Diffractive beam parameters of LP_{01} mode of fiber

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The diffractive beam parameters of LP_{01} mode of fiber are analyzed in detail. Based on solving linear equations, two formulas for two kinds of mode-field radii as functions of normalized frequency are presented, and relations between angular radius of far-field divergence, beam propagation factor, and normalized frequency are given. Numerical calculation indicates that the maximal relative error is smaller than 1% within a reasonable parameter range.

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The mode-field radius is closely related to various transmission characteristics, such as splice loss, microbending loss, two waveguides and source-to-fiber coupling efficiency, and so on[1−4]. Thus, it is always a research focus[5−7]. However, beam parameters include mode-field radius in the near-field, angular radius of far-field divergence, and beam propagation factor referring to International Organization for Standardization (ISO) 11146-1:2005[8]. Thus, the other two parameters, angular radius of far-field divergence and beam propagation factor, are also very important. But researches on them are often neglected.

As for circular-symmetric single-mode fiber, Petermann proposed two well-known mode-field radius definitions based on the near-field second moment and differential operator[9,10]. The angular radius of far-field divergence can also be seen as two types according to the two definitions of mode-field radius. Beam propagation factor, which was introduced by Siegman[11], is the evaluation criteria for optical beam and still a research interest[12−14]. As for diffractive beam of LP_{01} mode of circular-symmetric step refractive index fiber, the relation among beam parameters is presented in Ref. [5], and the formula of each of beam parameters as function of normalized standing wave parameter and normalized evanescent wave parameter has been derived.

Because of the characteristic of normalized frequency itself, relation between beam parameters and normalized frequency is investigated widely for the convenience of analysis and calculation. In spite of that, accurate calculating expressions of beam parameters as functions of normalized frequency are not found yet. Only some approximate formulas for mode-field radius as functions of normalized frequency are presented and all of them are based on mathematical modeling, but their precision is not perfect. For example, an approximation of mode-field radius presented by Marcuse[15] is used very often. However, our research indicates that it is not accurate enough. Furthermore, equations of angular radius of far-field divergence and beam propagation factor as functions of normalized frequency have not been reported yet.

In this letter, the relation between beam parameters and normalized frequency is analyzed in detail with numerical calculation. Two more accurate formulas for two kinds of mode-field radii as functions of normalized frequency are derived. Furthermore, the relations among angular radius of far-field divergence, beam propagation factor, and normalized frequency are presented based on solving linear equations. These conclusions have important meaning in practical applications.

For diffractive beam of LP_{01} mode of circular-symmetric step refractive index fiber, the second moment and differential operator mode-field radius are respectively expressed as[5]

$$\omega_{\rm SM} = \frac{\sqrt{6}a}{3}\left[1-\frac{2}{U^2}+\frac{2}{W^2}+\frac{2J_0(U)}{UJ_1(U)}\right]^{1/2}, \quad (1)$$

$$\omega_{\rm DO} = \frac{\sqrt{2}aJ_1(U)}{WJ_0(U)}, \quad (2)$$

where $U = a[(k_0n_1)^2-\beta^2]^{1/2}$ and $W = a[\beta^2-(k_0n_2)^2]^{1/2}$ are the normalized standing wave parameter and the normalized evanescent wave parameter, respectively, $k_0 = 2\pi/\lambda$ is the wave number in vacuum, λ is the wavelength of electromagnetic wave in vacuum, β is the propagation constant, a is the radius of core layer, n_1 and n_2 are the refractive indices of core and cladding, respectively.

For convenience, Marcuse proposed a well-known approximation for mode-field radius as a function of normalized frequency from least mean square fitting of a Gaussian function to the actual mode field[15]:

$$\omega_{\rm M} = a[0.65+1.619V^{-3/2}+2.879V^{-6}], \quad (3)$$

where V is the normalized frequency. For comparison, the results obtained from Eqs. (1) and (3) are given in Fig. 1. We can clearly see that gaps between them are significant. Although Eq. (3) is convenient for calculation, it is not enough accurate, because it is introduced based on Gaussian approximation.

In view of this, an approach of solving linear equations is presented based on the mathematical model as

$$y = A+\frac{B}{x^{3/2}}+\frac{C}{x^6}, \quad (4)$$

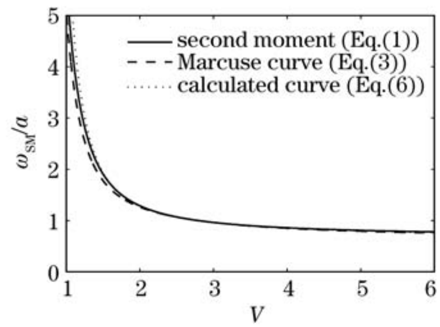

Fig. 1. Relation between the second moment mode-field radius ω_{SM} and normalized frequency V.

where A, B, and C are coefficients. In order to solve Eq. (4), we take three values of x into account only and assume that they are x_1, x_2, and x_3 respectively; accrodingly, three values of y_1, y_2, and y_3 are taken for y. Consequently, Eq. (4) can be translated into matrix form as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & x_1^{-\frac{3}{2}} & x_1^{-6} \\ 1 & x_2^{-\frac{3}{2}} & x_2^{-6} \\ 1 & x_3^{-\frac{3}{2}} & x_3^{-6} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}, \tag{5}$$

and coefficients A, B, and C can be obtained by solving the matrix function.

