Accuracy analysis on Rayleigh lidar measurements of atmospheric temperature based on spectroscopy

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We make a detailed analysis on the linearity and accuracy of the relationship between the full-width at half-height (FWHH) of the atmosphere molecules Rayleigh scattering spectrum and the square root of the atmospheric temperature. A simulation of the FWHH of the atmosphere molecules Rayleigh scattering spectrum is made based on the S6 Atmosphere Model and U.S. Standard Atmosphere Model. The calculated temperature is compared with the initial simulation temperature. The result shows that the FWHH of the atmosphere molecules Rayleigh scattering spectrum is nearly proportional to the atmospheric temperature. The goodness-of-fit index of the fitting curve is 0.9977 and the maximum absolute error of measured atmospheric temperature is about 2 K.

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Accurate remote sensing of atmospheric temperature is of major importance to atmospheric science. Traditionally, the low elevation atmospheric temperature profile is measured by lidar based on echo energy detection[1−3]. It suffers from error due to the aerosol's Mie scattering effect. This effect can be eliminated in the lidar system based on spectroscopy detection. It is assumed that the atmosphere Rayleigh scattering spectrum is Gaussian. The full-width at half-height (FWHH) of the spectrum is proportional to the square root of the atmospheric temperature[4−6]. The atmospheric temperature can be derived from the FWHH. However, the hypothesis is not accurate. The atmosphere pressure will result in molecular collision, which leads to spectral broadening. As the pressure increases, the effect will be worse and the Rayleigh scattering spectrum will deviate from the Gaussian shape.

The relationship between the FWHH of atmosphere Rayleigh scattering spectrum and the square root of the atmospheric temperature is not strictly linear. Therefore, the inaccuracy of the method will result in error of atmospheric temperature measurement. The accuracy analysis of the lidar system is of great importance. However, to the best of our knowledge, such work has not been reported yet. In this letter, we will analyze the theoretical error in the measurements of the atmospheric temperature in detail, and give some helpful predictions for actual applications.

Yip and Nelkin *et al.* have studied the Rayleigh-Brillouin scattering theoretically and experimentally[7,8]. Based on their experimental results, Tenti's S6 model is regarded as the best model available for atmospheric applications[9−11]. Krueger *et al.* used the Tenti's S6 model as the theoretical foundations for atmospheric temperature measurement accuracy analysis[12]. In this letter, we also use the theoretical Tenti's S6 model for the analysis of Rayleigh-Brillouin scattering spectrum. And the 1976 U.S. Standard Atmosphere Model[13] is used for the molecular backscattering and the globe north median distribution for the aerosol model.

Thirteen samples are taken in the elevation range of $0-11$ km, i.e., 0, 0.5, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 km. The corresponding temperature T and pressure P are calculated by the U.S. Standard Atmosphere Model 1976[13]. Then the atmosphere Rayleigh-Brillouin scattering spectrum is got by using the S6 Atmosphere Model. The FWHH for actual atmosphere is figured out according to these results, as shown in Table 1.

From the data in Table 1, one can obtain the relationship between Rayleigh-Brillouin scattering spectrum and $\sqrt{T}$, as shown in Fig. 1.

It can be concluded that the temperature increases as the FWHH increases. Though not strictly linear, $\sqrt{T}$ is almost proportional to FWHH. Part of the line is slightly bending.

Table 1. FWHH-Elevation-Pressure-Temperature Relationship in the Actual Atmosphere Condition

Elevation H (km)	Pressure P ($\times10^5$ Pa)	Temperature T (K)	$\sqrt{T}$ ($K^{1/2}$)	FWHH Γ (GHz)
0	1.01325	288.15	16.975	3.120
0.5	0.95461	284.90	16.879	3.086
1	0.89876	281.65	16.782	3.052
2	0.79501	275.15	16.588	2.985
3	0.70121	268.66	16.391	2.916
4	0.61660	262.17	16.192	2.848
5	0.54480	255.68	15.990	2.780
6	0.47218	249.19	15.786	2.713
7	0.41105	242.70	15.579	2.652
8	0.35651	236.22	15.369	2.589
9	0.30800	229.73	15.157	2.530
10	0.26499	223.25	14.942	2.471
11	0.22699	216.77	14.723	2.412

