

# A new family of two-dimensional triple-codeweight asymmetric optical orthogonal code for OCDMA networks

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A new generation algorithm of two-dimensional triple-codeweight asymmetric optical orthogonal codes for optical code division multiple access (OCDMA) networks is proposed. The code cardinality is obtained and the error-probability performance for corresponding OCDMA system is analyzed. The codes with two constraints (i.e., auto- and cross-correlation properties) being unequal are taken into account. On the premise of fixed system resources, the code cardinality can be significantly improved. By analysis of the error-probability performance, it is shown that the codes with different parameters have different performances. Therefore, this type of codes can be applied to support diverse quality of service (QoS) and satisfy the quality requirement of different multimedia or distinct users, and simultaneously make the better use of bandwidth resources in optical networks.

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Optical code division multiple access (OCDMA) has been considered as a competitive candidate for the multiple access scheme in the future all-optical networks, especially optical access networks, due to its attractive features such as asynchronous access, dynamic bandwidth assignment, ability to support multimedia services, bursty traffic, and so on. The user address code with better performance is the basis for implementing an OCDMA network. The result in previous work on optical orthogonal codes (OOCs) considers the code with the same auto- and cross-correlation constraints, until a new asymmetric OOC has been developed<sup>[1]</sup>. The two constraints have different effects on a system performance, because the auto-correlation constraint contributes only to system synchronization and the cross-correlation constraint affects both synchronization and operation. According to the distinction between these two constraints, Yang *et al.* developed a new algorithm to generate one-dimensional (1D) constant-weight OOC with unequal correlation constraints (i.e.,  $\lambda_a > \lambda_c$ )<sup>[1]</sup> through letting the auto-correlation constraint exceed the cross-correlation constraint, which can ensure the system dependability by improving the code cardinality.

To support diverse quality of service (QoS) requirements in OCDMA networks, a double-codeweight OOC technique<sup>[2]</sup> was proposed. However, this double-weight OOC only supports two classes of services and cannot satisfy QoS of different multimedia (e.g., data, voice, and video). In addition, the codes mentioned above are all 1D codes. Although their cardinalities have been improved, compared with that of 1D symmetric code, they still have some defects such as smaller cardinalities and worse correlation performances.

In this letter, we extend the theories of 1D asymmetric OOC and variable-weight OOC, and break through the limitation of double-codeweight. As a result, a new

family of triple-codeweight 1D OOC is obtained, which acts as time spreading pattern. Meanwhile, we employ one-coincident frequency-hopping code (OCFHC) as wavelength-hopping pattern<sup>[3]</sup> and then a new two-dimensional (2D) triple-codeweight asymmetric OOC is gained. Using this code in OCDMA network, the different QoS requirements for different multimedia or distinct users can be satisfied. This scheme can make the better use of network resources so that 2D triple-codeweight asymmetric OOC has the potential to be widely applied.

According to the effect of the two constraints on system performance, in order to improve code cardinality, the auto-correlation constraint of 1D OOC is relaxed such that we can construct a triple-codeweight code where the codewords with large weights have auto-correlation constraints equal to 2 and the codewords with smaller weights still keep auto- and cross-correlation of at most one. By extending double-codeweight codewords to triple-codeweight codewords, an  $(n, \{w_l, w_m, w_s\}, \{2, 1, 1\}, 1, D)$  1D OOC can be obtained, where “l” represents large codeweight, “m” indicates medium codeweight, “s” signifies small codeweight, and  $D = \{t_0/(t_0 + t_1 + t_2), t_1/(t_0 + t_1 + t_2), t_2/(t_0 + t_1 + t_2)\}$  with  $t_0$ ,  $t_1$ , and  $t_2$  denoting the numbers of codewords with large, medium, and small codeweights, respectively. The small codeweight  $w_s = w$ , medium codeweight  $w_m = w + 1$ , and large codeweight  $w_l = 2w$ , are chosen, and then the set of codeweight is  $W = \{2w, w + 1, w\}$ . The 1D triple-codeweight code is constructed as follows.