We set V as 1.6013, 2.2366, and 3.9276, respectively, and then ω_{SM} takes the values of $1.7142a$, $1.1652a$, and $0.8665a$ accordingly from Eq. (1). Substituting these data into Eq. (5) and through solving linear equations, we can obtain

$$\omega'_{SM} = a\left[0.6685 + \frac{1.5304}{V^{3/2}} + \frac{4.8955}{V^6}\right]. \tag{6}$$

The results obtained from Eq. (6) are shown as the dotted line in Fig. 1. It is obviously in agreement with the solid line. The relative error is introduced for further explanation,

$$\delta = \frac{\alpha - \gamma}{\alpha} \times 100\%, \tag{7}$$

where α refers to ω_{SM}, ω_{DO}, θ_{SM}, θ_{DO}, and M^2 respectively, γ depends on ω'_{SM}, ω'_{DO}, θ'_{SM}, θ'_{DO}, and $M^{2\prime}$ accordingly.

Further study shows that relative error between ω_{SM} and ω'_{SM} fluctuates with increasing normalized frequency. When $1.50 < V < 5.67$, the maximal relative error satisfies $|\delta_{max}| < 1\%$. Therefore, the accuracy of Eq. (6) is high and can be used to calculate the second moment mode-field radius.

Through a similar procedure, let $V = 1.1074$, 1.6625, and 3.7202 separately, then $\omega_{DO} = 2.9539a$, $1.4899a$, and $0.8644a$ correspondingly according to Eq. (2). Substitution of the data into Eq. (5) yields

$$\omega'_{DO} = a\left[0.6300 + \frac{1.6778}{V^{3/2}} + \frac{1.6307}{V^6}\right]. \tag{8}$$

Numerical data prove that the relative error between ω_{DO} and ω'_{DO} varies with the increase of normalized frequency, but $|\delta|$ varies within 1% as V changes from 1.04 to 11.13 all the same. The result reveals that Eq. (8) also has high accuracy.

The two kinds of angular radius of far-field divergence for diffractive beam of LP_{01} mode of circular-symmetric fiber, that is, the second moment and differential operator angular radii of far-field divergence, are as functions of U and W under paraxial approximation[5]:

$$\theta_{SM} = \frac{\sqrt{2}\lambda W J_0(U)}{2\pi a J_1(U)}, \tag{9}$$

$$\theta_{DO} = \frac{\sqrt{6}\lambda}{2\pi a}\left[1 - \frac{2}{U^2} + \frac{2}{W^2} + \frac{2J_0(U)}{UJ_1(U)}\right]^{-1/2}. \tag{10}$$

There are not accurate relations between these two parameters and normalized frequency. In view of this, the method mentioned above is taken based on another mathematical model[4]:

$$y = A + \frac{B}{\sqrt{x}} + \frac{C}{x^3} + \frac{D}{x^5}, \tag{11}$$

where A, B, C, and D are coefficients also. All the same, in order to obtain the coefficients, four values of x, x_1, x_2, x_3, and x_4, must be taken into account, and y takes y_1, y_2, y_3, and y_4 accordingly. Substitution of these data into Eq. (11) yields a matrix function as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & x_1^{-\frac{1}{2}} & x_1^{-3} & x_1^{-5} \\ 1 & x_2^{-\frac{1}{2}} & x_2^{-3} & x_2^{-5} \\ 1 & x_3^{-\frac{1}{2}} & x_3^{-3} & x_3^{-5} \\ 1 & x_4^{-\frac{1}{2}} & x_4^{-3} & x_4^{-5} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}. \tag{12}$$

The coefficients A, B, C, and D can be obtained by solving the matrix function.

