Equal-power 4m-th power sum squeezing of generalized magnetic-field component in vacuum state immiting four-state superposition multimode entangled state light field

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By utilizing multimode squeezed states theory, we study the generalized nonlinear equal-power higher-power sum squeezing properties of the generalized magnetic-field component in four-state superposition multimode entangled state light field $\left|\Psi_1{}^{(4)}\right\rangle_q$. The state is composed of multimode vacuum state, multimode coherent state and its contrary state, multimode imaginary coherent state. It is found that the state $\left|\Psi_1{}^{(4)}\right\rangle_q$ is a type of four-state superposition multimode nonclassical light field, and under certain fixed conditions, the generalized magnetic-field component in the state $\left|\Psi_1{}^{(4)}\right\rangle_q$ can display generalized nonlinear equal-power 4m-th power sum squeezing effects which change periodically.

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Since the concept of "squeezed states"[1] was proposed in 1970, many researches have been done in squeezed states[2−14]. In the 1990s, the establishment of the multimode squeezed states theory[10] deepens the theoretical studies in the area of squeezed states. Multimode squeezed states light field[10−14] is a type of nonclassical light field. Because of its special pure quantum property, it is widely applied in many scientific fields such as multi-longitudinal-mode light-quanta communication. And it is a pioneering research project in current field of quantum optics[4−14]. Multimode squeezed states theory[10] has been ultilized in the studies of the generalized nonlinear squeezed properties in two-, three-, and five-state superposition multimode squeezed state light field and multimode functional coherent state superposition state light field[11−13], and some special four-state superposition multimode squeezed state light field and multimode functional coherent state superposition state light field[14]. The studies show a series of new physical phenomena like "contrary squeezing" effect, which are significantly helpful to the further study of multi-longitudinal-mode light-quanta communication based on the theory of multimode squeezed state. However, those studies focus little attention on the generalized nonlinear higher-order squeezed characteristics in general four-state superposition multimode superposition state light field. And no attention is even paid to the generalized nonlinear higher-order squeezed characteristics in vacuum immiting general four-state superposition multimode superposition state light field. In fact, the further research of this field not only deepens the understanding of quantum essences of light by revealing the nonclassical nature of multimode squeezed state light field, but also provides theoretical guide to prepare, produce, and control the multimode light field in experiments.

In this letter, the characteristics of generalized nonlinear equal-power higher-power sum squeezing (equal-power Nth-power H-squeezing) of the generalized magnetic-field component in the vacuum state immiting four-state superposition multimode entangled state light-field $\left|\Psi_1{}^{(4)}\right\rangle_q$ are studied by utilizing the multimode squeezed states theory mentioned above, and a series of results and conclusions are obtained.

The general definition[10] of generalized nonlinear equal-power Nth-power H-squeezing is given below. In the multimode radiation light field (MRLF) with frequency ω_j ($j = 1, 2, 3, \cdots, q$), a_j^+ and a_j represent the creation operator and annihilation operator of the jth-mode of MRLF, respectively[10]. And then two pairs of Hermitian conjugate operators $B_q^+(N)$ and $B_q(N)$ can be defined as

$$B_q^+(N) = \prod_{j=1}^{q} a_j^{+N}, \quad B_q(N) = \prod_{j=1}^{q} a_j^N.$$

Based upon the above two operators, we introduce the following two quadrature phase Hermitian operators:

$$H_1^q(N) = \frac{1}{2}[B_q^+(N) + B_q(N)],$$

$$H_2^q(N) = \frac{\mathrm{i}}{2}[B_q^+(N) - B_q(N)].$$

By utilizing the Cauchy-Schwarz inequality, the uncertainty principle can be expressed as

$$\left\langle \Delta H_1^2(N)_q \right\rangle \left\langle \Delta H_2^2(N)_q \right\rangle \geq \frac{1}{16} \left|\left\langle [B_q(N), B_q^+(N)] \right\rangle\right|^2,$$

where $\left\langle \Delta H_m^2(N)_q \right\rangle = \left\langle [H_m^q(N)]^2 \right\rangle - \langle [H_m^q(N)] \rangle^2$, $m = 1, 2$; $\left\langle \Delta H_1^2(N)_q \right\rangle$ and $\left\langle \Delta H_2^2(N)_q \right\rangle$ stand for the quantum fluctuations of generalized magnetic- and electric-field components in the MRLF, respectively. According to the equation mentioned above, if this inequality is satisfied by one of the quantum fluctuations of the two quadrature phase components in the MRLF, then

$$4\left\langle \Delta H_m^2(N)_q\right\rangle - \langle\, [B_q(N), B_q^+(N)]\,\rangle < 0 \quad (m=1,2),$$

or

$$h_m(N) = 4\left\langle \Delta H_m^2(N)_q\right\rangle - \left\langle [B_q(N), B_q^+(N)]\right\rangle < 0.$$

This would lead to the existence of generalized nonlinear equal-power Nth-power H-squeezing effect that happens in the mth ($m=1,2$) quadrature phase component in MRLF.

