Analysis of normalized throughput in WDM-based coherent time-spreading OCDMA system

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The performance of normalized throughput in wavelength division multiplexing (WDM) based coherent time-spreading optical code division multiple access (TS-OCDMA) system is studied. The upper bound and lower bound of normalized throughput are obtained with 8 wavelength channels and 127-length Gold code, and with 4 wavelength channels and 511-length Gold code respectively. It is shown that when all simultaneous users are equally allocated to different wavelength channels, WDM+TS-OCDMA has much better performance. However, if there is no central control to allocate wavelength channels equally, WDM+TS-OCDMA system has the in-between performance of normalized throughput.

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Optical code-division multiple access (OCDMA) is a promising candidate for the next-generation broad-band access network. According to the working principle, OCDMA systems can be divided into incoherent and coherent OCDMA ones. Incoherent OCDMA is based on amplitude coding, which can be performed in time domain[1], frequency domain[2], or in time and frequency domains simultaneously[3]. Coherent OCDMA is based on phase coding, which can be performed in frequency domain[4] or time domain[5−7]. Among these schemes, coherent time-spreading OCDMA is the most attractive one, which is encoded by superstructure fiber Bragg grating (FBG) or planar lightwave circuits. On the other hand, wavelength-division multiple access (WDMA) is a nature approach to enhance the system capacity. Therefore, wavelength division multiplexing (WDM) based coherent time-spreading OCDMA (WDM+TS-OCDMA) is proposed as a solution for the gigabit-symmetric optical access networks[5]. However, system performance of WDM+TS-OCDMA has not been evaluated. In this letter, we analyze the upper bound and lower bound of normalized throughput in a WDM+TS-OCDMA system.

In WDM+TS-OCDMA systems, each wavelength channel can employ the same set of address codes (such as Gold codes), as shown in Fig. 1. Let q denote the number of available wavelength channels in WDM+TS-OCDMA, and C is the number of available Gold codes. Therefore, the total number of users in the WDM+TS-OCDMA system is $q \times C$.

To analyze the system performance, the discrete aperiodic cross-correlation function is defined as[8]

$$C_{k,i}(l) = \begin{cases} \frac{1}{N}\sum\limits_{j=0}^{N-1-l} a_j^k \cdot a_{j+l}^i & 0 \le l \le N-1 \\ \frac{1}{N}\sum\limits_{j=0}^{N-1+l} a_{j-l}^k \cdot a_j^i & 1-N \le l < 0 \\ 0 & \text{else} \end{cases}, \quad (1)$$

where a_j^k $(j = 0, 1, \cdots, N-1)$ is the kth user's address code, $a_j^k \in \{1, -1\}$, and N is the code length.

If there are m interfering users with the same wavelength, the received optical field at the photon detector of the target user is

$$\begin{aligned} E(t) = {} & \sqrt{P} b_{0,0} \exp j(\omega_0 t + \theta_0(t)) + \sqrt{P} \sum_{k=1}^{m} \{[b_{k,-1} C_{k,0}(l_k) \exp j(\omega_k (t + l_k T_c) + \theta_k (t + l_k T_c))] \\ & + [b_{k,0} C_{k,0}(l_k - N) \exp j(\omega_k (t + l_k T_c - T) + \theta_k (t + l_k T_c - T))]\}, \end{aligned} \quad (2)$$

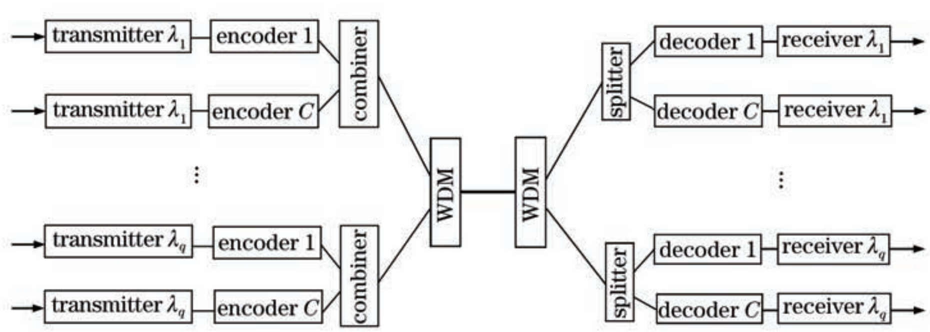

Fig. 1. Architecture of WDM+TS-OCDMA.

where P is the optical intensity of signal; ω_0 and ω_k are optical frequencies; $\theta_0(t)$ and $\theta_k(t)$ are phase noises; $\tau_k = l_k T_c$ is the relative transmit delay of the kth user, l_k is an integer, T_c is chip duration, T is bit duration; $b_{0,0}$ is the target user's data bit, $b_{k,-1}$ and $b_{k,0}$ are the adjacent data bits of the kth interfering user, respectively.

