Scheme for entangling atom-photon pairs via an input light in superposition state

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We propose a feasible scheme to create macroscopically entangled atom-photon pairs by preparing an input optical superposition state. Several interesting non-classical quantum statistical effects like the atomic squeezed effects are clearly demonstrated. The making and manipulation of entangled atom-photon pairs are useful for, e.g., high-precision interferometry and quantum information science.

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The experimental realizations of Bose-Einstein condensation (BEC) in the alkali atomic gas have led to many remarkable advances over the traditional framework of atomic, molecular, and optical physics[1,2]. One of these advances having close relation with the present work is to make and probe an atom laser, since the first pulsed atom laser was created by the MIT group in 1997 by using a radio frequency (RF) pulse to out-couple an initially trapped Bose condensate into a propagating state[3]. The novel properties of the atom laser have been investigated both theoretically and experimentally. For example, in analogy with an optical laser, the atomic wave mixing and superradiance[4] were demonstrated and more recently, the remarkable effect of Hanbury-Brown-Twiss (HBT) interference was observed by several groups with both ultracold bosonic and fermionic atoms[5].

The possibility of coherent control of the quantum properties of the atom laser beam is one of the active research issues for the community of ultracold matter-wave physics. Several schemes have been proposed by exploiting the nonlinear atomic interactions[6−10], such as the creation of quantum squeezing and entanglement in two atomic beams by using the nonlinear spin-exchange collisions[10]. The making and manipulation of the atom lasers with non-classical properties will be extremely useful for various fields from high-precision atom interferometry to quantum information science[1,2].

Recently, another novel scheme has also attracted much interest, the basic idea of which is coherent conversion of non-classicality from the initial input light to the output atoms and thereby is very different from the nonlinear-collision-based scheme[11−15]. In particular, by using a squeezed input light[11−13], the quantum statistical properties of the light can be efficiently converted to the output fields, which was first predicted with a single-mode approach[11−13] and then in a multi-mode configuration. This scheme was also generalized to create a squeezed light from spin-squeezed atoms by Poulsen *et al*. or to entangle the output atom-photon or atom-atom pairs via an input squeezed light by Haine *et al*.[14,15], and thus was promising to find potential applications in the fields of, e.g., quantum memory and quantum cryptography.

In this letter, we demonstrate theoretically that the macroscopic entangled atom-photon pairs can be created by applying a RF field initially prepared in a superposition state (through, e.g., a nonlinear Kerr medium[16]). The basic mechanism is to transfer the nonclassicality of the entrance-channel photons to the closed-channel atom-photon pairs (for this purpose, the nonlinear inter-atomic interactions should be tuned to near zero by using a Feshbach resonance technique or by starting from a dilute atomic sample[17]). Our study shows several interesting non-classical properties in the output fields, indicating that this so-called quantum conversion process can be realized by almost any kind of input non-classical light, not only by a squeezed light.

For simplicity, we still adopt the simplest single-mode approach by assuming a two-state atom (states $|1\rangle$ and $|2\rangle$) with the initial condensation occurring in a trapping state $|1\rangle$. State $|2\rangle$, which has different trapping properties and is typically unconfined by the magnetic trap, is coupled to $|1\rangle$ by a RF field tuned near the $|1\rangle \rightarrow |2\rangle$ transition with frequency ω_0. Thus, the interaction of the field may generate condensate in state $|2\rangle$ from an initial condensate that is totally in state $|1\rangle$. The Hamiltonian we consider here is therefore a linear-coupled atom-photon system, for which the interacting part is ($\hbar = 1$): $H_{\rm int} = \omega_R^{'}(ab_1b_2^\dagger + a^\dagger b_1^\dagger b_2)$, in terms of the creation and annihilation operators, $b_1^\dagger$, $b_2^\dagger$, b_1, and b_2, of bosonic atoms for the magnetically trapped state $|1\rangle$ and the untrapped state $|2\rangle$ with level difference ω_0, $a^\dagger$ and a are the creation and annihilation operators of the optical field with frequency ω_a. Here $\omega_R^{'} = \sqrt{\omega_a/2\varepsilon_0 V}$, V is the effective mode volume and ε_0 is the vacuum permittivity[17].

