Spectral efficiency analysis of OCDMA systems

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We discuss several kinds of code schemes and analyze their spectral efficiency, code utilizing efficiency, and the maximal spectral efficiency. Error correction coding is used to increase the spectral efficiency, and it can avoid the spectral decrease with the increase of the length. The extended primer code (EPC) has the highest spectral efficiency in the unipolar code system. The bipolar code system has larger spectral efficiency than unipolar code system, but has lower code utilizing efficiency and the maximal spectral efficiency. From the numerical results, we can see that the spectral efficiency increases by 0.025 (b/s)/Hz when the bit error rate (BER) increases from 10^{-9} to 10^{-7}.

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Optical code division multiple-access (OCDMA) that allows multiple users to share a common optical channel simultaneously and asynchronously, is considered as one of the most promising technologies for the next generation broadband access network[1−4]. And it may play an important role in the future photonic network, because it has the unique features of all-optical processing, full asynchronous transmission, and potentially excellent security[5]. The optical medium is superbly suited for the spread spectrum multiple-access communications due to its extremely large bandwidth, however, the spectral efficiency of OCDMA system is an import factor.

The spectral efficiency is a fundamental measure of the performance of an optical communication system. It specifies the overall throughput per unit of optical bandwidth associated with a fixed bit error rate (BER),

$$\eta = \frac{N_{\rm ber}R_{\rm b}}{B}, \tag{1}$$

where $N_{\rm ber}$ is the number of simultaneous users emitting at a bit rate $R_{\rm b}$ in a certain BER requirement and B is the optical bandwidth occupied by the system. Because of the limited bandwidth resource, the highest performance systems are generally those that achieve the largest spectral efficiency at a certain BER. The spectral efficiency of OCDMA systems with coherent pulsed sources was evaluated in Ref. [6]. It is demonstrated that the spectral efficiency of an OCDMA system with coherent sources is at least a factor of 5 higher than OCDMA systems with incoherent sources. The spectral efficiency of incoherent OCDMA systems was introduced in Ref. [7]. But this paper considered the $N_{\rm ber}$ equivalent to the cardinality C of codes, while the number of available codes C always satisfies $C \gg N_{\rm ber}$ at any given time for an asynchronous OCDMA system. So its results are larger than practical systems. It is possible to exploit these unused codes to increase the spectral efficiency of the system by using a multi-dimensional modulation[8]. A novel method for increasing the spectral efficiency of OCDMA by using multi-dimensional modulation was introduced in Ref. [9]. Reference [10] evaluated the spectral efficiency of bipolar phase-encoded and direct-sequence OCDMA systems and proposed a new approach to achieve high spectral efficiency in the wavelength-time OCDMA network. In this letter, we discuss several kinds of code schemes. Comparisons on spectral efficiency, code utilizing efficiency, and the maximal spectral efficiency of these schemes are carried out.

We know that the maximal spectral efficiency of a communication system is

$$\eta_{\max} = \log_2(1 + {\rm SNR}), \tag{2}$$

where SNR is the signal-to-noise ratio of the system. The practical spectral efficiency $\eta \ll \eta_{\max}$, the spectral efficiency of a practical OCDMA system can approach $\eta_{\max}$ by using error correction coding and multi-dimensional modulation. Since the number of available codes C is always far larger than the number of users $N_{\rm ber}$ for any OCDMA scheme, there is always a set of unused codes. We define the code utilizing efficiency as

$$\gamma = \frac{N_{\rm ber}}{C}. \tag{3}$$

High code utilizing efficiency and high spectral efficiency mean high resource utilizing efficiency. We will analyze the spectral efficiency of OCDMA systems which use the unipolar codes and bipolar codes.

The optical orthogonal code (OOC) can be characterized by $(L, w, 1, 1)$, where L and w are the length and weight, the values of auto-correlation and cross-correlation are both 1. Due to its excellent correlation properties, OOC is used in the unipolar optical domain. But in general, there are very few codes in the family of OOC. Large cardinality can be achieved by either increasing the length of the codes or decreasing the weight of the codes. Its mean and variance are

$$\mu = \frac{w^2}{2L}, \tag{4a}$$

$$\sigma^2 = \frac{w^2}{2L}(1 - \frac{w^2}{2L}). \tag{4b}$$

The cardinality of OOC is $\lceil (L-1)/w(w-1) \rceil$, where the symbol $\lceil x \rceil$ denotes the integer portion of the real value x.

