Teleportation of a two-particle four-component squeezed vacuum state by linear optical elements

Huina Chen (陈慧娜)[1] and Jinming Liu (刘金明)[1,2*]

[1]*State Key Laboratory of Precision Spectroscopy, Department of Physics, East China Normal University, Shanghai 200062, China*

[2]*Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117542, Singapore*

**E-mail: jmliu@phy.ecnu.edu.cn*

Received August 22, 2008

We present a linear optical scheme for achieving a unity fidelity teleportation of a two-particle four-component squeezed vacuum state using two entangled squeezed vacuum states as quantum channel. The devices used are beam splitters and ideal photon detectors capable of distinguishing between odd and even photon numbers. Moreover, we also obtain the success probability of the teleportation scheme.

OCIS codes: 270.0270, 270.6570, 000.1600.

doi: 10.3788/COL20090705.0440.

Quantum entanglement plays an essentially central role in accomplishing current tasks of quantum information processing[1−4]. One of the important exhibitions of entangled states is quantum teleportation[5], which is a process that a sender wants to transfer an unknown state to a remote receiver via a priorly shared entangled state with the assistance of some classical information. In the past few years, teleportation has attracted a large number of attention due to its remarkable applications in quantum communication and quantum computation. In experiment, the earlier implementations of teleportation have mainly focused on the discrete-variable states described by a Hilbert space of finite dimension, such as the superposed photon polarization state in optical systems[6,7] or the superposition of atomic internal states in trapped ions[8]. Furthermore, discrete-variable teleportation has been generalized to the cases of continuous-variable corresponding to quantum states of a system with an infinite-dimensional state space[9−11]. Based on the theoretical protocol of Ref. [9], experimental teleportation of a single coherent mode of a radiation field has been demonstrated with a two-mode squeezed vacuum state[10,11].

Recently, Enk *et al*. proposed a novel linear optical scheme for teleporting a coherent superposition state by using an entangled coherent state[12]. Wang presented how to teleport an entangled coherent state with only linear optical devices[13]. Subsequently, optical schemes for the teleportation of a single-mode superposed coherent state or of an entangled coherent state have been widely investigated[14−18]. Cai *et al*. described a proposal to teleport a superposition state of two equal-amplitude and opposite-phase squeezed vacuum states (SVSs) through an entangled SVS[19]. This scheme was then extended to teleport a two-mode two-component entangled SVS[20]. Following the ideas of Refs. [21,22], in this letter, we present an optical scheme to teleport a bipartite four-component SVS with a unity fidelity using only linear optical devices like beam splitters and photon detectors, and two entangled SVSs are utilized as the quantum channel.

Suppose that an unknown two-particle arbitrary SVS that a sender Alice wants to transmit to a receiver Bob is in the following form

$$\begin{aligned}|\phi\rangle_{12}=&\frac{1}{\sqrt{N_{12}}}(x_1|\xi\rangle_1|\xi\rangle_2+x_2|\xi\rangle_1|-\xi\rangle_2\\&+x_3|-\xi\rangle_1|\xi\rangle_2+x_4|-\xi\rangle_1|-\xi\rangle_2),\end{aligned}\qquad(1)$$

where x_1, x_2, x_3, and x_4 are unknown complex numbers, $|\xi\rangle$ and $|-\xi\rangle$ are two single-mode SVSs, $\xi=r\mathrm{e}^{\mathrm{i}\theta}$ is a complex number with squeezing amplitude r and squeezing angle θ. Generally, on the basis of Fock state, $|\xi\rangle$ can be expanded as

$$|\xi\rangle=\sqrt{\mathrm{sech}r}\sum_{m=0}^{\infty}\frac{\sqrt{(2m)!}}{m!2^m}(-\mathrm{e}^{\mathrm{i}\theta}\tanh r)^m|2m\rangle.\qquad(2)$$