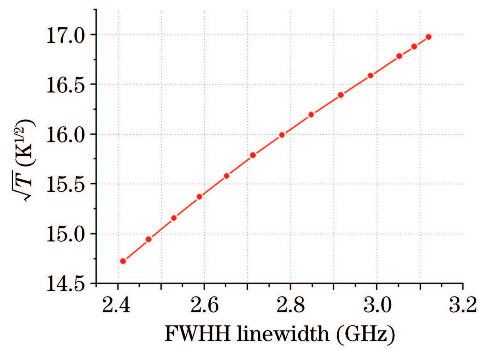

Fig. 1. Relationship between the FWHH of Rayleigh-Brillouin scattering spectrum and square root of temperature $\sqrt{T}$ in actual atmosphere condition.

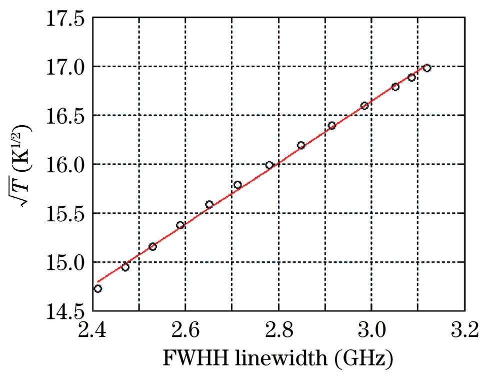

Fig. 2. Comparison of regression data (line) and original data (circles).

The root mean square (RMS) method is applied to the data set to verify the linear relationship again. The regression formula is

$$\sqrt{T} = 3.1426 \times \Gamma + 7.2105. \tag{1}$$

The reference data in Table 1 are compared with the data calculated from the regression formula, as shown in Fig. 2. It is clearly seen that the two sets of data fit well with each other, and only little deviation exists between them.

The regression formula is verified for goodness-of-fit. The RMS is taken to measure the quantity[14]. Assume that the actual measured value is Y, take Y_1 as its average, and the value taken from the linear regression formula is Y_2. The RMS is given by the formula $\Sigma(Y-Y_2)^2$. The variance is given by the formula $\Sigma(Y-Y_1)^2$. The goodness-of-fit increases as the RMS-to-variance ratio decreases. A normalized index R^2 is defined as

$$R^2 = 1 - \frac{\sum(Y-Y_2)^2}{\sum(Y-Y_1)^2}. \tag{2}$$

The linearity index is got as $R^2 = 0.9977$. It can be concluded that the goodness-of-fit is satisfying. From the above analysis, $\sqrt{T}$ and FWHH follows the linear relationship.

The regression Eq. (1) can be taken as the empirical formula describing the relationship between Γ and T. Once the value of the Rayleigh-Brillouin scattering spectrum Γ at a point in the atmosphere is detected, the temperature T can be figured out by the formula. It is convenient for engineering applications.

We calculate the values of $\sqrt{T}$ for different Γ by Eq. (1) as shown in Table 2. For comparison, the data calculated from the S6 Atmosphere Model are also given.

The fitted values calculated from the regression formula and the reference values got from the U.S. Standard Atmosphere Model[13] are compared in Fig. 3.

From Table 2 and Fig. 3, the maximum absolute error between the initial $\sqrt{T}$ and the fitted value is $\max \sigma_{\sqrt{T}} = 0.0675\ \mathrm{K}^{1/2}$, and the minimum is $\min \sigma_{\sqrt{T}} = 0.0032\ \mathrm{K}^{1/2}$. The mean value of the absolute value is $\bar{\sigma}_{\sqrt{T}} = 0.0305\ \mathrm{K}^{1/2}$.

Similarly, the fitted values of T and the reference values are compared in Fig. 4.

From Table 2 and Fig. 4, the maximum absolute error between the initial value and the fitted value of T is $\max \sigma_T = 1.9889$ K, and the minimum is $\min \sigma_T = 0.1179$ K. The mean value of the absolute value is $\bar{\sigma}_T = 0.9640$ K.