Let  $w = 2m + 1$  and choose  $n$  to be a prime number such that  $n = 2w^2t_0 + (w + 1)wt_1 + w(w - 1)t_2$ . If  $r$  denotes the greatest common divisor of  $t_0$ ,  $t_1$ , and  $t_2$ , let  $\alpha$  be a primitive element of the Galois field  $GF(n)$  such that

$$\begin{aligned} & \{\log_{\alpha}[\alpha^{x_j}(\alpha^{[2wt_0+(w+1)t_1+(w-1)t_2]k} - 1)] : 1 \leq k \leq m, j = 0, \dots, (t_2/r) - 1\}, \\ & \{\log_{\alpha}[\alpha^{y_s}(\alpha^{[2wt_0+(w+1)t_1+(w-1)t_2]k} - 1)] : 1 \leq k \leq m, s = 0, \dots, (t_1/r) - 1\}, \\ & \{\log_{\alpha}[\alpha^{z_l}(\alpha^{[2wt_0+(w+1)t_1+(w-1)t_2]k} - 1)] : 1 \leq k \leq m, l = 0, \dots, (t_0/r) - 1\} \end{aligned}$$

are all distinct modulo  $((2m+1)t_0 + (m+1)t_1 + mt_2)/r$ , where  $x_j, y_s$ , and  $z_l$  are integers between 0 and  $(2m+1)t_0 + (m+1)t_1 + mt_2 - 1$ . Then, the blocks

$$\begin{aligned} & \{\alpha^{[(2m+1)t_0+(m+1)t_1+mt_2]i/r+z_l}, \alpha^{[(2m+1)t_0+(m+1)t_1+mt_2]i/r+z_l+2(2m+1)t_0+2(m+1)t_1+2mt_2}, \dots, \\ & \alpha^{[(2m+1)t_0+(m+1)t_1+mt_2]i/r+z_l+2m(2m+1)t_0+2m(m+1)t_1+4m^2t_2} : i = 0, \dots, r-1, l = 0, \dots, (t_0/r) - 1\}, \\ & \{[0, \alpha^{[(2m+1)t_0+(m+1)t_1+mt_2]i/r+y_s}, \alpha^{[(2m+1)t_0+(m+1)t_1+mt_2]i/r+y_s+2(2m+1)t_0+2(m+1)t_1+2mt_2}, \dots, \\ & \alpha^{[(2m+1)t_0+(m+1)t_1+mt_2]i/r+y_s+2m(2m+1)t_0+2m(m+1)t_1+4m^2t_2} : i = 0, \dots, r-1, l = 0, \dots, (t_0/r) - 1\}, \\ & \{\alpha^{[(2m+1)t_0+(m+1)t_1+mt_2]i/r+x_j}, \alpha^{[(2m+1)t_0+(m+1)t_1+mt_2]i/r+x_j+(2m+1)t_0+(m+1)t_1+mt_2}, \dots, \\ & \alpha^{[(2m+1)t_0+(m+1)t_1+mt_2]i/r+x_j+(4m+1)(2m+1)t_0+(4m+1)(m+1)t_1+(4m+1)mt_2} : i = 0, \dots, r-1, l = 0, \dots, (t_0/r) - 1\} \end{aligned}$$

form an 1D triple-codeweight OOC.

As an example, we consider an  $(n, \{6, 4, 3\}, \{2, 1, 1\}, 1, \{1/3, 1/3, 1/3\})$ OOC. Let  $n$  be a prime number such that  $n = 36t + 1$  for an integer  $t$  ( $t$  is the number of codewords for each type of codeweight). Let  $\alpha$  be a primitive element of the Galois field  $GF(n)$  such that  $\alpha^q = \alpha^x(\alpha^{6t} - 1)$ ,  $\alpha^r = \alpha^y(\alpha^{6t} - 1)$ , and  $\alpha^v = \alpha^z(\alpha^{6t} - 1)$  with  $x, y$ , and  $z$  any integers between 1 and  $6t - 1$ ;  $q, r$ , and  $v$  are integers that satisfy all distinct modulo  $6t - 1$ .