In Bogoliubov approximation, we can ignore the slow change of the large number N_c of condensed atoms in the trap by replacing operators b_1 and $b_1^\dagger$ with a c-number $\sqrt{N_c}$. Hence the trapped component initially in a coherent state $|\alpha\rangle$, $b_1|\alpha\rangle = \sqrt{N_c}{\rm e}^{-{\rm i}\theta}|\alpha\rangle$, remains in such a state while another component $|\Phi(0)\rangle$ is governed by the Bogoliubov approximate Hamiltonian[11−13]. After averaging over the coherent state $|\alpha\rangle$, we get the Heisenberg equations for the operators b_2 and a, which can be solved analytically by diagonalizing the coefficient matrix. For the specific case with a resonance frequency ($\omega_a = \omega_0 = \omega$), one can obtain the solutions

$$\begin{pmatrix} b(t) \\ a(t) \end{pmatrix} = \begin{pmatrix} \cos(\omega_R t) & -\mathrm{i}\sin(\omega_R t)\mathrm{e}^{-\mathrm{i}\theta} \\ -\mathrm{i}\sin(\omega_R t)\mathrm{e}^{\mathrm{i}\theta} & \cos(\omega_R t) \end{pmatrix} \times \begin{pmatrix} b(0) \\ a(0) \end{pmatrix} \mathrm{e}^{-\mathrm{i}\omega t}, \quad (1)$$

where $\omega_R = \omega_R^{'}\sqrt{N_c}$ and b_2 is rewritten as b. Now we suppose that the initial state of the system is theoretically described as $|\psi(0)\rangle = |\alpha\rangle \otimes |\Phi(0)\rangle$ with $|\Phi(0)\rangle = |0\rangle_b \otimes |\mathrm{SP}\rangle_a$. Here $|0\rangle_b$ represents that the initial untrapped state $|2\rangle$ is a vacuum state since there is no occupying atom in it. And the initial RF field is prepared in a macroscopic superposition of distinct coherent state as an optical analog to Schrödinger cat state: $|\mathrm{SP}\rangle_a = c_0|p_0\rangle + c_1|p_1\rangle$, which could also be considered as a special kind of qubit consisting of two Glauber coherent states, i.e., $|p_i\rangle = \exp[p_i a^\dagger(0) - p_i^* a(0)]|0\rangle_a \quad (i = 1,\ 2)$. Through the time evolution operator $U(t) = \mathrm{e}^{-\mathrm{i}Ht}$, the state of the system is derived as

$$\begin{aligned} |\Phi(t)\rangle &= U(t)[c_0 \mathrm{e}^{p_0 a^\dagger(0) - p_0^* a(0)} + c_1 \mathrm{e}^{p_1 a^\dagger(0) - p_1^* a(0)}]|0\rangle_a \otimes |0\rangle_b \\ &= c_0|\lambda p_0\rangle_a \otimes |\mathrm{i}\eta p_0\rangle_b + c_1|\lambda p_1\rangle_a \otimes |\mathrm{i}\eta p_1\rangle_b, \end{aligned} \quad (2)$$

where $\lambda = \cos(\omega_R t)$, $\eta = \sin(\omega_R t)e^{-i\theta}$, and we have used the properties of the evolution operator, i.e., $U^\dagger(t)SU(t) = S(t)$ and $U(t)|0\rangle = |0\rangle$ for any operator S. This state is essentially an entanglement or non-local superposition of the quasi-classical product state of two coherent states, i.e., the entangled coherent state pointed out earlier in the context of nonlinear Mach-Zehnder interferometer[18]. There is a similarity between this state and the entangled photon-number state which has been discussed in testing Bell's inequalities[19]. Besides, it is noted that the displacement amplitude of the output atomic field conditionally depends on the "logic values" of RF field, if one takes it as a special kind of qubit. After obtaining the state of system at any time, one could also conveniently study several interesting problems, such as the decoherence factor due to the environmental noise or the Pancharatnam phase[20].