For a chip-rate square-law photodetector, the output signal is

$$\begin{aligned} Z &= \int_0^{T_c} \Re\cdot(E\cdot E^*)\mathrm{d}t + \int_0^{T_c} n_0(t)\mathrm{d}t \\ &= \Re T_c P b_{0,0} + \Re T_c P \sum_{k=1}^{m} (b_{k,-1}\left|C_{k,0}(l_k)\right|^2 + b_{k,0}\left|C_{k,0}(l_k - N)\right|^2) + B_1 + B_2 + \int_0^{T_c} n_0(t)\mathrm{d}t, \end{aligned} \quad (3)$$

where $\Re$ is the responsivity of the photodetector, and $n_0(t)$ is the receiver noise. In this expression, the first term is user data signal, the second term is multi-access interference, the third term is beat noise between user and interferers, and the fourth term is beat noise between interferers

$$\begin{aligned} B_1 &= 2\Re P \sum_{k=1}^{m} b_{0,0}b_{k,-1}C_{k,0}(l_k) \int_0^{T_c} \cos\left[(\omega_0 - \omega_k)t - \omega_k l_k T_c + \theta_0(t) - \theta_k(t + l_k T_c)\right]\mathrm{d}t \\ &+ 2\Re P \sum_{k=1}^{m} b_{0,0}b_{k,0}C_{k,0}(l_k - N) \int_0^{T_c} \cos\left[(\omega_0 - \omega_k)t - \omega_k(l_k T_c - T) + \theta_0(t) - \theta_k(t + l_k T_c - T)\right]\mathrm{d}t. \end{aligned} \quad (4)$$

We define $\Delta\phi_{k1} = (\omega_0 - \omega_k)t - \omega_k l_k T_c + \theta_0(t) - \theta_k(t + l_k T_c)$, $\Delta\phi_{k2} = (\omega_0 - \omega_k)t - \omega_k(l_k T_c - T) + \theta_0(t) - \theta_k(t + l_k T_c - T)$ as overall phase noises, which can be constant within T_c and vary over $[-\pi, \pi]$ from bit to bit. Similarly, B_2 can be expressed as

$$\begin{aligned} B_2 &= 2\Re P \sum_{k=1}^{m-1} \sum_{j=k+1}^{m} b_{k,-1}b_{j,-1}C_{k,0}(l_k)C_{j,0}(l_j) \int_0^{T_c} \cos\Delta\phi_{k3}\mathrm{d}t \\ &+ 2\Re P \sum_{k=1}^{m-1} \sum_{j=k+1}^{m} b_{k,0}b_{j,0}C_{k,0}(l_k - N)C_{j,0}(l_j - N) \int_0^{T_c} \cos\Delta\phi_{k4}\mathrm{d}t \\ &+ 2\Re P \sum_{k=1}^{m} \sum_{j=1}^{m} b_{k,-1}b_{j,0}C_{k,0}(l_k)C_{j,0}(l_j - N) \int_0^{T_c} \cos\Delta\phi_{k5}\mathrm{d}t, \end{aligned} \quad (5)$$

Where $\Delta\phi_{k3}$, $\Delta\phi_{k4}$, $\Delta\phi_{k5}$ are overall phase noises, which can be constant within T_c and vary over $[-\pi, \pi]$ from bit to bit.

We define the mean intensity of aperiodic cross-correlation as

$$\psi_{k,0}(l_k) = \frac{1}{2}\left(\left|C_{k,0}(l_k)\right|^2 + \left|C_{k,0}(l_k - N)\right|^2\right). \quad (6)$$

Considering the equal probability of user data '0' and '1', the averaged output for data '1' of transmit delays $l_1, l_2, \cdots, l_m$ is

$$Z_1 = \Re T_c P \left[1 + \sum_{k=1}^{m} \psi_{k,0}(l_k)\right]. \quad (7)$$

The variance of beat noise for data '1' is

$$\sigma_{b1}^2 \approx 2\Re^2 P^2 T_c^2 \sum_{k=1}^{m} \psi_{k,0}(l_k). \quad (8)$$

The averaged output for data '0' in transmit delays $l_1, l_2, \cdots, l_m$ is

$$Z_0 = \Re T_c P \left[\sum_{k=1}^{m} \psi_{k,0}(l_k)\right]. \quad (9)$$

The variance of beat noise variance for data '0' is

$$\sigma_{b0}^2 \approx 2\Re^2 P^2 T_c^2 \sum_{k=1}^{m-1} \sum_{j=k+1}^{m} \left[\psi_{k,0}(l_k)\psi_{j,0}(l_j)\right]. \quad (10)$$