Code cardinality  $\Phi = t_0 + t_1 + t_2 = 3t$  achieves the upper bound in theory. Then, the code consists of the blocks

$$\begin{aligned} & \{\alpha^{6i}, \alpha^{6i+12t}, \alpha^{6i+24t} : i = 0, 1, \dots, t-1\}, \{[0, \alpha^{6i+y}, \alpha^{6i+y+12t}, \alpha^{6i+y+24t}] : i = 0, 1, \dots, t-1\}, \\ & \{\alpha^{6i+x}, \alpha^{6i+x+6t}, \alpha^{6i+x+12t}, \alpha^{6i+x+18t}, \alpha^{6i+x+24t}, \alpha^{6i+x+30t} : i = 0, 1, \dots, t-1\}. \end{aligned}$$

Let  $n = 37$  and  $t = 1$ . Choose  $\alpha = 2$  as a primitive element of  $GF(37)$  and then  $2^6 - 1 = 26 = 2^{12}$ ,  $2(2^6 - 1) = 15 = 2^{13}$ ,  $2^2(2^6 - 1) = 30 = 2^{14}$ , where 12, 13, 14 are all distinct modulo-5. The code consists of the blocks  $\{[1, 10, 26], [0, 2, 15, 20], \text{ and } [3, 4, 7, 30, 33, 34]\}$  and therefore, the three codewords are

$$\begin{aligned} x_0 &= [010000000010000000000000001000000000], \\ y_0 &= [10100000000000001000010000000000000000], \\ z_0 &= [0001100100000000000000000000001001100]. \end{aligned}$$

Similarly, let  $n = 73$  and the code cardinality be 6. Let  $\alpha = 5$ , then there exist  $5^{12} - 1 = 5^{24}$ ,  $(5^{12} - 1)5 = 5^{25}$ , and  $(5^{12} - 1)5^2 = 5^{26}$ . The code consists of the blocks  $\{[25, 54, 67], [2, 16, 55], [0, 5, 28, 40], [0, 15, 11, 47], [1, 98, 72, 64, 65], \text{ and } [3, 27, 24, 70, 46, 49]\}$ . The codewords pattern has been omitted for the sake of simplification.

Employing OCFHC with the number of wavelengths  $m = p^k$  as wavelength-hopping patterns and 1D triple-codeweight OOC constructed above as time-spreading patterns, namely, mapping OCFHC with  $m$  wavelengths into 1D time-spreading triple-codeweight OOC based on the construction of 2D OCFHC/OOC<sup>[3,4]</sup>, 2D  $(p^k \times n, W, A, \lambda_c, D)$  weight-hopping/time-spreading (WH/Ts) variable-weight OOC (VWOOC) with the cardinality of  $\Phi_{\text{OOC}}m^2$  can be obtained.

Based on the extension Galois field  $GF(2^3)$ , the constructed  $(2^3, 7, 1)$  OCFHC with codelength  $L = 2^3 - 1$  is shown in Table 1. Employing OCFHC as wavelength-hopping patterns and 1D  $(73, \{6, 4, 3\}, \{2, 1, 1\}, 1, \{1/3, 1/3, 1/3\})$  OOC as time-spreading patterns, a 2D variable-weight  $(2^3 \times 73, \{6, 4, 3\}, \{2, 1, 1\}, 1, \{1/3, 1/3, 1/3\})$ OOC could be constructed. The codewords constructed in the basis of these eight wavelengths mapping into the codeword  $x_0$  (the block is  $[25, 54, 67]$ ) are shown in Table 2, where we employ  $(x, y)$  to denote an element of a 2D OOC matrix ( $x$  is the number of time slot and  $y$  is the number of wavelength). In addition,  $n = 73$ ,  $m = 2^3$ , this code has three types of hamming weight, which are  $w_1 = 6$ ,

**Table 1.  $(2^3, 7, 1)$  OCFHC**

$i$	$S_i$						
0	1	2	4	3	6	7	5
1	0	3	5	2	7	6	4
2	3	0	6	1	4	5	7
3	5	6	0	7	2	3	1
4	2	1	7	0	5	4	6
5	7	4	2	5	0	1	3
6	6	5	3	4	1	0	2
7	4	7	1	6	3	2	0

**Table 2. Codewords of 2D ( $2^3 \times 73, \{6,4,3\}, \{2,1,1\}, 1, \{1/3, 1/3, 1/3\}$ )OOC Based on  $x_0 = [25, 54, 67]$** 