The multimode entangled state light field $\left|\Psi_1{}^{(4)}\right\rangle_q$ constructed in this letter is a kind of amplitude-phase-mixed multimode entangled state light field, which is formed by linear superposition of four macroscopically different multimode quantum states — multimode vacuum state $\left|\{0_j\}\right\rangle_q$, multimode coherent state $\left|\{Z_j\}\right\rangle_q$ and its contrary state $\left|\{-Z_j\}\right\rangle_q$, and multimode imaginary coherent state $\left|\{\mathrm{i}Z_j\}\right\rangle_q$. It can be produced in experiments by nonlinear Mach-Zehnder interferometer. It can be expressed as

$$\left|\Psi_1^{(4)}\right\rangle_q = C_1\left|\{Z_j\}\right\rangle_q + C_2\left|\{0_j\}\right\rangle_q + C_3\left|\{-Z_j\}\right\rangle_q + C_4\left|\{\mathrm{i}Z_j\}\right\rangle_q, \tag{1}$$

where

$$C_n = r_n\exp[\mathrm{i}\theta_n] \quad (n=1,2,3,4), \tag{2}$$

$$Z_j = R_j\exp[\mathrm{i}\varphi_j] \quad (j=1,2,3,\cdots,q), \tag{3}$$

$$\left.\begin{array}{l}
\left|\{Z_j\}\right\rangle_q = \exp\{-\frac{1}{2}[\sum\limits_{j=1}^{q}|z_j|^2]\}\sum\limits_{\{n_j\}=0}^{\infty}\{\prod\limits_{j=1}^{q}\frac{Z_j^{n_j}}{\sqrt{n_j!}}\}\left|\{n_j\}\right\rangle_q,\\
\left|\{-Z_j\}\right\rangle_q = \exp\{-\frac{1}{2}[\sum\limits_{j=1}^{q}|z_j|^2]\}\sum\limits_{\{n_j\}=0}^{\infty}\{\prod\limits_{j=1}^{q}\frac{(-1)^{n_j}Z_j^{n_j}}{\sqrt{n_j!}}\}\left|\{n_j\}\right\rangle_q,\\
\left|\{\mathrm{i}Z_j\}\right\rangle_q = \exp\{-\frac{1}{2}[\sum\limits_{j=1}^{q}|z_j|^2]\}\sum\limits_{\{n_j\}=0}^{\infty}\{\prod\limits_{j=1}^{q}\frac{\mathrm{i}^{n_j}Z_j^{n_j}}{\sqrt{n_j!}}\}\left|\{n_j\}\right\rangle_q,\\
\left|\{0_j\}\right\rangle_q = \left|0_1,0_2,0_3,\cdots,0_j,\cdots,0_{q-1},0_q\right\rangle_q.
\end{array}\right\} \tag{4}$$

And r_n ($n=1,2,3,4$) is the superposition probability amplitude of each coherent state in $\left|\Psi_1{}^{(4)}\right\rangle_q$, θ_n ($n=1,2,3,4$) is the initial phase of each coherent state in $\left|\Psi_1{}^{(4)}\right\rangle_q$, $\left|\{n_j\}\right\rangle_q = \left|n_1,n_2,n_3,\cdots,n_{q-1},n_q\right\rangle$ is the multimode photon number state, $\left|\{0_j\}\right\rangle_q$ is the multimode vacuum state, and q is the total number of light field's cavity-modes (longitudinal-mode).

The normalization condition of state $\left|\Psi_1{}^{(4)}\right\rangle_q$ is

$$\begin{aligned}
{}_q\langle\Psi_1{}^{(4)}|\Psi_1{}^{(4)}\rangle_q &= r_1^2+r_2^2+r_3^2+r_4^2+2r_1r_3\cos(\theta_1-\theta_3)\exp\{-2\sum_{j=1}^{q}R_j^2\}\\
&+2r_2\exp\{-\frac{1}{2}\sum_{j=1}^{q}R_j^2\}[r_1\cos(\theta_1-\theta_2)+r_3\cos(\theta_2-\theta_3)+r_4\cos(\theta_2-\theta_4)]\\
&+2\exp\{-\sum_{j=1}^{q}R_j^2\}\{r_1r_4\cos[(\theta_1-\theta_4)-\sum_{j=1}^{q}R_j^2]+r_3r_4\cos[(\theta_3-\theta_4)+\sum_{j=1}^{q}R_j^2]\}=1.
\end{aligned} \tag{5}$$