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Table 1. SNR of Optical Codes

Code	Character	SNR	Cardinality
OOC	$(L, w, 1, 1)$	$\frac{4L^2}{(m-1)(2L-w^2)}$	$\lceil (L-1)/w(w-1) \rceil$
Prime Code	$(p^2, p, p-1, 2)$	$\frac{p^2}{(m-1)(\frac{5}{12}-\frac{1}{6p}-\frac{1}{3p^2})}$	p
EPC	$(2p^2-p, p, p-1, 1)$	$\frac{p^2}{(m-1)(\frac{3}{8}+\frac{1}{4(2p-1)}-\frac{1}{8(2p-1)^2}}$	p
Gold Sequence	$L = 2^n - 1$	$\frac{4L^3}{(m-1)(L^2+L-1)}$	$2^n + 1$

Prime codes have been the focus of research on new codes in many spreading incoherent OCDMA systems. The prime codes can be characterized by $(p^2, p, p-1, 2)$, where p^2 and p are the length and weight, $p-1$ and 2 are the auto-correlation value and the cross-correlation value. The auto-correlation and cross-correlation of prime codes are much worse than OOC. For prime codes, the cardinality is p and the mean and variance are

$$\mu = \frac{1}{2}, \tag{5a}$$

$$\sigma^2 = \frac{5p^2 - 2p - 4}{12p^2}. \tag{5b}$$

In order to decrease the cross-correlation of prime codes, we can increase the length of the code. The extended prime codes (EPCs) can be characterized by $(2p^2 - p, p, p-1, 1)$, where $2p^2 - p$ and p are the length and weight, $p-1$ and 1 are the auto-correlation value and the cross-correlation value. The EPC's cardinality is p and its mean and variance are

$$\mu = \frac{p}{2(2p-1)}, \tag{6a}$$

$$\sigma^2 = \frac{p(3p-2)}{4(2p-1)^2}. \tag{6b}$$

The cardinality of bipolar codes is larger than that of unipolar ones, if the length of the Gold sequence L is $2^n - 1$, its cardinality is $L+2$. Due to the good auto-correlation and cross-correlation of Gold sequences, it is used in most coherent OCDMA systems. Its mean and variance are

$$\mu = \frac{1}{L}, \tag{7a}$$

$$\sigma^2 = \frac{L^3 + L^2 - L - 1}{L^2}. \tag{7b}$$

According to the mean and variance of the optical codes, we can evaluate the SNR of the codes, as shown in Table 1.

For an on-off keying (OOK) OCDMA system using unipolar codes, if the number of active users is large enough, the multiple access interference (MAI) may be approximated as Gaussian distribution. The BER may then be calculated using the Q-function as

$$\mathrm{BER}_{\mathrm{unipolar}} = Q(\sqrt{\mathrm{SNR}/2}). \tag{8}$$

For a binary phase shift keying (BPSK) OCDMA system using bipolar codes, the BER may be calculated using

$$\mathrm{BER}_{\mathrm{bipolar}} = Q(\sqrt{\mathrm{SNR}}). \tag{9}$$

The SNR and the capacity of different codes are also shown in Table 1.

Error correction coding is shown to be useful for improving the performance of OCDMA systems[11,12]. An (n, k, t) Bose-Chaudhuri-Hocquenhem (BCH) code of length n and information-bit length k can correct up to t words error. The expression of BER while using BCH coding is derived as

$$\mathrm{BER}_{\mathrm{bch}} \le \sum_{i=t+1}^{n} \frac{i+t}{n} C_n^i \mathrm{BER}^i (1 - \mathrm{BER})^{n-i}, \tag{10}$$

where N_{bch} is the active user number when using BCH coding at the same BER, and $N_{\mathrm{bch}} > N_{\mathrm{ber}}$, so the spectral efficiency is increased.

Now we compare the spectral efficiency of OOC, prime code, EPC, and Gold sequence. The parameters used in our numerical calculation are summarized in Table 2.

Figure 1 illustrates the spectral efficiency η versus the length L used in the OOC, prime code, and EPC system at a BER of 10^{-9} and a fixed bit rate. As the length L is increased, the spectral efficiency decreases quickly. Additionally, the spectral efficiency of EPC system is

Table 2. Notations in Numerical Calculation

Symbol	Definition	Value
R_{b}	Bit Rate	2.5 Gb/s
w	Weight of OOC	3
L	Length of Codes	100:600
B	Optical Bandwidth	$R_{\mathrm{b}} \times L$
$\eta_{\max}$	Maximal Spectral Efficiency	$\log_2(1 + \mathrm{SNR})$

The fixed BER is 10^{-9}.