	Group 0	Group 1	Group 2	...	Group 8
0	$[(25, \lambda_0), (54, \lambda_0), (67, \lambda_0)]$	$[(25, \lambda_1), (54, \lambda_2), (67, \lambda_4)]$	$[(25, \lambda_0), (54, \lambda_3), (67, \lambda_5)]$	...	$[(25, \lambda_4), (54, \lambda_7), (67, \lambda_1)]$
1	$[(25, \lambda_1), (54, \lambda_1), (67, \lambda_1)]$	$[(25, \lambda_2), (54, \lambda_4), (67, \lambda_3)]$	$[(25, \lambda_3), (54, \lambda_5), (67, \lambda_2)]$	...	$[(25, \lambda_7), (54, \lambda_1), (67, \lambda_6)]$
2	$[(25, \lambda_2), (54, \lambda_2), (67, \lambda_2)]$	$[(25, \lambda_4), (54, \lambda_3), (67, \lambda_6)]$	$[(25, \lambda_5), (54, \lambda_2), (67, \lambda_7)]$	...	$[(25, \lambda_1), (54, \lambda_6), (67, \lambda_3)]$
3	$[(25, \lambda_3), (54, \lambda_3), (67, \lambda_3)]$	$[(25, \lambda_3), (54, \lambda_6), (67, \lambda_7)]$	$[(25, \lambda_2), (54, \lambda_7), (67, \lambda_6)]$	...	$[(25, \lambda_6), (54, \lambda_3), (67, \lambda_2)]$
4	$[(25, \lambda_4), (54, \lambda_4), (67, \lambda_4)]$	$[(25, \lambda_6), (54, \lambda_7), (67, \lambda_5)]$	$[(25, \lambda_7), (54, \lambda_6), (67, \lambda_4)]$	...	$[(25, \lambda_3), (54, \lambda_2), (67, \lambda_6)]$
5	$[(25, \lambda_5), (54, \lambda_5), (67, \lambda_5)]$	$[(25, \lambda_7), (54, \lambda_5), (67, \lambda_1)]$	$[(25, \lambda_6), (54, \lambda_4), (67, \lambda_0)]$	...	$[(25, \lambda_2), (54, \lambda_0), (67, \lambda_4)]$
6	$[(25, \lambda_6), (54, \lambda_6), (67, \lambda_6)]$	$[(25, \lambda_5), (54, \lambda_1), (67, \lambda_2)]$	$[(25, \lambda_0), (54, \lambda_4), (67, \lambda_7)]$	...	$[(25, \lambda_0), (54, \lambda_4), (67, \lambda_7)]$
7	$[(25, \lambda_7), (54, \lambda_7), (67, \lambda_7)]$				

$w_m = 4$  and  $w_s = 3$ , such that the overall cardinality is  $(8 + 8 \times 7) \times 6 = 384$ .

The cardinality of 1D VWOOC mentioned above is

$$\Phi(n, \{2w, w+1, w\}, A, 1, D) = \frac{(n-1)}{d_1(2w)^2/\lambda_a^1 + d_m(w+1)w/\lambda_a^m + d_s w(w-1)/\lambda_a^s}, \quad (1)$$

where  $A = \{\lambda_a^1, \lambda_a^m, \lambda_a^s\}$ ,  $D = \{d_1, d_m, d_s\}$ . Therefore, the code cardinality of 2D VWOOC is given by

$$\Phi(m \times n, \{2w, w+1, w\}, A, 1, D) = \frac{m(mn-1)}{d_1(2w)^2/\lambda_a^1 + d_m(w+1)w/\lambda_a^m + d_s w(w-1)/\lambda_a^s}. \quad (2)$$

It can be seen that the code cardinality of 2D VWOOC

has been significantly improved.

