Nanoscale displacement of the image of an atomic source of radiation

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Light emitted by an atomic source of radiation appears to travel along a straight line (ray) from the location of the source to the observer in the far field. However, when the energy flow pattern of the radiation is resolved with an accuracy better than an optical wavelength, it turns out that the field lines are usually curved. We consider electric dipole radiation, a prime example of which is the radiation emitted by an atom during an electronic transition, and we show that the field lines of energy flow are in general curves. Near the location of the dipole, the field lines exhibit a vortex structure, and in the far field they approach a straight line. The spatial extension of the vortex in the optical near field is of nanoscale dimension. Due to the rotation of the field lines near the source, the asymptotic limit of a field line is not exactly in the radially outward direction and as a consequence, the image in the far field is slightly shifted. This sub-wavelength displacement of the image of the source should be amenable to experimental observation with contemporary nanoscale-precision techniques.

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When light is emitted by a localized source and detected in the far field (many wavelengths from the source), it appears that the radiation travels along a straight line, usually referred to as an optical ray. Such a ray is a field line of the Poynting vector, representing the direction of energy flow in the electromagnetic field, and therefore it seems that the energy propagates along a straight path from the source to the observer. Moreover, in the geometrical optics limit of light propagation[1], the optical rays in a homogeneous medium are straight, no matter the distance to the source. Therefore the field lines of the Poynting vector are straight, even close to the source. In the geometrical optics limit, details of the energy flow pattern on the scale of a wavelength are neglected. When the exact solution of Maxwell's equations for the radiation emitted by a source is considered, the field lines of the Poynting vector are in general curves, rather than straight lines, and they will approach a straight line only asymptotically when observed in the far field. For instance, the radiation emitted by an electric or magnetic dipole may exhibit a vortex structure near the location of the dipole[2], and such vortices may also appear in multipole radiation of any order[3]. The first prediction of the existence of an optical vortex was made by Braunbek *et al.*[4]. They considered the diffraction of a plane wave around the edge of a conducting half-plane and found that a vortex could appear at the illuminated side, located a fraction of a wavelength from the sheet and near the edge. Optical vortices of this type are a result of interference and diffraction and they can also be found in diffraction, for instance, through a slit in a conducting material[5,6]. The most common type of optical vortices are the vortices in the field of a Laguerre-Gaussian laser beam[7−9]. With state-of-the-art contemporary high-precision experimental techniques, it has become feasible to detect such sub-wavelength vortices in field line patterns of light[10,11].

A vortex in the field line pattern near the location of a source is of a different nature. It is not due to diffraction or interference but reflection of the angular momentum carried away by the radiation field[2]. Also, in diffraction or interference vortices, the field lines of the Poynting vector swirl around a singular point of the radiation field. At such a point, the Poynting vector itself vanishes and the energy circulates around the singularity. When a source is located at the center of a vortex, as in the case we consider here, the energy emanates from the center and rotates about an axis for numerous times before radiating away to the far field. We shall consider the electromagnetic radiation emitted by an electric dipole, oscillating harmonically with an angular frequency ω. The dipole moment is written as $\mathbf{d}(t) = d_{\mathrm{O}}\mathrm{Re}[\boldsymbol{\varepsilon}\exp(-\mathrm{i}\omega t)]$, where d_{O} is an amplitude factor and $\boldsymbol{\varepsilon}$ is a complex-valued vector. It can be shown[12] that in its most general state of oscillation, the dipole moment $\mathbf{d}(t)$ traces out an ellipse. When we take the plane of the ellipse as the xy plane, we can parametrize the vector $\boldsymbol{\varepsilon}$ as

$$\boldsymbol{\varepsilon} = -\frac{1}{\sqrt{\beta^2+1}}(\beta\mathbf{e}_x + \mathrm{i}\mathbf{e}_y), \tag{1}$$

with β being real, and here $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors along the x and y axes, respectively. For $\beta > 0$, the dipole moment rotates counterclockwise when viewed from the positive z axis, and for $\beta < 0$, the rotation is clockwise. When $\beta = \pm 1$, the ellipse reduces to a circle, and for $\beta = 0$, the oscillation becomes linear along the y axis. With the known expressions[13] for the electric and magnetic fields of an electric dipole, the Poynting vector $\mathbf{S}(\mathbf{r})$ can be evaluated and the result in spherical coordinates (r, θ, ϕ) is

$$\mathbf{S}(\mathbf{r}) = \frac{3P_{\mathrm{O}}}{8\pi r^2}\left[\hat{\mathbf{r}}\zeta(\theta,\phi) + \mathbf{e}_\phi \frac{2}{q}\left(1+\frac{1}{q^2}\right)\frac{\beta}{\beta^2+1}\sin\theta\right], \tag{2}$$