To evaluate the mean performance in an OCDMA system, we should calculate the mean value of $\psi_{k,0}(l)$ over $l = 0, 1, \cdots, N-1$:

$$\xi_{k0} = \frac{1}{N} \sum_{l=0}^{N-1} \psi_{k,0}(l). \quad (11)$$

Obviously, $\xi_{k,0}$ depends on two different codes, which indicates that different users have different performance. However, it is useful to evaluate the mean performance for all users. For N-length Gold code, it can be evaluated $\xi_{k,0} \approx 1/(2N)$. Therefore,

$$\begin{aligned} Z_1 &= \Re T_c P \left(1 + \frac{m}{2N}\right), \quad &\sigma_{b1}^2 &= \frac{m\Re^2 P^2 T_c^2}{N}, \\ Z_0 &= \frac{m\Re T_c P}{2N}, \quad &\sigma_{b0}^2 &= \frac{m(m-1)\Re^2 P^2 T_c^2}{4N^2}. \end{aligned} \quad (12)$$

The variance of multi-access interference (MAI) is[7]

$$\sigma_{\mathrm{MAI}}^2 = m\Re^2 P^2 T_{\mathrm{c}}^2 \sigma_{\mathrm{MAI-0}}^2, \tag{13}$$

where $\sigma_{\mathrm{MAI-0}}^2$ is the variance of single interfering signal, for $N = 127$ Gold code, $\sigma_{\mathrm{MAI-0}}^2 \approx 6.5 \times 10^{-5}$. For $N = 511$ Gold code, $\sigma_{\mathrm{MAI-0}}^2 \approx 3.88 \times 10^{-6}$.

In this letter, we will focus on beat noise (BN) and MAI, which are the main system limitations. Other receiver noise such as shot noise and thermal noise will be discussed in succeeding papers. Thus, if we neglect the receiver noise, the total noise variances with '0' and '1' are

$$\sigma_0^2 = \sigma_{b0}^2 + \sigma_{\mathrm{MAI}}^2, \quad \sigma_1^2 = \sigma_{b1}^2 + \sigma_{\mathrm{MAI}}^2. \tag{14}$$

Using time gating device, the bit error rate (BER) of OCDMA is

$$\mathrm{BER}(m) = \frac{1}{2}\left[p\,(1/0) + p\,(0/1)\right], \tag{15}$$

where $p\,(0/1)$ and $p\,(1/0)$ are conditional error probabilities with '0' and '1', respectively

$$p(1/0) = \frac{1}{2}\mathrm{erfc}\left[\frac{\mathrm{Th} - Z_0}{\sqrt{2}\sigma_0}\right],$$
$$p(0/1) = \frac{1}{2}\mathrm{erfc}\left[\frac{Z_1 - \mathrm{Th}}{\sqrt{2}\sigma_1}\right], \tag{16}$$

where Th is the optimum decision threshold based on $\mathrm{Th} = \frac{\sigma_1 Z_0 + \sigma_0 Z_1}{\sigma_0 + \sigma_1}$.

Because MAI and BN only exist in the users with the same wavelength channel, we should consider two extreme cases in the WDM+TS-OCDMA system. The first case is that when all simultaneous users are equally allocated to different wavelength channels, the WDM+TS-OCDMA system has the best performance. The second case is that when all simultaneous users are first allocated to the same wavelength channels, the WDM+TS-OCDMA system has the worst performance.

In this letter, we consider the synchronous, random-access, packet broadcast network described in Ref. [9]. Users begin transmissions on common clock instances and the length of a slot corresponds to a packet of the length of L bits. Due to the effect of MAI and BN, some of the packets will arrive at the receiver with bit errors. The probability of receiving a packet without errors is given by

$$p_c(m) = [1 - \mathrm{BER}(m)]^L. \tag{17}$$

According to Ref. [9], the steady-state throughput β of random-access and packet broadcast network is

$$\beta = \mathrm{e}^{-\lambda T} \sum_{k=1}^{\infty} k p_c(k) \frac{(\lambda T)^k}{k!}, \tag{18}$$

where λ is arrival rate, T is the duration of per time slot, λT is the offered load (average number of attempted transmissions per time slot).