Using different address code-matrix, diverse QoS can be achieved by different subscribers. Coming up the next, the system performance of 2D VWOOCs with  $l = 3$  (i.e., supporting three types of QoS) is taken into account. Here we suppose that the number of available wavelength is  $m$ , the set of hamming weights is  $W = \{w_1, w_m, w_s\}$ ; the cardinality of the 2D VWOOC is  $\Phi_C = \Phi_1 + \Phi_m + \Phi_s$  ( $\Phi_s$ ,  $\Phi_m$ , and  $\Phi_1$  indicate the cardinalities of matrices with small, medium, and large codeweights, respectively); the set of cardinality distributions is  $D = \{d_1, d_m, d_s\}$ . Let  $q^0$  and  $q^i$  denote the probabilities of one hit between an address matrix originated from group 0 or group  $i$  (for  $i = \{1, 2, \dots, l-1\}$ ) and any arriving address matrix in the same code set. Then, the probability of one hit between two same-weight matrices is obtained as

$$q_s = \frac{w_s \cdot \Phi_s \cdot (w_s m d_s \Phi_{OOC} - 1) - w_s m (w_s - 1)}{2n(\Phi_s - 1) \cdot \Phi_s}, \quad (3)$$

$$q_m = \frac{w_m \cdot \Phi_m \cdot (w_m m d_m \Phi_{OOC} - 1) - w_m m (w_m - 1)}{2n(\Phi_m - 1) \cdot \Phi_m}, \quad (4)$$

$$q_1 = \frac{w_1(w_1 - 2)(\Phi_1 - m) + w_1^2 m d_1 \Phi_{OOC} \Phi_1 + m w_1^2 (w_1 - 2)}{2n\Phi_1(\Phi_1 - 1)}. \quad (5)$$

Based on the number of hits between a large-weight matrix and a medium-weight matrix being at most 1, the probability of one hit between a large-weight matrix and a medium-weight arriving matrix,  $q_{1,m}$ , can be obtained. Meanwhile, by analyzing all probabilities of one hit between two unequal weight matrices, we find that the medium-weight matrix has the same effect on large-weight matrix and small-weight matrix. Thus, there exists

$$q_{1,m} = q_{s,m} = \frac{m w_m^2 d_m \Phi_{VWOOC}}{2n(\Phi_m - 1)}. \quad (6)$$

By the similar analysis, we have

$$q_{1,s} = q_{m,s} = \frac{m w_s^2 d_s \Phi_{VWOOC}}{2n(\Phi_s - 1)}, \quad (7)$$

$$q_{s,1} = q_{m,1} = \frac{m w_1^2 d_1 \Phi_{VWOOC}}{2n(\Phi_1 - 1)}. \quad (8)$$

The error probabilities  $P_{e,s}$ ,  $P_{e,m}$ , and  $P_{e,1}$  of the users with address matrices of weights  $w_s$ ,  $w_m$ , and  $w_1$  are given by

$$P_{e,s} = \frac{1}{2} \sum_{l_s + l_m + l_1 = w_s}^{K_s + K_m + K_1 - 1} \binom{K_s - 1}{l_s} (q_s)^{l_s} (1 - q_s)^{K_s - 1 - l_s} \cdot \binom{K_m}{l_m} (q_{s,m})^{l_m} (1 - q_{s,m})^{K_m - l_m} \cdot \binom{K_1}{l_1} (q_{s,1})^{l_1} (1 - q_{s,1})^{K_1 - l_1}, \quad (9)$$

$$P_{e,m} = \frac{1}{2} \sum_{l_s+l_m+l_1=w_m}^{K_s+K_m+K_1-1} \binom{K_m-1}{l_m} (q_m)^{l_m} (1-q_m)^{K_m-1-l_m} \cdot \binom{K_s}{l_s} (q_{m,s})^{l_s} (1-q_{m,s})^{K_s-l_s} \cdot \binom{K_1}{l_1} (q_{m,1})^{l_1} (1-q_{m,1})^{K_1-l_1}, \quad (10)$$

$$P_{e,1} = \frac{1}{2} \sum_{l_s+l_m+l_1=w_1}^{K_s+K_m+K_1-1} \binom{K_1-1}{l_1} (q_1)^{l_1} (1-q_1)^{K_1-1-l_1} \cdot \binom{K_m}{l_m} (q_{1,m})^{l_m} (1-q_{1,m})^{K_m-l_m} \cdot \binom{K_s}{l_s} (q_{1,s})^{l_s} (1-q_{1,s})^{K_s-l_s}, \quad (11)$$

respectively, where  $K_s$ ,  $K_m$ , and  $K_1$  are the numbers of simultaneous users using matrices of codeweights  $w_s$ ,  $w_m$ , and  $w_1$ .