Now we start to consider the interesting quantum conversions of the non-classical effects, such as the super-Poisson distribution and the quadrature squeezing effect, between the RF field and the output atomic field. Just for the convenience, here we suppose $p_i \quad (i = 1,\ 2)$ being real numbers. Using the above solutions, it is easy to compute the average numbers and the fluctuations of the output atoms, e.g., $\langle N_b(t)\rangle = C_0 \sin^2(\omega_R t)$ and $\langle N_b^2(t)\rangle = C_1 \sin^4(\omega_R t) + C_0 \sin^2(\omega_R t)\cos^2(\omega_R t)$, where $C_0 := \langle \mathrm{SP}|a^\dagger(0)a(0)|\mathrm{SP}\rangle = \sum_{i=1}^2 c_i^2 p_i^2 + \gamma_1\gamma_2$, and

$$\begin{aligned} C_1 :=& \langle \mathrm{SP}|a^\dagger(0)a(0)a^\dagger(0)a(0)|\mathrm{SP}\rangle \\ =& \sum_{i=1}^{2} c_i^2 p_i^2 (1 + p_i^2) + \gamma_1(1 + p_0 p_1)\gamma_2, \end{aligned} \quad (3)$$

with $\gamma_1 = 2c_0c_1p_0p_1,\ \gamma_2 = \exp[-\frac{1}{2}(p_0 - p_1)^2]$. Similar results can be also obtained for the out-state photons. In order to decide the statistical properties of a quantum field, we can define the Mandel's Q parameter[21]

$$Q_b(t) = \frac{\langle \Delta N_b^2(t)\rangle}{\langle N_b(t)\rangle} - 1 \begin{cases} < 0: & \text{sub–Poisson distribution,} \\ = 0: & \text{Poisson distribution,} \\ > 0: & \text{super–Poisson distribution.} \end{cases} \quad (4)$$

From the obtained general solutions, the Q parameters of the RF field and the output atomic field can be found to have the following structure:

$$\begin{pmatrix} Q_a(t) \\ Q_b(t) \end{pmatrix} = [\frac{C_1 - C_0^2}{C_0} - 1] \begin{pmatrix} \cos^2(\omega_R t) \\ \sin^2(\omega_R t) \end{pmatrix}. \quad (5)$$

For the case $c_0 = c_1 = 1/\sqrt{2}$, $\quad p_0 = -p_1 = p$, the initial state ($t = 0$) is $|\Phi(0)\rangle = |0\rangle \otimes |\mathrm{SP}\rangle$ with $|\mathrm{SP}\rangle = \frac{1}{\sqrt{2}}(|p\rangle + |-p\rangle)$, and the above structure factor is

$$F(m) := \frac{C_1 - C_0^2}{C_0} - 1 = \frac{1 + \gamma_2}{1 - \gamma_2}\gamma_2 p^2 > 0, \quad (6)$$

where we have defined $\gamma_2 = \mathrm{e}^{-2p^2}$. Thereby in the initial state ($t = 0$), we can obtain the results $Q_a(t) > 0$ and $Q_b(t) = 0$, which means that the initial state of the RF field is in a super-Poisson distribution and the initial atomic field is in a vacuum state, as they should be. When the evolution time t_0 satisfies $\cos(\omega_R t_0) = 0$ or $\omega_R t_0 = (n + \frac{1}{2})\pi \quad (n = 0, 1, 2, \cdots)$, we have $Q_a(t) = 0,\ \ Q_b(t) > 0$, which means that the quantum state of the RF field transforms from the initial superposition state into a coherent state while the output matter wave is now in a superposition state. This result is actually a general feature of the quantum conversion scheme, which in this case restores the factorized structure as the initial state of the system. In addition, the revealed periodically oscillating behaviors of the quantum fields hold the promise to be observed, at least in principle, in the next generation of matter-wave experiments.