We suppose that V takes 1.6040, 2.0690, 3.4420, and 5.3960, respectively, then θ_{SM} is $1.2869/(ka)$, $1.6575/(ka)$, $2.2415/(ka)$, and $2.6077/(ka)$ accordingly based on Eq. (9). Substituting V and θ_{SM} into Eq. (12) and solving linear algebraic equations, the second moment angular radius of far-field divergence as function of normalized frequency is obtained:

$$\theta'_{SM} = \frac{1}{ka}\left[3.9701 - \frac{3.1422}{\sqrt{V}} - \frac{1.5859}{V^3} + \frac{1.9342}{V^5}\right]. \tag{13}$$

In order to explain the accuracy, the results given in terms of Eq. (9) is shown in Fig. 2 as the solid line. The results obtained from Eq. (13) are also shown in Fig. 2

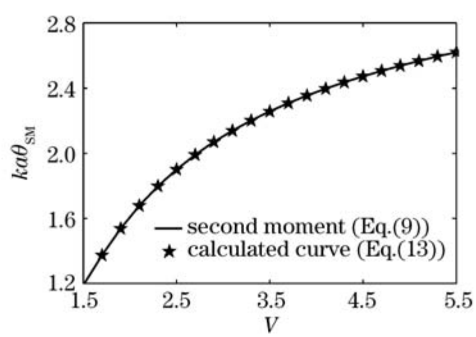

Fig. 2. Relation between second moment angular radius of far-field divergence θ_{SM} and normalized frequency V.

as pentagrams and it is conspicuous that they agree with the solid line well.

Further study manifests that the relative error between $\theta_{\rm SM}$ and $\theta'_{\rm SM}$ fluctuates with changing normalized frequency, while if the normalized frequency satisfies $1.56 < V < 6.695$, the maximal relative error $|\delta|$ is always within 0.4%.

As for the differential operator angular radius of far-field divergence, suppose that $V = 1.5894$, 2.0708, 3.3173, and 5.4158, consequently, $\theta_{\rm DO} = 1.1530/(ka)$, $1.6030/(ka)$, $2.1691/(ka)$, and $2.5087/(ka)$ correspondingly by Eq. (10). Then the relation between $\theta_{\rm DO}$ and V can be presented based on Eq. (12) as

$$\theta'_{\rm DO} = \frac{1}{ka}\left[3.4470 - \frac{2.1944}{\sqrt{V}} - \frac{4.1856}{V^3} + \frac{4.6564}{V^5}\right]. \quad (14)$$

All the same, numerical calculation verifies that the relative error between $\theta_{\rm DO}$ and $\theta'_{\rm DO}$ varies also with the increasing normalized frequency, however, when $1.54 < V < 7.32$, the maximal error $|\delta_{\max}| < 0.5\%$. Numerical data also support the correctness of Eq. (14).

Under the condition of paraxial approximation, beam propagation factor that is defined by the second moment method can be expressed as[11]

$$M^2 = \frac{\pi\omega_{\rm SM}\theta_{\rm SM}}{\lambda}. \quad (15)$$

With regard to diffractive beam of LP_{01} mode of fiber, it is based on normalized standing wave parameter and normalized evanescent wave parameter also[5]:

$$M^2 = \frac{\sqrt{3}WJ_0(U)}{3J_1(U)}\left[1 - \frac{2}{U^2} + \frac{2}{W^2} + \frac{2J_0(U)}{UJ_1(U)}\right]^{1/2}. \quad (16)$$

It is crucial to give a relation between M^2 and V. Considering that $V = 1.5610$, 2.2350, 3.7510, and 5.4840 for example, then $M^2 = 1.1114$, 1.0259, 1.0212, and 1.0413 can be obtained based on Eq. (16). Thus, after substituting the data of V and M^2 into Eq. (12) and solving linear equations, beam propagation factor as function of normalized frequency can be given as

$$M^{2\prime} = 1.1844 - \frac{0.3474}{\sqrt{V}} + \frac{0.8669}{V^3} - \frac{0.2121}{V^5}. \quad (17)$$

To show the accuracy of Eq. (17), the results obtained by Eqs. (16) and (17) are shown in Fig. 3. We can draw an conclusion that the results from Eqs. (17) are in excellent agreement with that from Eqs. (16).

Further survey illuminates that relative error between M^2 and $M^{2\prime}$ varies with the increase of normalized frequency. On the other hand, $|\delta|$ varies within 0.1% as V changes from 1.16 to 7.17. Obviously, these numerical results show that the accuracy of Eq. (17) is very good.

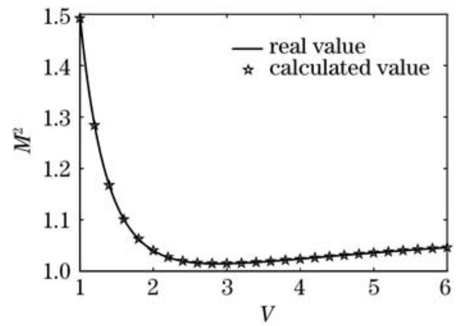

Fig. 3. Beam propagation factor M^2 as function of normalized frequency V.

In conclusion, the mode-field radius, the angular radius of far-field divergence, and the beam propagation factor are investigated numerically. The equations of beam parameters as functions of normalized frequency are given. Numerical calculation proves that the equations have high accuracy and the maximal relative error is always smaller than 1% within a reasonable parameter range. These conclusions may provide theoretical support for analyzing beam parameters quickly in practical applications.

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