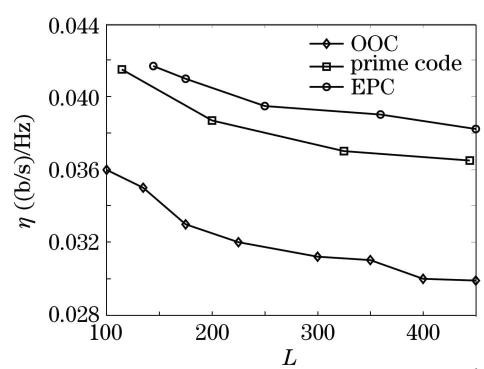

Fig. 1. Spectral efficiency η versus length L used in unipolar codes systems (BER = 10^{-9}).

higher than those of the other two systems. The spectral efficiency of EPC is 0.042 (b/s)/Hz at the length $L = 150$.

The relationship between η and L in the Gold sequence system is illustrated in Fig. 2. The spectral efficiency decreases with the increase of length. And in this case, the spectral efficiency of Gold sequence is the highest. When the length $L = 150$, the spectral efficiency η is 0.125 (b/s)/Hz at the fixed BER 10^{-9}. If we increase the BER to 10^{-7}, the spectral efficiency increases to 0.15 (b/s)/Hz. From Fig. 2, the spectral efficiency approaches to a fixed higher value when using BCH code, and does not change with the increase of the length L. When the number of active users is large, BCH coding can be used to increase the spectral efficiency. When the length L arrives a certain value, the spectral efficiency of Gold sequence system changes slowly.

Figure 3 shows the code utilizing efficiency γ versus the length L. The code utilizing efficiencies of EPC and prime code systems are higher than that of Gold sequence system, although the Gold sequence system has the highest spectral efficiency.

In order to gain more insight into these results in a numerical comparison, we assume some parameters for the above codes, and calculate their spectral efficiencies, as shown in Table 3.

We can see that the spectral efficiency of Gold sequence system is the highest, but its code utilizing efficiency is the lowest, and its maximal spectral efficiency is also the lowest. With the same length L, the spectral efficiency η of OOC system is 0.0302 (b/s)/Hz while that of Gold sequence system is 0.1115 (b/s)/Hz at a BER of 10^{-9}. The code utilizing efficiency γ of OOC system is 61.54%

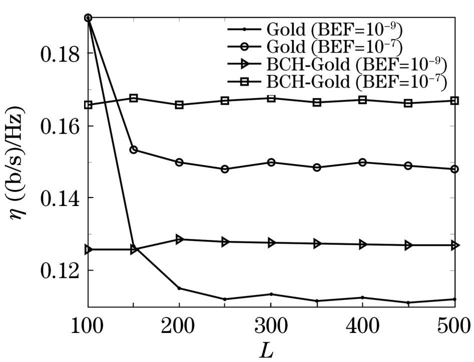

Fig. 2. Spectral efficiency η of Gold sequence versus length L.

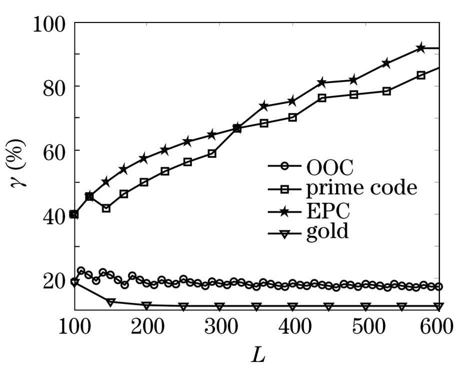

Fig. 3. Code utilizing efficiency γ versus length L (BER = 10^{-9}).

Table 3. Spectral Efficiencies of OCDMA Codes

Code	L	η ((b/s)/Hz)	γ (%)	$\eta_{\max}$ ((b/s)/Hz)
OOC	529	0.0302	61.54	6.1028
Prime Code	529	0.0304	78.26	6.1877
EPC	561	0.0196	64.71	6.1021
Gold Sequence	511	0.1115	11.11	5.2012

The fixed BER is 10^{-9}.

while that is 11.11% for Gold sequence system, and the maximal spectral efficiency of OOC system is 0.9 (b/s)/Hz higher than that of Gold sequence system. From Figs. 1, 2 and Table 3, the spectral efficiencies of all such OCDMA systems remain very low as a whole. To a certain extent, low efficiencies militate against practical development of OCDMA systems in spite of their tremendous potentials. The spectral efficiency in coherent OCDMA systems is higher than that in incoherent OCDMA systems. Therefore, the research effort to improve the spectral efficiency of OCDMA systems should be further focused on the growth of code cardinality and the decrease of BER.

In this letter, we discuss the spectral efficiency in the unipolar code and bipolar code OCDMA systems. BCH error correction coding is used to increase the spectral efficiency. We define the code utilizing efficiency and compare several kinds of code schemes. Results show that the bipolar code has a large spectral efficiency and the unipolar code has a large code utilizing efficiency. The maximal spectral efficiency of unipolar code systems is higher than that of bipolar code systems. When the BER requirement of system is decreased, the spectral efficiency can be increased.

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