The normalization factor N_{12} of the state $|\phi\rangle_{12}$ is described by

$$\begin{aligned}N_{12}=&(|x_1|^2+|x_2|^2+|x_3|^2+|x_4|^2\\&+2x_\xi\mathrm{Re}[x_1^*x_3+x_2^*x_4+x_1^*x_2+x_3^*x_4]\\&+2x_\xi^2\mathrm{Re}[x_1^*x_4+x_2^*x_3]),\end{aligned}\qquad(3)$$

with $x_\xi=\langle\xi|-\xi\rangle=\sqrt{\mathrm{sech}(2r)}$. To achieve the teleportation, we also suppose that Alice and Bob previously share the quantum channel consisting of the following two entangled SVSs:

$$|\phi\rangle_{34}=\frac{1}{\sqrt{M}}(|\xi\rangle_3|\xi\rangle_4-|-\xi\rangle_3|-\xi\rangle_4),\qquad(4)$$

$$|\phi\rangle_{56}=\frac{1}{\sqrt{M}}(|\xi\rangle_5|\xi\rangle_6-|-\xi\rangle_5|-\xi\rangle_6),\qquad(5)$$

where $M=2\left[1-x_\xi^2\right]$. Note that the quantum states $|\phi\rangle_{34}$ and $|\phi\rangle_{56}$ are both maximally entangled states with the amount of entanglement being exactly one ebit. At the beginning of the teleportation process, the initial state of the whole system including the modes 1, 2, 3,

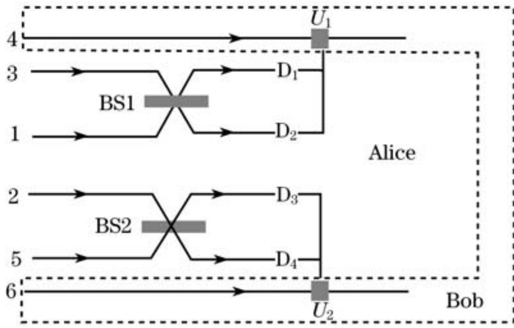

Fig. 1. Schematic illustration of teleporting a bipartite four-component SVS, where BS1 and BS2 denote 50/50 beam splitters, D_1, D_2, D_3, and D_4 denote photon detectors, and U_1 and U_2 denote unitary operation.

4, 5, and 6 is given by $|\psi\rangle = |\phi\rangle_{12} \otimes |\phi\rangle_{34} \otimes |\phi\rangle_{56}$, where modes 1, 2, 3, and 5 belong to Alice while modes 4 and 6 belong to Bob. The schematic diagram for teleportation is depicted in Fig. 1. From the figure, one can see that six modes of light, two beam splitters and four photon detectors are needed to achieve our teleportation scheme.

Now, Alice lets modes 1, 3 and modes 2, 5 at her side enter the input ports of the lossless 50/50 beam splitters BS1 and BS2, respectively. The 50/50 beam splitter is described by $A_{jk} = e^{i\pi(a_j^+ a_k + a_k^+ a_j)/4}$, here a_l^+ and a_l ($l = j,\ k$) are the bosonic creation and annihilation operators for the two light beams entering the two input ports of the beam splitter. B_l^+ and B_l are assumed to denote the bosonic creation and annihilation operators leaving the two output ports of the beam splitter. Then we can get the following input-output relations between a_j, a_k and B_j, B_k:

$$B_j = \frac{1}{\sqrt{2}}(a_j + ia_k),\ B_k = \frac{1}{\sqrt{2}}(a_k + ia_j). \tag{6}$$

A straightforward calculation yields that the input state $|\psi\rangle_{in} = S_j(\xi_j)S_k(\xi_k)\,|00\rangle_{jk}$ through the beam splitter can be transformed into

$$\begin{aligned}|\psi\rangle_{\text{out}} = \ & \exp[\frac{1}{4}(\xi_k - \xi_j)B_j^{+2} \\ & + \frac{1}{4}(\xi_j^* - \xi_k^*)B_j^2 + \frac{1}{4}(\xi_j - \xi_k)B_k^{+2} \\ & + \frac{1}{4}(\xi_k^* - \xi_j^*)B_k^2 - \frac{i}{2}(\xi_j + \xi_k)B_j^+ B_k^+ \\ & - \frac{i}{2}(\xi_j^* + \xi_k^*)B_j B_k]\,|00\rangle_{jk}\,,\end{aligned} \tag{7}$$

where $S_l(\xi_l) = \exp(-\frac{\xi_l}{2}a_l^{+2} + \frac{\xi_l^*}{2}a_l^2)$ denotes the single-mode squeezing operator for mode l ($l = j,\ k$) with the parameter $\xi_l = r_l e^{i\theta_l}$.