Based on the definition of generalized nonlinear equal-power Nth-power H-squeezing in multimode state light field mentioned in Ref. [10] and Eqs. (1)—(5), we can obtain the general theoretical result of equal-power higher-power sum squeezing of generalized magnetic-field component (1st quadrature phase) in vacuum state immiting four-state superposition multimode entangled state light field $\left|\Psi_1{}^{(4)}\right\rangle_q$:

$$\begin{aligned}
h_1(N) &= 4\left\langle \Delta H_1^2(N)_q\right\rangle - \left\langle [B_q(N), B_q^+(N)]\right\rangle\\
&= 2\{\prod_{j=1}^{q}R_j^{2N}\}\{r_1^2+r_3^2+r_4^2+2(-1)^{qN}r_1r_3\cos(\theta_1-\theta_3)\exp[-2\sum_{j=1}^{q}R_j^2]\\
&+2\exp[-\sum_{j=1}^{q}R_j^2]\{r_1r_4\cos[(\theta_1-\theta_4)-qN\frac{\pi}{2}-\sum_{j=1}^{q}R_j^2]+r_3r_4\cos[(\theta_3-\theta_4)+qN\frac{\pi}{2}+\sum_{j=1}^{q}R_j^2]\}\\
&+\{r_1^2+r_3^2+(-1)^{qN}r_4^2+2r_1r_3\cos(\theta_1-\theta_3)\exp[-2\sum_{j=1}^{q}R_j^2]\}\cos[2N(\sum_{j=1}^{q}\varphi_j)]
\end{aligned}$$

$$+r_2\exp[-\frac{1}{2}\sum_{j=1}^{q}R_j^2]\{r_1\cos[2N(\sum_{j=1}^{q}\varphi_j)+(\theta_1-\theta_2)]+r_3\cos[2N(\sum_{j=1}^{q}\varphi_j)-(\theta_2-\theta_3)]$$
$$+(-1)^{qN}r_4\cos[2N(\sum_{j=1}^{q}\varphi_j)-(\theta_2-\theta_4)]\}+\exp[-\sum_{j=1}^{q}R_j^2]\{r_1r_4\{\cos[2N(\sum_{j=1}^{q}\varphi_j)+(\theta_1-\theta_4)-\sum_{j=1}^{q}R_j^2]$$
$$+(-1)^{qN}\cos[2N(\sum_{j=1}^{q}\varphi_j)-(\theta_1-\theta_4)+\sum_{j=1}^{q}R_j^2]\}+r_3r_4\{\cos[2N(\sum_{j=1}^{q}\varphi_j)+(\theta_3-\theta_4)+\sum_{j=1}^{q}R_j^2]$$
$$+(-1)^{qN}\cos[2N(\sum_{j=1}^{q}\varphi_j)-(\theta_3-\theta_4)-\sum_{j=1}^{q}R_j^2]\}\}-2\{[r_1^2+(-1)^{qN}r_3^2]\cos[N(\sum_{j=1}^{q}\varphi_j)]$$
$$+r_4^2\cos[N(\sum_{j=1}^{q}\varphi_j)+qN\frac{\pi}{2}]+\exp[-2\sum_{j=1}^{q}R_j^2]\cdot r_1r_3\{\cos[N(\sum_{j=1}^{q}\varphi_j)]+(\theta_1-\theta_3)]$$
$$+(-1)^{qN}\cos[N(\sum_{j=1}^{q}\varphi_j)-(\theta_1-\theta_3)]\}+\exp[-\frac{1}{2}\sum_{j=1}^{q}R_j^2]\cdot r_2\{r_1\cos[N(\sum_{j=1}^{q}\varphi_j)+(\theta_1-\theta_2)]$$
$$+(-1)^{qN}r_3\cos[N(\sum_{j=1}^{q}\varphi_j)-(\theta_2-\theta_3)]+r_4\cos[N(\sum_{j=1}^{q}\varphi_j)-(\theta_2-\theta_4)+qN\frac{\pi}{2}]\}$$
$$+\exp[-\sum_{j=1}^{q}R_j^2]\cdot\{r_1r_4\{\cos[N(\sum_{j=1}^{q}\varphi_j)+(\theta_1-\theta_4)-\sum_{j=1}^{q}R_j^2]+\cos[N(\sum_{j=1}^{q}\varphi_j)-(\theta_1-\theta_4)+\sum_{j=1}^{q}R_j^2+qN\frac{\pi}{2}]\}$$
$$+r_3r_4\{(-1)^{qN}\cos[N(\sum_{j=1}^{q}\varphi_j)+(\theta_3-\theta_4)+\sum_{j=1}^{q}R_j^2]+\cos[N(\sum_{j=1}^{q}\varphi_j)-(\theta_3-\theta_4)-\sum_{j=1}^{q}R_j^2+qN\frac{\pi}{2}]\}\}\}^2\}, \quad (6)$$

where R_j^2 is the mean photon-number of the jth coherent single-mode light, φ_j is its initial phase and N is the sum squeezing-power-number.