When all simultaneous users (k) are equally allocated to different wavelength channels,

$$k = q \times r + s, \quad 0 \leq r < C, 0 \leq s \leq q. \tag{19}$$

In this case, each wavelength channel has $(r+1)$ users of all s different wavelength channels, and each wavelength channel has r users in the other $(q-s)$ different wavelength channels. Therefore, the upper bound of throughput in a WDM+TS-OCDMA system is

$$\beta_{\mathrm{upper}} = \sum_{k=1}^{qC} \frac{(\lambda T)^k \mathrm{e}^{-\lambda T}}{k!} \times \left\{[s \times (r+1) \times p_c(r+1)] + [(q-s) \times r \times p_c(r)]\right\}. \tag{20}$$

When all simultaneous users are first allocated to the same wavelength channels,

$$k = C \times r + s, \quad 0 \leq r \leq q, 0 \leq s < C. \tag{21}$$

In this case, each wavelength channel has C users of all r different wavelength channels, one different wavelength channel has s users, and the other wavelength channels have no users. Therefore, the lower bound of throughput in a WDM+TS-OCDMA system is

$$\beta_{\mathrm{lower}} = \sum_{k=1}^{qC} \frac{(\lambda T)^k \mathrm{e}^{-\lambda T}}{k!} \times \left\{[r \times C \times p_c(C)] + [s \times p_c(s)]\right\}. \tag{22}$$

We define the normalized throughput as

$$\beta^{\circ} = \frac{\beta}{n_{\mathrm{w}} n_t}, \tag{23}$$

Where n_{w} is the number of wavelength channels, n_t is the code length.

Figure 2 shows the performance of normalized throughput in WDM+TS-OCDMA systems when 127-length

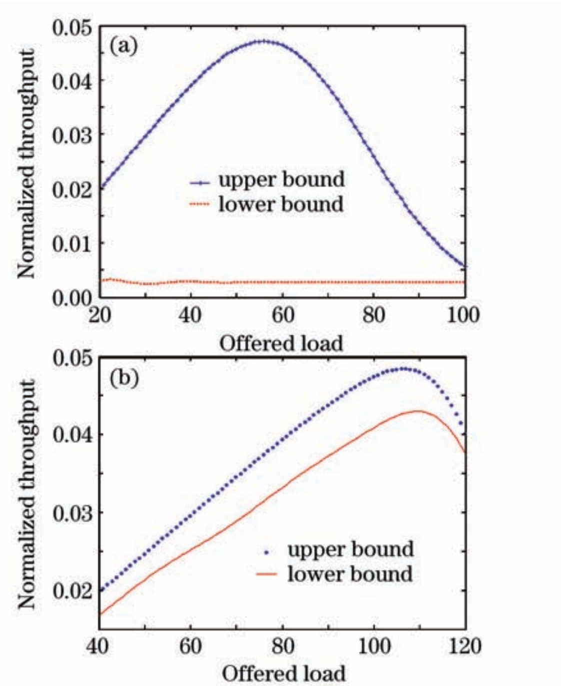

Fig. 2. Performance of normalized throughput in WDM+TS-OCDMA. (a) For 127-length Gold code; (b) for 511-length Gold code. Packet length $L = 1024$ bits.

Gold sequence (16 subset codes are selected, $q = 8$) and 511-length Gold sequence (32 subset codes are selected, $q = 4$) are used. It is shown that when all simultaneous users are equally allocated to different wavelength channels, WDM+TS-OCDMA has much better performance. However, if there is no central control to allocate wavelength channels equally, the WDM+TS-OCDMA system has the in-between performance.

Figure 3 shows the lower bound of normalized throughput under different wavelength channels in the WDM+TS-OCDMA system, employing 127-length sequence, $q = 8$ (16 subset codes are selected) and $q = 16$ (8 subset codes are selected). It is shown that the normalized throughput can be enhanced under large wavelength channel number and small subset code number.

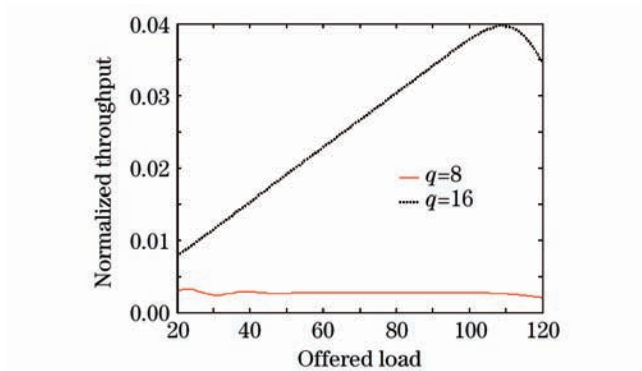

Fig. 3. Lower bound of normalized throughput under different wavelength channels.

In summary, we have obtained the upper bound and lower bound of normalized throughput in a WDM+TS-OCDMA system, considering the effects of MAI and BN. When the simultaneous users are equally allocated to different wavelength channels, the upper bound of normalized throughput is obtained. When the simultaneous users are firstly allocated to the same wavelength channels, the lower bound of normalized throughput is obtained. However, if there is no central control to allocate wavelength channels equally, the WDM+TS-OCDMA system has the in-between performance of normalized throughput.

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