For the sake of convenience, we describe the following two specific cases of beam-splitter transformation which are useful for the implementation of our teleportation scheme. Case 1: if the two input light beams have the equal squeezing amplitudes and the equal phases, i.e., $r_j = r_k = r$, $\theta_j = \theta_k = \theta$, then Eq. (**??**) can be rewritten as

$$|\psi\rangle_{\text{out}} = \exp[-ir(e^{i\theta}B_j^+ B_k^+ + e^{-i\theta}B_j B_k)]\,|00\rangle_{jk}\,. \tag{8}$$

Case 2: if the two input light beams have the equal squeezing amplitudes but the opposite phases, i.e., $r_j = r_k = r$, $\theta_k - \theta_j = \pi$, the output state leaving the beam splitter becomes a superposition of two single-mode squeezed vacuum states:

$$\begin{aligned}|\psi\rangle_{\text{out}} = \ & \exp[-\frac{1}{2}r(e^{i\theta}B_j^{+2} - e^{-i\theta}B_j^2)] \\ & \times\exp[\frac{1}{2}r(e^{i\theta}B_k^{+2} - e^{-i\theta}B_k^2)]\,|00\rangle_{jk}\,.\end{aligned} \tag{9}$$

According to Eqs. (8) and (9), after modes 1, 3 and modes 2, 5 pass through the beam splitters BS1 and BS2, respectively, the initial state $|\psi\rangle$ of the total system evolves to

$$\begin{aligned}\left|\psi'\right\rangle_{\text{out}} = \ & \frac{1}{M\sqrt{N_{12}}}[x_1 S_{13}(i\xi)S_{25}(i\xi)\,|00\rangle_{13}\,|00\rangle_{25}\,|\xi\rangle_4\,|\xi\rangle_6 + x_2 S_{13}(i\xi)S_2(-\xi)S_5(\xi)\,|00\rangle_{13}\,|0\rangle_2\,|0\rangle_5\,|\xi\rangle_4\,|\xi\rangle_6 \\ & + x_3 S_1(-\xi)S_3(\xi)S_{25}(i\xi)\,|0\rangle_1\,|0\rangle_3\,|00\rangle_{25}\,|\xi\rangle_4\,|\xi\rangle_6 + x_4 S_1(-\xi)S_3(\xi)S_2(-\xi)S_5(\xi)\,|0\rangle_1\,|0\rangle_3\,|0\rangle_2\,|0\rangle_5\,|\xi\rangle_4\,|\xi\rangle_6 \\ & - x_1 S_1(\xi)S_3(-\xi)S_{25}(i\xi)\,|0\rangle_1\,|0\rangle_3\,|00\rangle_{25}\,|-\xi\rangle_4\,|\xi\rangle_6 - x_2 S_1(\xi)S_3(-\xi)S_2(-\xi)S_5(\xi)\,|0\rangle_1\,|0\rangle_3\,|0\rangle_2\,|0\rangle_5\,|-\xi\rangle_4\,|\xi\rangle_6 \\ & - x_3 S_{13}(-i\xi)S_{25}(i\xi)\,|00\rangle_{13}\,|00\rangle_{25}\,|-\xi\rangle_4\,|\xi\rangle_6 - x_4 S_{13}(-i\xi)S_2(-\xi)S_5(\xi)\,|00\rangle_{13}\,|0\rangle_2\,|0\rangle_5\,|-\xi\rangle_4\,|\xi\rangle_6 \\ & - x_1 S_{13}(i\xi)S_2(\xi)S_5(-\xi)\,|00\rangle_{13}\,|0\rangle_2\,|0\rangle_5\,|\xi\rangle_4\,|-\xi\rangle_6 - x_2 S_{13}(i\xi)S_{25}(-i\xi)\,|00\rangle_{13}\,|00\rangle_{25}\,|\xi\rangle_4\,|-\xi\rangle_6 \\ & - x_3 S_1(-\xi)S_3(\xi)S_2(\xi)S_5(-\xi)\,|0\rangle_1\,|0\rangle_3\,|0\rangle_2\,|0\rangle_5\,|\xi\rangle_4\,|-\xi\rangle_6 - x_4 S_1(-\xi)S_3(\xi)S_{25}(-i\xi)\,|0\rangle_1\,|0\rangle_3\,|00\rangle_{25}\,|\xi\rangle_4\,|-\xi\rangle_6 \\ & + x_1 S_1(\xi)S_3(-\xi)S_2(\xi)S_5(-\xi)\,|0\rangle_1\,|0\rangle_3\,|0\rangle_2\,|0\rangle_5\,|-\xi\rangle_4\,|-\xi\rangle_6 + x_2 S_1(\xi)S_3(-\xi)S_{25}(-i\xi)\,|0\rangle_1\,|0\rangle_3\,|00\rangle_{25}\,|-\xi\rangle_4\,|-\xi\rangle_6 \\ & + x_3 S_{13}(-i\xi)S_2(\xi)S_5(-\xi)\,|00\rangle_{13}\,|0\rangle_2\,|0\rangle_5\,|-\xi\rangle_4\,|-\xi\rangle_6 + x_4 S_{13}(-i\xi)S_{25}(-i\xi)\,|00\rangle_{13}\,|00\rangle_{25}\,|-\xi\rangle_4\,|-\xi\rangle_6],\end{aligned} \tag{10}$$