A  $(2^3 \times n, \{6, 4, 3\}, \{2, 1, 1\}, 1, \{1/3, 1/3, 1/3\})$  triple-weight OOC is employed as an example. Shown in Fig. 1 are the error probabilities versus the codeword length  $n$ . As a whole, the system performance of users with larger codeweights outperforms that of users with small codeweights. For instance, the resulting error probabilities of the system for 10 users with weights 3 and 4 are  $3.26 \times 10^{-3}$  and  $2.78 \times 10^{-4}$ , respectively, when the length  $n = 200$ . Hence, the error probability for the large codeword matrix is  $1.03 \times 10^{-8}$  that surpasses the error probability for the small and medium codeword matrices two or three orders of magnitude.

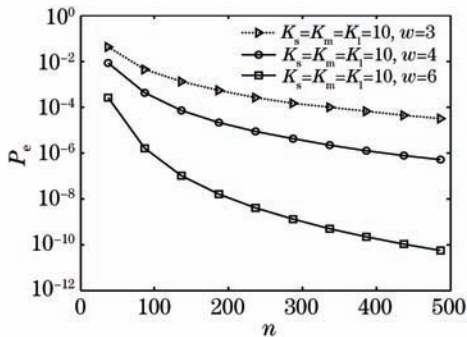


Fig. 1. Bit error probabilities  $P_e$  versus codeword length  $n$  for triple-codeword 2D VWOOC.

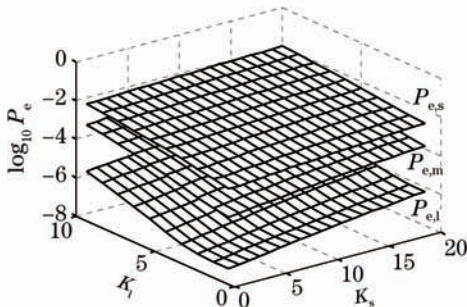


Fig. 2. Error probabilities versus the number of simultaneous users  $K_1$  and  $K_s$  for triple-codeword 2D VWOOC with  $K_m = 10$ .

Figure 2 shows the error probability versus the numbers of simultaneous users  $K_s$  transmitting small codeword matrices and  $K_1$  transmitting large codeword matrices when the number of users transmitting medium codeword matrices is fixed at  $K_m = 10$ . The lowest, middle, and topmost surfaces correspond to the performances (i.e.,  $P_{e,s}$  in Eq. (9),  $P_{e,m}$  in Eq. (10), and  $P_{e,1}$  in Eq. (11)) of users with small-, medium-, large-codeword matrices, respectively. As a whole, the performance worsens as the total number of simultaneous users (i.e.,  $K_s + K_m + K_1$ ) increases. The users with the largest codeweights always perform the best.

When an OCDMA network is constructed by employing 2D VWOOC, the different QoS can be achieved and the usage of network resource can be optimized by assigning larger weight codewords to the services with higher requirements of QoS and smaller weight codewords to the services with lower requirements of QoS.

In conclusion, a new generation algorithm of 2D triple-codeword asymmetric optical orthogonal codes has been proposed in this letter. The cardinality of code produced in this way can be significantly improved. It is shown that the systems with different weight codewords have distinct performances, therefore, they can be employed to support different services with distinct types of QoS and satisfy different requirements of QoS from diverse multimedia or distinct subscribers, which makes the better use of bandwidth resources in optical networks. Thus, 2D triple codeword asymmetric OOCs proposed in this letter have the potential to be widely used.

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## References

1. G.-C. Yang and T. E. Fuja, IEEE Trans. Inform. Theory **41**, 96 (1995).
2. G.-C. Yang, IEEE Trans. Commun. **44**, 47 (1996).
3. S. Sun, H. Yin, Z. Wang, and A. Xu, J. Lightwave Technol. **24**, 1646 (2006).
4. W. Liang, H. Yin, L. Qin, Z. Wang, and A. Xu, Photon. Netw. Commun. **16**, 53 (2008).