In order to study the quadrature squeezing of the output atomic beam, we can define the field quadratures X_{1b} and X_{2b} as $X_{1b} = \frac{1}{2}(b + b^\dagger),\ \ X_{2b} = \frac{1}{2\mathrm{i}}(b - b^\dagger)$. Following Bužek *et al.*[22], we introduce the squeezed coefficients

$$S_i = \frac{\langle (\Delta X_i)^2\rangle - \frac{1}{2}|\langle [X_1, X_2]\rangle|}{\frac{1}{2}|\langle [X_1, X_2]\rangle|}, \quad i = 1, 2, \quad (7)$$

which, by taking into account of ($\theta = 0,\ p \in R$)

$$\langle b^2(t)\rangle = -\sin^2(\omega_R t)\mathrm{e}^{-2\mathrm{i}\omega t}\langle \mathrm{SP}|a^2(0)|\mathrm{SP}\rangle,$$

$$\langle \mathrm{SP}|a^2(0)|\mathrm{SP}\rangle = \sum_{i=1}^{2} c_i^2 p_i^2 + c_0 c_1 \gamma_2 \sum_{i=1}^{2} p_i^2, \quad (8)$$

finally leads to the simple results

$$\begin{aligned} S_{1b}(t) &= 2p^2 \sin^2(\omega_R t)[1 - \gamma_2 - \cos(2\omega t)], \\ S_{2b}(t) &= 2p^2 \sin^2(\omega_R t)[1 - \gamma_2 + \cos(2\omega t)], \end{aligned} \quad (9)$$

for $c_0 = c_1 = 1/\sqrt{2}$, $p_0 = -p_1 = p$, with $\gamma_2 = \mathrm{e}^{-2p^2} \in (0, 1]$. Obviously, for the initial state of the atomic field, it yields $S_{1b}(0) = S_{2b}(0) = 0$, which means that there is no squeezing, as it should be. When the evolution time satisfies $\cos(2\omega t) > 1 - \gamma_2$, we can get $S_{1b} < 0$, $S_{2b} > 0$, which means that the quadrature component X_{1b} is squeezed.

But when it satisfies $\cos(2\omega t) < \gamma_2 - 1$, we will have $S_{1b} > 0$, $S_{2b} < 0$, which means that the squeezing effect transfers to X_{2b} component. The similar behavior also happens for the RF field. Clearly, the conditions of super-Poisson distribution and quadrature squeezing are quite different, as they should be generally.

However, if the initial state of the RF field is chosen as an optical superposition state: $|\mathrm{SP}\rangle = \frac{1}{\sqrt{2}}(|p\rangle - |-p\rangle)$, one can reach

$$S_{1b}(t) = 2p^2 \sin^2(\omega_R t)[1 + \gamma_2 - \cos(2\omega t)] \geq 0,$$
$$S_{2b}(t) = 2p^2 \sin^2(\omega_R t)[1 + \gamma_2 + \cos(2\omega t)] \geq 0. \quad (10)$$

This means that there is no squeezing effect at any time, which is actually a well-known fact for such type of initial input states.

The peculiar behaviors about the quadrature squeezing clearly show the key impact of the initial state of the RF field on the quantum feature of the output atomic beam. Of course, the basic structure of the Mandel's Q parameter still holds right since it is directly from the linear-coupling model and therefore independent of the concrete state form of the initial light, which should be noted also for that using a squeezing input light[11−13].

In addition, one could also analyze the quantum correlation[1,17] of the RF field and the output atomic beam by computing the second-order zero-time correlation function[21] and thereby study the interesting possible violation of the classical Cauchy-Schwarz inequality (CSI) which, according to Reid *et al.*[23], could be accompanied with the violation of Bell's inequality[21]. By following the similar calculations as above, this calculation will be quite straightforward.

The realization of atom-photon entanglement is of interest for current study of high-precision interferometry and quantum information physics[11−13,24]. Thereby, we expect our scheme can be adjusted and improved so that one could generate the maximally entangled state between the RF field and the output atomic beam, and even the entangled atom-atom pairs by applying a squeezed or superposition light to different parts of an atomic BEC[16,24]. In our future work, we plan to study the creation of entangled atom-photon pairs by using a fermionic atomic sample in an optical lattice[2,25−27] or a multi-species Bose-Fermi mixture[28], with or without binary collisions between the atoms of the propagating mode[8].

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