here $S_{jk}(\xi) = \exp(-\xi B_j^+ B_k^+ + \xi^* B_j B_k)$ is a two-mode squeezed operator ($j,\ k = 1,\ 3$ or $2,\ 5$). Then Alice performs photon number measurement on the four modes 1, 3, 2 and 5 with the four detectors D_2, D_1, D_3, and D_4, respectively. From Eq. (**??**), we find that the term $S_j(-\xi)S_k(\xi)\,|0\rangle_j\,|0\rangle_k$ or $S_j(\xi)S_k(-\xi)\,|0\rangle_j\,|0\rangle_k$ stands for two single-mode squeezed states only including even-number photon states in their number-state expansions, the term $S_{jk}(i\xi)\,|00\rangle_{jk}$ or $S_{jk}(-i\xi)\,|00\rangle_{jk}$ denotes a two-mode SVS which contains the same photon numbers in each mode j and k with the photon numbers having both odd and even numbers in their number-state expansions. When each result of the measurement made by the four detectors is odd number of photons at the same time, the state $\left|\psi'\right\rangle_{\text{out}}$ collapses into

$$\left|\psi'\right\rangle_{46} = \frac{-1}{M\sqrt{N_{12}}}\mathrm{sech}^2 r(\mathrm{e}^{\mathrm{i}\theta}\tanh r)^{2(2m+1)} (x_1 \left|\xi\right\rangle_4 \left|\xi\right\rangle_6 + x_2 \left|\xi\right\rangle_4 \left|-\xi\right\rangle_6 + x_3 \left|-\xi\right\rangle_4 \left|\xi\right\rangle_6 + x_4 \left|-\xi\right\rangle_4 \left|-\xi\right\rangle_6). \quad (11)$$

Clearly, the above state of modes 4 and 6 is exactly the same as the initial state $|\phi\rangle_{12}$. In this case, Bob needs to do nothing and the teleportation works perfectly with a unity fidelity. The probability of detecting odd photon numbers simultaneously by the four detectors can be obtained as

$$P(2m+1) = \frac{1}{M^2}\mathrm{sech}^4 r(\tanh r)^{4(2m+1)}, \quad (12)$$

which does not depend on the superposition coefficients of the state to be teleported. Summing up all the measurement outcomes of odd photon numbers, we can derive the probability of successful teleportation as

$$P = \sum_{m=0}^{\infty} P(2m+1) = \frac{\cosh(2r)}{16(\sinh^4 r + \cosh^4 r)}. \quad (13)$$

It is easy to find that the success probability P only depends on the squeezing amplitude r. For the other cases when any one of the four detectors obtains even number photons, the teleportation scheme cannot succeed due to the fact that it is impossible to reconstruct the initial state $|\phi\rangle_{12}$ from the corresponding collapsed states of modes 4 and 6.

