESD increase as the ratio of two atom-field coupling constants increases, as shown in Figs. 1(a), (c), and (d).

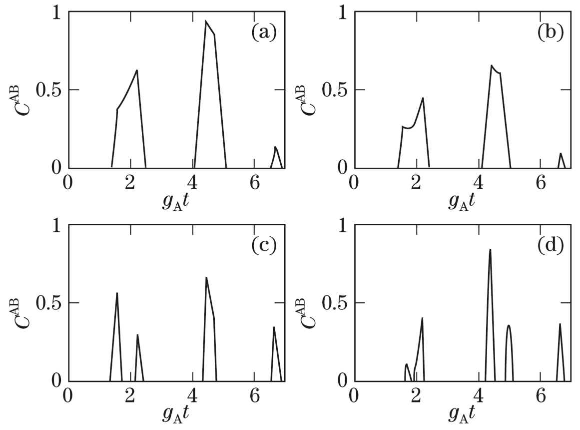

Fig. 1. Time evolution of the concurrence C for the atom-atom entanglement with the atomic initial state $|\Psi_{\mathrm{atom}}(0)\rangle = |ee\rangle$ and (a) $\alpha = \pi/4$, $g_{\mathrm{B}}/g_{\mathrm{A}} = 1$, (b) $\alpha = \pi/8$, $g_{\mathrm{B}}/g_{\mathrm{A}} = 1$, (c) $\alpha = \pi/4$, $g_{\mathrm{B}}/g_{\mathrm{A}} = 3$, and (d) $\alpha = \pi/4$, $g_{\mathrm{B}}/g_{\mathrm{A}} = 4$.

Alternatively, we assume that the two atoms are initially in the ground states $|gg\rangle$, so the total system is

$$
|\Psi_{(0)}\rangle = \cos\alpha|gg01\rangle + \sin\alpha|gg10\rangle. \tag{12}
$$

Then we can obtain the following expression for the state vector of the total system at time t,

$$
|\Psi(t)\rangle = c_1|gg01\rangle + c_2|ge00\rangle + c_3|gg10\rangle + c_4|eg00\rangle, \tag{13}
$$

and the corresponding coefficients are

$$
\begin{aligned}
c_1 &= f_{\mathrm{B}}(t)h_{\mathrm{A}}(t)\cos(\alpha),\\
c_2 &= g_{\mathrm{B}}(t)h_{\mathrm{A}}(t)\cos(\alpha),\\
c_3 &= f_{\mathrm{A}}(t)h_{\mathrm{B}}(t)\sin(\alpha),\\
c_4 &= g_{\mathrm{A}}(t)h_{\mathrm{B}}(t)\sin(\alpha),
\end{aligned} \tag{14}
$$

with

$$
\begin{aligned}
f_n(t) &= \mathrm{e}^{-\mathrm{i}\nu_n t}\left[\cos(\Omega_n t) + \frac{\mathrm{i}\Delta_n \sin(\Omega_n t)}{2\Omega_n}\right],\\
g_n(t) &= \mathrm{e}^{-\mathrm{i}\nu_n t}\frac{-\mathrm{i}g_n \sin(\Omega_n t)}{\Omega_n},\\
h_n(t) &= \mathrm{e}^{\frac{\mathrm{i}\Delta_n t}{2}},
\end{aligned} \tag{15}
$$

where the Rabi frequencies are $\Omega_n = (g_n^2 + \frac{\Delta_n^2}{4})^{\frac{1}{2}}$, $\Delta_n = \omega_n - \nu_n$. For simplicity, we only consider the resonant case ($\Delta_{\mathrm{B}} = \Delta_{\mathrm{A}} = 0$).

In the basis of $|ee\rangle$, $|ge\rangle$, $|eg\rangle$, and $|gg\rangle$, the reduced density matrix ρ^{AB} is given by

$$
\rho^{\mathrm{AB}} = \begin{pmatrix}
0 & 0 & 0 & 0\\
0 & |c_2|^2 & c_2 c_4^* & 0\\
0 & c_2^* c_4 & |c_4|^2 & 0\\
0 & 0 & 0 & |c_1|^2 + |c_3|^2
\end{pmatrix}, \tag{16}
$$

The concurrence associated with the density matrix ρ^{AB} is given by

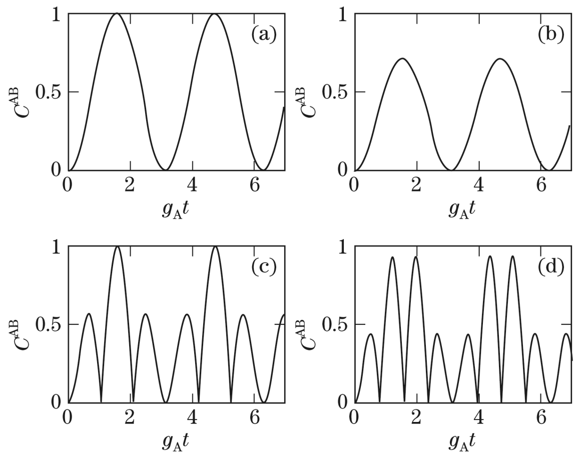

Fig. 2. Time evolution of the concurrence C for the atom-atom entanglement with the atomic initial state $|\Psi_{\rm atom}(0)\rangle = |gg\rangle$ and (a) $\alpha = \pi/4$, $g_B/g_A = 1$, (b) $\alpha = \pi/8$, $g_B/g_A = 1$, (c) $\alpha = \pi/4$, $g_B/g_A = 3$, and (d) $\alpha = \pi/4$, $g_B/g_A = 4$.

$$C^{\rm AB}(t) = 2\max\{0, |c_2 c_4^*|\}. \quad (17)$$

The numerable results of Eq. (17) are presented in Fig. 2. It is evident that the concurrence C of two atoms evolve periodically (period is π) and there is no ESB and ESD between the two atoms.

In conclusion, we investigate the ESB between two initially separate atoms in a double JC model. We find that the ESB between the two atoms depends on the initial state of two atoms. When the two atoms are initially in excited states, the ESB between two atoms can occur. When the two atoms are initially in ground states, the periodic entanglement dynamics of two atoms can be produced, but there is no ESB between the two atoms. We also find that the ratio of two atom-field coupling constants has influence on the ESB between the two atoms. The larger the ratio is, the more times the ESB can occur.

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