where we have introduced $q = \omega r/c$ as the dimensionless distance between the field point (r, θ, ϕ) and the dipole. Here c is the speed of light in vacuum and ω/c is the wave number. In this way, a distance of 2π corresponds to one wavelength. In Eq. (2), P_O is the total emitted power and the function

$$\zeta(\theta, \phi) = 1 - \frac{1}{2}\sin^2\theta\left[1 + \frac{\beta^2 - 1}{\beta^2 + 1}\cos(2\phi)\right] \tag{3}$$

is the dimensionless power per unit solid angle. The Poynting vector has an $\hat{\mathbf{r}}$ and an $\mathbf{e}_\phi$ component, but no $\mathbf{e}_\theta$ component. Therefore, along a field line the value of θ is constant, say θ_O, and consequently the field line lies on the cone $\theta = \theta_\mathrm{O}$. Close to the source, the term proportional to $\mathbf{e}_\phi$ dominates, and this gives a swirling of the field line around the z axis. In the far field, this term becomes small compared with the term proportional to $\hat{\mathbf{r}}$, and therefore far away from the source the field lines run in the radially outward direction. A typical field line is shown in Fig. 1. It is seen that the spatial extension of the vortex is a fraction of a wavelength.

Figure 2 shows three field lines for $\theta_\mathrm{O} = \pi/2$, for which the cone reduces to the xy plane. The difference between the various field lines is that asymptotically they run into different directions. Therefore, for a given θ_O, we can specify a field line further by its final azimuthal angle ϕ_O. Conversely, when the radiation is observed in the far field in a given direction $(\theta_\mathrm{O}, \phi_\mathrm{O})$, this corresponds uniquely to a field line of the Poynting vector into this direction. Figure 3 shows two field lines with $\phi_\mathrm{O} = \pi$ for a circular dipole with $\beta = 1$. Each field line approaches asymptotically a straight line, indicated by ℓ in the figure, and when a field line is observed in the far field, there appears no difference between the field line and the asymptote ℓ. For an observer far away it therefore seems that the radiation comes from a point in the xy plane, which is displaced with respect to the position of the source. We represent this virtual displacement by the vector $\mathbf{q}_\mathrm{d}$ and for the cases shown in the figure, this vector lies along the positive y axis. The equation for the line ℓ can be found from Eq. (2) by asymptotic expansion, after which we can compute the intersection with the xy plane. This yields

$$\mathbf{q}_\mathrm{d} = \frac{\sin\theta_\mathrm{O}}{\zeta(\theta_\mathrm{O}, \phi_\mathrm{O})}\frac{2\beta}{\beta^2 + 1}(\mathbf{e}_x \sin\phi_\mathrm{O} - \mathbf{e}_y\cos\phi_\mathrm{O}) \tag{4}$$

for the displacement vector. The displacement depends on the direction of observation $(\theta_\mathrm{O}, \phi_\mathrm{O})$ and parametrically on the parameter β of the ellipse. For a linear dipole

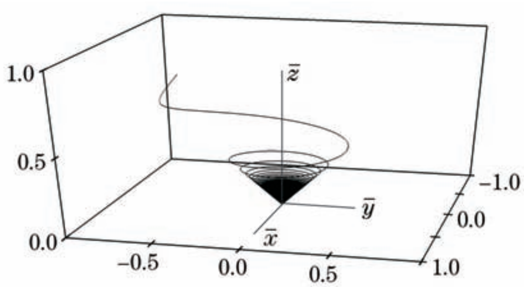

Fig. 1. Typical field line of the Poynting vector for the radiation emitted by an electric dipole with $\beta = 1$, located at the origin of coordinates. The field line lies on a cone (of 45° with the z axis for this example). Near the source, the field line rotates numerous times around the z axis and this gives the vortex structure in the near-field emission pattern. We use dimensionless variables $\bar{x} = \omega x/c$, etc., so that a distance of 2π corresponds to one optical wavelength.

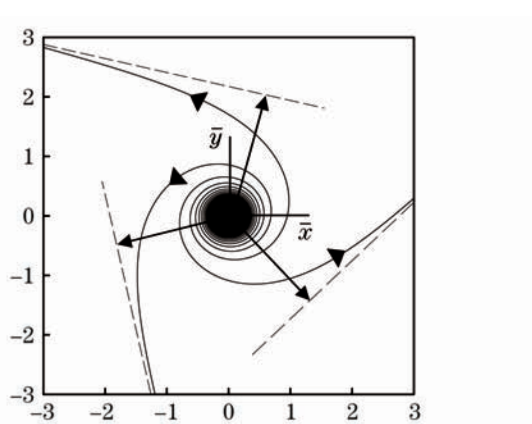

Fig. 2. Three field lines in the xy plane ($\theta_\mathrm{O} = \pi/2$) for a circular dipole ($\beta = 1$). The field lines approach asymptotically the dashed lines, corresponding to various values of the observation angle ϕ_O.