In summary, we have proposed a scheme to teleport a two-particle four-component SVS using linear optical elements such as beam splitters and photon detectors with the help of classical information. To accomplish our teleportation scheme, the consumed quantum resource is a pair of entangled SVSs, and the photon detectors are required to have the ability to distinguish the odd and even photon numbers, which is relatively difficult to realize especially when the photon number becomes large, but in principle, it can be done. One possible method is that odd and even photon numbers in the Fock state $|n\rangle$ can be discriminated by coupling the cavity field with a two-level atom through dispersive interaction[13,23,24]. Compared with the initial state only including two components of $\{|\xi\rangle\,|\xi\rangle,\ |-\xi\rangle\,|-\xi\rangle\}$ in Ref. [20], the state to be teleported in our scheme is a more general state, i.e., a bipartite four-component SVS. Besides, no matter whether the two-particle pure SVS to be teleported is entangled or not, our teleportation scheme can always be probabilistically realized with unity fidelity. Thus our scheme may be helpful to understand the applications of SVS for quantum information processing.

This work was supported by the National Natural Science Foundation of China (Nos. 60708003 and 60578050), the National "973" Program of China (No. 2006CB921604), the Shanghai Science and Technology Committee (No. 07JC14017), and the Director Fund of State Key Laboratory of Precision Spectroscopy.

References

1. A. Abliz, H. J. Gao, X. C. Xie, Y. S. Wu, and W. M. Liu, Phys. Rev. A **74,** 052105 (2006).
2. J. M. Zhang, W. M. Liu, and D. L. Zhou, Phys. Rev. A **77,** 033620 (2008).
3. M. Chen, S. Ma, and X. Zhang, Chin. Opt. Lett. **5,** 364 (2007).
4. J. Qian, X. Feng, and S. Gong, Chin. Opt. Lett. **5,** 484 (2007).
5. C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70,** 1895 (1993).
6. D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature **390,** 575 (1997).
7. D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80,** 1121 (1998).
8. M. Riebe, H. Häffner, C. F. Roos, W. Hänsel, J. Benhelm, G. P. T. Lancaster, T. W. Körber, C. Becher, F. Schmidt-Kaler, D. F. V. James, and R. Blatt, Nature **429,** 734 (2004).
9. S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80,** 869 (1998).
10. A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282,** 706 (1998).
11. T. C. Zhang, K. W. Goh, C. W. Chou, P. Lodahl, and H. J. Kimble, Phys. Rev. A **67,** 033802 (2003).
12. S. J. van Enk and O. Hirota, Phys. Rev. A **64,** 022313 (2001).
13. X. Wang, Phys. Rev. A **64,** 022302 (2001).
14. H. Jeong, M. S. Kim, and J. Lee, Phys. Rev. A **64,** 052308 (2001).
15. N. B. An, Phys. Rev. A **69,** 022315 (2004).
16. Y. W. Cheong, H. Kim, and H.-W. Lee, Phys. Rev. A **70,** 032327 (2004).
17. S. J. van Enk, Phys. Rev. Lett. **91,** 017902 (2003).
18. J. Liu, J. Li, and G. Guo, Chin. Opt. Lett. **1,** 114 (2003).
19. X.-H. Cai and L.-M. Kuang, Phys. Lett. A **300,** 103 (2002).
20. X.-H. Cai, Chin. Phys. **12,** 1066 (2003).
21. J.-Q. Liao and L.-M. Kuang, J. Phys. B **40,** 1183 (2007).
22. H. N. Phien and N. B. An, Phys. Lett. A **372,** 2825 (2008).
23. M. Brune, S. Haroche, J. M. Raimond, L. Davidovich, and N. Zagury, Phys. Rev. A **45,** 5193 (1992).
24. C. C. Gerry, Phys. Rev. A **55,** 2478 (1997).