When $qN=4m$ $(m=1,2,\cdots)$, if the sum of the modes' initial phases $(\sum_{j=1}^{q}\varphi_j)$ satisfies

$$\sum_{j=1}^{q}\varphi_j=\pm(2K_\varphi+1)(q\pi/8m)\quad(K_\varphi=0,1,2,\cdots), \quad (7)$$

and the initial phase difference among different states satisfies

$$\theta_i-\theta_2\in[2l\pi-\pi/2,2l\pi+\pi/2]\quad(i=1,3,4;l=0,\pm1,\pm2,\cdots), \quad (8)$$

then Eq. (6) can be rewritten as

$$h_1(4m)=-2\{\prod_{j=1}^{q}R_j^{8m/q}\}\cdot\exp[-\frac{1}{2}\sum_{j=1}^{q}R_j^2]\cdot\{[r_1r_2\cos(\theta_1-\theta_2)+r_2r_3\cos(\theta_2-\theta_3)+r_2r_4\cos(\theta_2-\theta_4)]$$
$$+\exp[-\frac{1}{2}\sum_{j=1}^{q}R_j^2]\cdot 2r_2^2[r_1\sin(\theta_1-\theta_2)+r_3\sin(\theta_3-\theta_2)+r_4\sin(\theta_4-\theta_2)]^2\}<0. \quad (9)$$

It can be seen that there exist equal-power $4m$-th power sum squeezing effects of generalized magnetic-field component in state $\left|\Psi_1{}^{(4)}\right\rangle_q$, and these effects change periodically. Its squeezing result, squeezing depth, and squeezing degree are related nonlinearly to the total number of light field's cavity-modes q, to the sum squeezing-power-number $N=8m/q$, to the mean photon-number of the jth coherent single-mode light R_j^2 and its sum $\sum_{j=1}^{q}R_j^2$, to the product of the superposition probability amplitude r_ir_2 $(i=1,3,4)$ of each coherent state in $\left|\Psi_1{}^{(4)}\right\rangle_q$, to the sum of the modes' initial phase $(\sum_{j=1}^{q}\varphi_j)$, and to the initial phase difference $(\theta_i-\theta_2)$ $(i=1,3,4)$.

From the above analysis, we can draw the following conclusions. Firstly, the vacuum state immiting four-state superposition multimode entangled state light field $\left|\Psi_1{}^{(4)}\right\rangle_q$ constructed in this letter is a kind of typical multimode nonclassical light field, which displays in certain conditions equal-power $4m$-th power sum squeezed effects which change periodically.

Secondly, under the condition of $qN=4m$ $(m=$

$1, 2, \cdots$), if the sum of the modes' initial phase ($\sum_{j=1}^{q} \varphi_j$) satisfies Eq. (7) and the initial phase difference ($\theta_i - \theta_2$) ($i = 1, 3, 4$) satisfies the condition (8), then there exist equal-power $4m$-th power sum squeezed effects of generalized magnetic-field component in the state $\left|\Psi_1^{(4)}\right\rangle_q$ which change periodically.

Thirdly, the basic reasons for equal-power $4m$-th power sum squeezing effects of generalized magnetic-field component in the state $\left|\Psi_1{}^{(4)}\right\rangle_q$ are quantum interference effects of different states and of different modes. The quantum interference effect of different states is presented in theory that the initial phase difference among different states must satisfy certain and fixed conditions, that is, the different states in $\left|\Psi_1{}^{(4)}\right\rangle_q$ must satisfy fixed phase relations in the process of linear superposition. However, the quantum interference effect of different states in experiments is shown as macroscopical interference phenomenon of two or more light beams. The quantum interference effects of different modes are shown as follows. 1) There exists quantum probability interference in the same photon of the same longitudinal mode. 2) There exists quantum probability interference in different photons of the same longitudinal mode. 3) There exists quantum entanglement in different photons of the same or different longitudinal modes, and this quantum entanglement leads to mode competition effect in different longitudinal modes. Therefore, the quantum interference effect of different modes is presented in theory that the initial phase sum of each mode $\sum_{j=1}^{q} \varphi_j$ must satisfy certain and fixed conditions, that is, the different modes in $\left|\Psi_1{}^{(4)}\right\rangle_q$ must satisfy fixed phase relations. Yet in experiments, the quantum interference effect of different modes is closely related to the technology of preparing, producing, and controlling multimode quantum entangled light field.

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