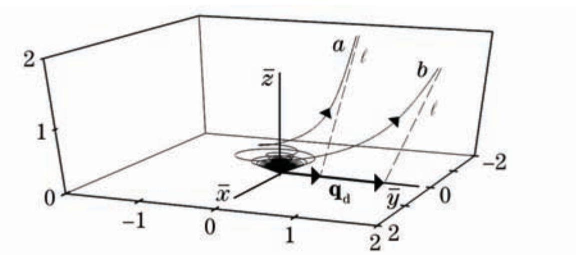

Fig. 3. Field lines for $\beta = 1$, $\phi_\mathrm{O} = \pi$. Curves a and b correspond to observation angles $\theta_\mathrm{O} = \pi/6$ and $\theta_\mathrm{O} = \pi/3$, respectively. Far away from the source, the field lines approach straight lines indicated by ℓ. When viewed from the far field, a curved field line is indistinguishable from the asymptotic line ℓ, and this gives rise to an apparent displacement of the source. The image point in the xy plane is the intersection between ℓ and the xy plane, and the location of this point is represented by the displacement vector $\mathbf{q}_\mathrm{d}$.

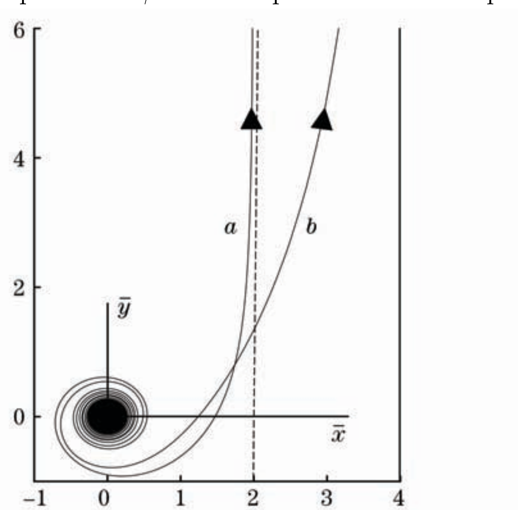

Fig. 4. Field lines for $\theta_\mathrm{O} = \pi/2$ and $\phi_\mathrm{O} = \pi/2$. Curves a and b correspond to $\beta = 1$ and $\beta = 0.5$, respectively. The dashed lines are the asymptotes. The displacement vectors are along the x axis, and their magnitudes are $q_\mathrm{d} = 2$ and $q_\mathrm{d} = 4$, respectively.

($\beta = 0$), there is no displacement, and the field lines run radially outward from the source to the observer. For $\beta \neq 0$, the displacement is zero for observation along the z axis, and its magnitude increases away from the z axis. The magnitude of the displacement is maximum for $\theta_{\mathrm{O}} = \pi/2$, so for observation along the xy plane, and given $\theta_{\mathrm{O}} = \pi/2$, it is maximum for observation along the major axis of the ellipse. For a circular dipole ($|\beta| = 1$), the magnitude of the displacement is $q_{\mathrm{d}} = 2$ for observation in the xy plane. For $|\beta| \neq 1$, we find from Eq. (4) that along the major axis of the ellipse the magnitude of the displacement is $2/|\beta|$ for $|\beta| < 1$ and $2|\beta|$ for $|\beta| > 1$. Therefore, the maximum value of q_{d} can grow without bounds as a function of β. Figure 4 shows the field lines for observation along the major axis for $\beta = 1$ and $\beta = 0.5$. For $|\beta| \neq 1$, the displacement can be large, and we see from the figure that the approach to the asymptote ℓ becomes slow.

In conclusion, the field lines of the Poynting vector for electric dipole radiation have a vortex structure near the location of the source. In the far field, each field line approaches asymptotically a straight line, reminiscent of an optical ray. Due to the rotation near the source, this asymptotic line does not go through the origin of coordinates, and hence it appears for an observer in the far field that the image is displaced in the xy plane. The displacement is of the order of an optical wavelength, and it depends on both the direction of observation and the state of rotation of the dipole moment. This nanoscale effect should be amenable to experimental observation, and in this fashion a near field property of the radiation could be detected in the far field.

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