Table 2. Comparison of Regression Data and Data Calculated from S6 Atmosphere Model

Elevation H (km)	FWHH Γ (GHz)	$\sqrt{T}$ (K$^{1/2}$)	Regression of $\sqrt{T}$ (K$^{1/2}$)	Abs. Error $\sigma_{\sqrt{T}}$ (K$^{1/2}$)	Temperature T (K)	Regression of T (K)	Abs. Error σ_T (K)
0	3.120	16.975	17.0154	0.0404	288.15	289.5238	1.3738
0.5	3.086	16.879	16.9086	0.0296	284.90	285.9008	1.0008
1	3.052	16.782	16.8017	0.0197	281.65	282.2971	0.6471
2	2.985	16.588	16.5912	0.0032	275.15	275.2679	0.1179
3	2.916	16.391	16.3743	0.0167	268.66	268.1177	0.5423
4	2.848	16.192	16.1606	0.0314	262.17	261.1650	1.0050
5	2.780	15.990	15.9469	0.0431	255.68	254.3036	1.3764
6	2.713	15.786	15.7364	0.0496	249.19	247.6343	1.5557
7	2.652	15.579	15.5447	0.0343	242.70	241.6377	1.0623
8	2.589	15.369	15.3467	0.0223	236.22	235.5212	0.6988
9	2.530	15.157	15.1613	0.0043	229.73	229.8650	0.1350
10	2.471	14.942	14.9759	0.0339	223.25	224.2776	1.0276
11	2.412	14.723	14.7905	0.0675	216.77	218.7589	1.9889

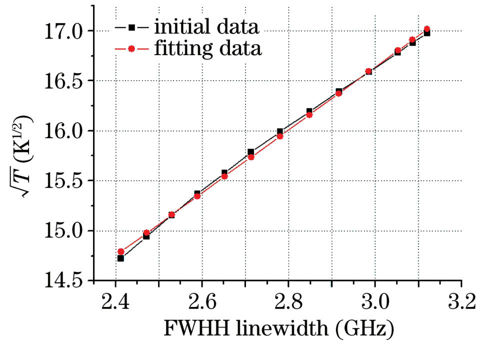

Fig. 3. Comparison of regression of $\sqrt{T}$ and the reference data.

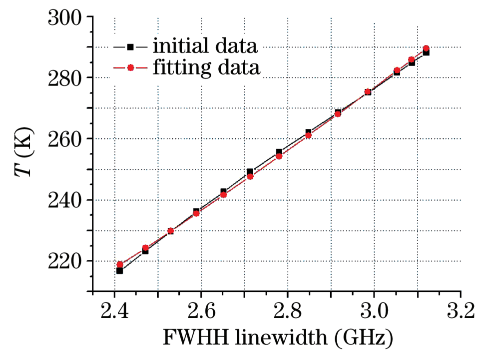

Fig. 4. Comparison of regression of T and the reference data.

We have found that the data calculated from the linear regression formula fits well with the data calculated from the S6 Atmosphere Model. The maximum absolute error is less than 2 K, and the minimum absolute error is merely 0.1 K. The error performance can satisfy the accuracy requirement.

It can be seen in Table 2 that the error decreases as the elevation increases from the sea level upwards to a fixed height. Then the error increases as the elevation increases. The phenomenon can be explained as follows. As the elevation increases, the pressure decreases. That will result in the broadening effect of the molecular collision. The Rayleigh scattering spectrum will be more like Gaussian distribution. The linearity and accuracy will be better. However, as the pressure decreases, the initial simulation data error will be dominant in the simulation process, which will increase the absolute error. It is our future work to eliminate the absolute error induced by the initial simulation data.

In conclusion, we have analyzed the linearity and accuracy of the relationship between the FWHH of the Rayleigh scattering spectrum and the square root of the atmospheric temperature for the actual atmosphere. We have also analyzed the error of temperature measurement of Rayleigh lidar caused by the inaccurate linearity. The goodness-of-fit index of the fitting curve is 0.9977 and the maximum absolute error of the measured atmospheric temperature is about 2 K.

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